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PRESSURE COVARIANCE TERMS IN THE HEAT AND MOISTURE FLUX EQUATIONS FOR LOCAL ADVECTION

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SUMMARY All terms in the flux equations for heat and water vapour were either measured directly, or, if sufficiently small, estimated, except the term containing fluctuating pressure, which was obtained by difference. The experimental site was a flooded rice field, which was bounded to windward by a semi-arid region. Local conditions over the rice were always stable, but the downward flux of heat and the upward flux of water vapour were large. Current parameterizations of the pressure covariance terms sometimes gave values which differed by factors of five or ten from the experimental results, and the interrelationships depended upon the stability.

1 INTRODUCTION

Where surfaces of our planet are homogeneous in the horizontal, there is no net horizontal transport of scalars, such as heat and water vapour, by the atmosphere. There one dimensional analyses of the temperature, humidity and velocity fields are appropriate, effective and useful. Where surfaces are heterogeneous, horizontal transport occurs, leading to local advection.

Here we use as an example an irrigated crop bounded upwind by a dry region. Heat is transported horizontally from the dry to the wet area and increases the evapotranspiration there. For extreme cases the evapotranspiration from the crop is about 1.75 times that from a crop that is infinite in extent (Lang et al., 1974).

Early analyses of local advection used two dimensions and eddy diffusion formulations (Sutton, 1943; Philip, 1959). These suffer from the dependence on similarity, which is inappropriate at least for extreme local advection (Lang et al., 1983a).

Rao, Wyngaard and Cote (1974) presented a second order closure analysis of the problem. It was based upon 16 simultaneous partial differential equations with 38 terms, which must be expressed as functions of the variables to be solved. The equations express the conservation of variances u, v, and w, and the scalars temperature θ and humidity q, together with four equations for the mean fields of these variables and an equation describing the energy dissipation rate. Some of these equations have been studied previously for the planetary surface layer (e.g., Wyngaard et al., 1971; Bradley et al., 1982).

We focus here on the two equations for the balance of vertical fluxes represented by:

$$\frac{\partial \overline{cw}}{\partial t} = - \overline{uw}\,\frac{\partial C}{\partial x} - \overline{w^2}\,\frac{\partial C}{\partial z} - \frac{U\partial \overline{cw}}{\partial x} - \frac{\partial}{\partial x}\,\overline{cuw} - \frac{\partial}{\partial z}\,\overline{cw^2} - \overline{c\,\frac{\partial p}{\partial z}} + \frac{g}{T}\,(\overline{\theta c} + 0.61\;T\;\overline{qc}) \qquad (1)$$

where c may be either θ or q, x and z are distances along streamwise and vertical axes, and capitals denote mean values and lower case fluctuations from the means.

Rao et al. (1974) assumed steady state for their second order closure analysis and obtained solutions for all terms in these two equations except $\partial\overline{cuw}/\partial x$, $\partial\overline{cww}/\partial z$ and $\overline{c\partial p/\partial z}$. To proceed, these last three terms must be expressed as more or less arbitrary functions of the variables solved in the system of 16 equations. That is, they must be parameterized; searches for suitable functions continue.

It is the purpose of this paper to compare some suggested functions (Launder, 1978; Warhaft, 1976) with empirical evidence.

We have evaluated all the terms in the flux equations experimentally except the pressure term, which was obtained by difference (Lang et al., 1983b).

2 EXPERIMENTS

Complete details of the site and instruments are given elsewhere (Lang et al., 1983a). Briefly, we studied a flooded rice field of size 500 × 600 m, which was in a semi-arid region and bounded on the northern, southern and western sides by dry fields. Thus, the crop was subjected to extreme local advection when the wind blew from the westerly quarter. The measured ratio of the vertical sensible heat flux to the latent heat flux (Bowen's ratio) for the arid region, during the course of the experiments, was about 18, while for the rice field it was about -0.3. The negative sign arises because the direction of the sensible heat flux in the region of local advection is towards the surface. The water in the rice field was 140 mm deep, and the crop height above the water was 600 mm.

Mean wind speeds using miniature cup anemometers at 7 levels between 0.5 and 6 m, and wet and dry bulb temperatures using aspirated psychrometers at 4 levels, were measured in the dry field and 310 m downwind from the edge of the rice field. Available energy at both sites was measured with net radiometers, heat flux plates and from the change in water temperature with time. Eddy covariances between the vertical component of the wind vector, temperature and humidity were obtained at a height of 1.5 m above the water surface with a single, vertical path, sonic anemometer, fine-wire thermocouple and Lyman-α hygrometer.

Variables were sampled at 10 Hz and signals destined for eddy covariance analyses were passed through low-pass, 2-pole Bessel filters with a half power point of 5 Hz.

At another site 130 m from the western boundary of the rice field, measurements were made using two, 3-component, sonic anemometers, associated with fast response resistance thermometers and usually one, but sometimes two, Lyman-α hygrometers. The instruments were set at 2 and 5.5 m above the water and were periodically interchanged.

3 RESULTS

Measured terms in the flux equation, in descending magnitude, were the production $\overline{w^2}\partial C/\partial z$, buoyancy $(g/T)(\overline{\theta c} + 0.61T\overline{cq})$ and the transport $\partial\overline{cww}/\partial z$. Data for the last term was quite scattered but averaged about 10% of the production term. The remaining terms were each less than 1% of the largest term. The residual, pressure covariance $\overline{c\partial p/\partial z}$, was found almost to balance the production term, as shown in Figure 1.

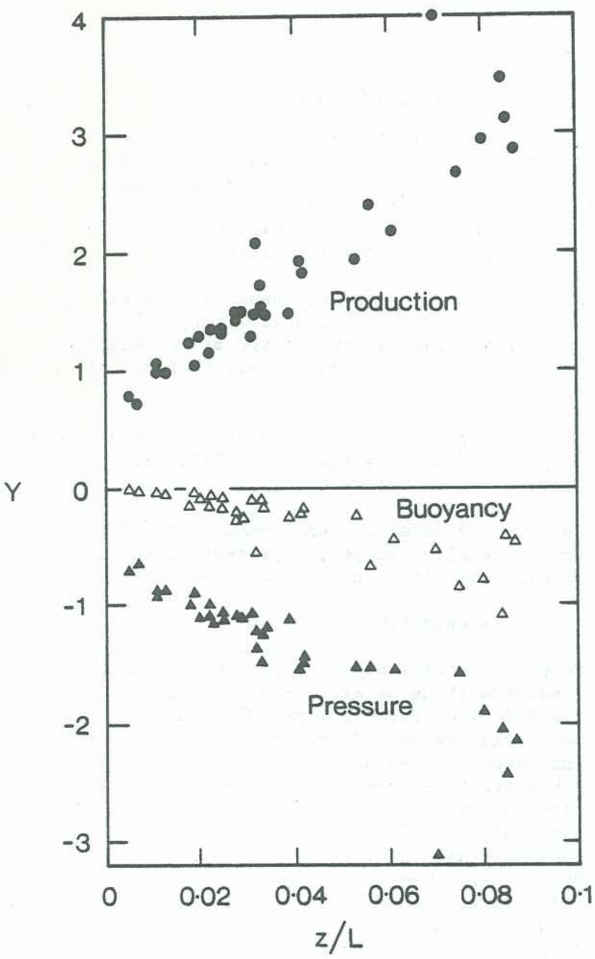

Figure 1 Nondimensionalized production, buoyancy and residual (pressure) terms in vertical heat flux equation.

$Y = \bullet, \overline{w^2}(\partial\Theta/\partial z)(kz/u_*^2\theta_*)$;

$\Delta, -(g/T)(\overline{\theta q} + 0.61\,T\,\overline{q^2})(kz/u_*^2\theta_*)$;

$\blacktriangle, \overline{\theta(\partial p/\partial z)}(kz/u_*^2\theta_*)$.

4 DISCUSSION

The pressure term is the largest requiring parameterization to close the system of equations to the second order. We now examine how well current predictions of it match our experimental values.

The pressure term is commonly written as

$$\overline{c\frac{\partial p}{\partial x_i}} = \frac{\partial\overline{cp}}{\partial x_i} - \overline{p\frac{\partial c}{\partial x_i}} \qquad (2)$$

(Launder, 1978; Warhaft, 1976). The term $\partial\overline{cp}/\partial x_i$ in Equation (2) is reasonably taken as zero (e.g., Launder, 1978, p. 260), although there is no direct evidence to support or refute this assumption. The term $\overline{p\partial c/\partial x_i}$ has been expanded using the Poisson equation for p, which gives three terms,

$$\overline{p\frac{\partial c}{\partial x_i}} = \phi_{ic} = \phi_{ic1} + \phi_{ic2} + \phi_{ic3} \qquad (3)$$

(e.g., Launder, 1978), where ϕ_{ic1} represents the return to isotropy and ϕ_{ic2} and ϕ_{ic3} are rapid terms. In general, ϕ_{icj} are functions of the wind and temperature fields; writing them as functions of the space variable strictly implies homogeneity of the turbulence, but is a necessary step in any practical parameterization. Useful forms for these three terms continue to emerge, but for the present purpose we selected the set given by Launder (1978), namely

$$\phi_{ic1} = -a_{1c}\frac{\varepsilon}{e}\overline{u_i c}\ , \qquad (4)$$

$$\phi_{ic2} = a_{2c}\overline{u_j c}\frac{\partial U_i}{\partial x_j} \qquad (5)$$

and

$$\phi_{ic3} = -a_{2c}\frac{\overline{\theta_v c}}{T}g_i\ , \qquad (6)$$

where a_{1c} and a_{2c} are constants, ε is the turbulence dissipation rate, e is the turbulence kinetic energy $(\overline{u_i u_i}/2)$, g_i is gravitational acceleration (note that $g_3 = -g$), and θ_v is the virtual temperature $(\theta + 0.61q)$.

The value of 1/3 for a_{2c} is exact for isotropic turbulence (Lumley, 1975, cited in Launder, 1978), but a value of 0.5 provided a better fit to wind tunnel data (Launder, 1978). Several values have been proposed for the constant a_{1c}, scattered about 3.4 (Launder, 1978). Accordingly, we have taken $a_{1\theta} = a_{1q} = 3.4$, and $a_{2\theta} = a_{2q} = 0.5$. The empirical constants incorporate any non-zero values of $\partial\overline{pc}/\partial x_i$, so that we could more legitimately rewrite Equation (3) as

$$-\overline{c\frac{\partial p}{\partial x_i}} = \phi_{ic} = \phi_{ic1} + \phi_{ic2} + \phi_{ic3}. \qquad (7)$$

The dissipation ε was estimated by assuming a local balance between shear production and dissipation, namely,

$$\varepsilon = u_*^2\frac{\partial U}{\partial z} \qquad \text{(Taylor, 1952).} \qquad (8)$$

Since

$$\frac{\partial U}{\partial z} = \frac{u_*^3\phi_u}{kz}\ , \qquad (9)$$

$$\varepsilon = \frac{u_*^3\phi_u}{kz}\ . \qquad (10)$$

Values of u_* were obtained from profiles of windspeed, temperature and humidity above the rice field, and from comparisons with values determined with the 3-component sonic anemometers were probably accurate to 10%. The turbulence kinetic energy was taken as $e = 6.25\ \sigma_{w2}$; the factor 6.25 was established from 19 runs using the 3-component sonic anemometers and is close to a value inferred from variances of wind components given by Lumley and Panofsky (1964, pp. 133, 155), for a one dimensional surface layer. The remaining variables in Equations (4) and (6) were measured for i = 3.

Figures 2a and 2b show comparisons of ϕ_{3c} from Equation (7) and the minor component ϕ_{3c3} from Equation (6), with pressure covariances obtained as residuals in the flux equation. With the chosen constants, Equation (7) usually gave lower values than the residuals for both heat and vapour terms, suggesting that Equation (10) underestimated ε, or that a_{1c} should be greater than 3.4; however, the apparent curvilinearity shown in Figures 2a and 2b precludes prescribing a single, alternative value for a_{1c}.

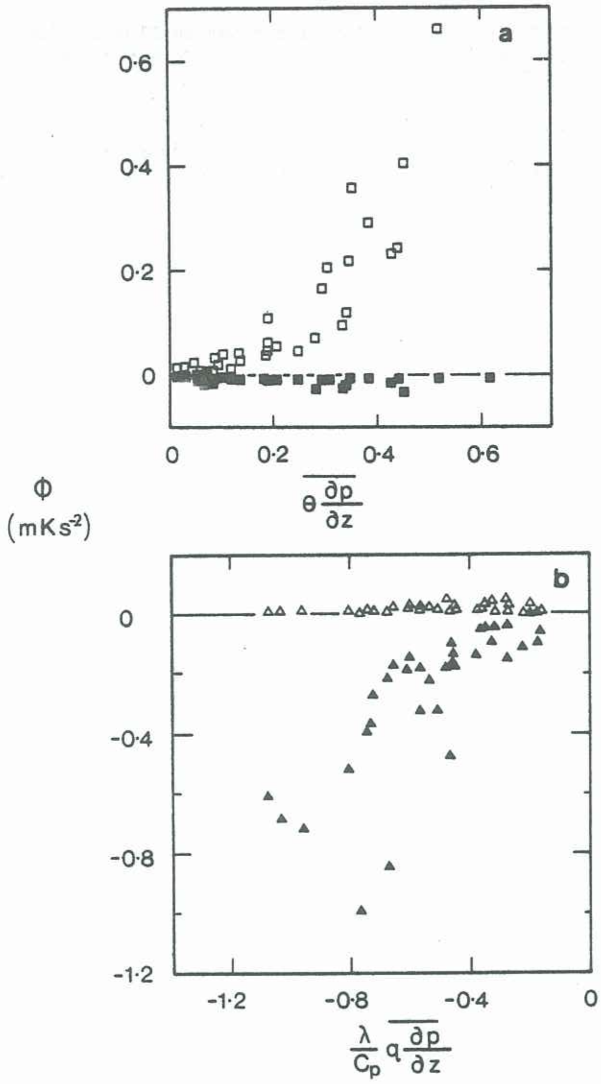

Figure 2 Comparison of parameterized pressure terms ϕ_{3c} (Launder, 1978) with estimate from residual of terms in flux equations. (a) heat: ■, $\phi_{3\theta 3}$; □, $\phi_{3\theta}$. (b) water vapour: ▲, ϕ_{3Q}, △, ϕ_{3Q3}.

Figures 3a and 3b show that the ratios of ϕ_{3c} to the residual approached 1 at neutral stability, but declined with increasing stability.

The selection of $a_{1\theta} = a_{1q} = 9.7$ and $a_{2\theta} = a_{2q} = 0$ by Rao et al. (1974), for their unique model of local advection using a second order closure, would only appear to give a good estimate of $\overline{c\partial p/\partial z}$ at stabilities of about z/L = 0.03. Assessment of the effect of errors in $\overline{c\partial p/\partial z}$ upon predictions from the model would require modifying and re-running the computer program.

These results show once again the importance of pressure covariance terms and the difficulty of modelling them; until they can be determined by direct experiment, the validity of their parameterization must remain in doubt.

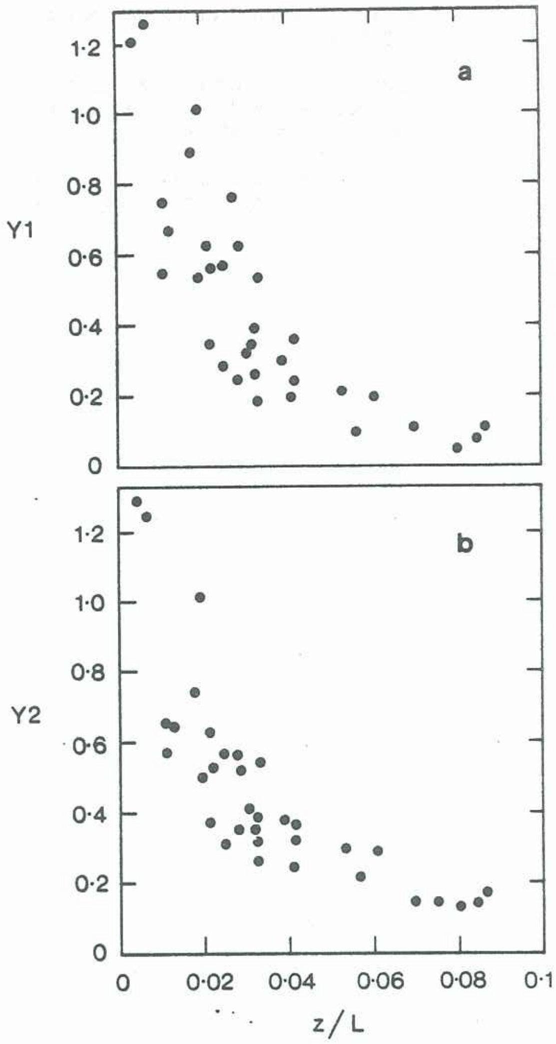

Figure 3 Ratio of parameterized pressure term to estimate from residual, as function of stability.

(a) heat: $Y1 = \phi_{3\theta}/\overline{\theta(\partial p/\partial z)}$

(b) water vapour: $Y2 = \phi_{3Q}/(\lambda/c_p)\overline{q(\partial p/\partial z)}$.

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