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COMPUTATION OF MIXED FLOWS IN PIPE NETWORKS

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SUMMARY A numerical model for simulating flows in pipe drainage systems is discussed. The model uses the implicit weighted four-point iterative non-linear solution technique for solving the gradually varied unsteady flow equations: these equations are used to simulate both open channel and pressurised flows in arbitary dendritic or looped drainage systems. The use of the implicit solution technique has allowed the development of a more efficient model which can use much larger time increments than an explicit solution process.

1 INTRODUCTION

Computer simulation is increasingly used to evaluate possible alternate sewer/storm drainage augmentation schemes. Several numerical simulation models have been developed during the last decade but these models are expensive to use because they use explicit computational schemes which require the use of extremely small time increments in the solution process.

One of the best known urban drainage simulation models is the Storm Water Management Model; this model uses two techniques for flow routing through pipes. The most complex of its routing processes, using a subroutine EXTRAN, uses an explicit technique for routing pressurised flows through the drainage system. Experience with EXTRAN has shown that to maintain numerical stability for pressurised flow calculations it is frequently necessary to use time increments which are a fraction of a second. Consequently, the use of this model for long period simulations is extremely expensive.

The use of implicit finite difference solution techniques, allowing larger time increments, is quite common for simulation of open channel flows in single channels. Few attempts have been made to extend these methods to simulate flows in channel networks.

A new simulation model for predicting flows in pipe networks is presented; this model used one of the efficient implicit solution techniques to simulate both open channel and pressurised flows. An application of the model demonstrates its versatility.

2. UNSTEADY FLOW EQUATIONS

The gradually varied one dimensional unsteady flow equations for an arbitary channel, in conservative form, are

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (1)$$

and

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial y}{\partial x} = g\,(S_o - S_f) \qquad (2)$$

where A is the cross sectional flow area, g is the gravitational acceleration constant, Q is the discharge, S_o is the channel slope, S_f is the frictional slope, t is time and x is the distance.

When the channels are prismatic, as generally occurs between manholes in artifical drainage systems, these equations can be reduced to a simpler form

$$\frac{\partial y}{\partial t} + \frac{A}{B}\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \qquad (3)$$

and

$$g\frac{\partial y}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} = g(S_o - S_f) \qquad (4)$$

Equations (3) and (4) are used as the basis of the proposed unsteady flow model.

An examination of the unsteady flow equations governing waterhammer transients reveals that they can also be reduced to the form of Equations (3) and (4) provided that a modified pipe cross section is used to evaluate the cross sectional area and free surface flow width. In addition it must be assumed that the water is incompressible and the pipe walls are rigid and inextensible.

The cross section modification, termed the Preissmann slot technique by Cunge and Wegner (1964), requires that a narrow slot, open to atmospheric pressure, be placed above the pipe, Figure 1. The slot width, B_s, is adjusted according to Equation (5)

$$B_s = \frac{A_s g}{a^2} \qquad (5)$$

where A_s is the cross sectional area of the pipe and a is the wavespeed of a pressure pulse.

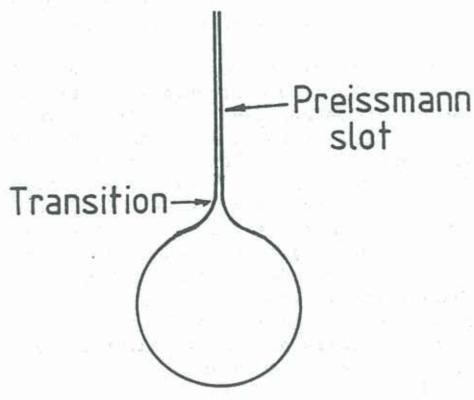

Figure 1. Modified cross sectional shape

Equations (3) and (4) can then be used to predict waterhammer transients when A and B in Equation (3) are replaced by A_s and B_s; in addition the flow depth y becomes analogous to the pressure head in the pipeline.

3. SOLUTION OF UNSTEADY FLOW EQUATIONS

Several implicit finite difference techniques exist for solving the one dimensional unsteady flow equations. The available implicit solution techniques have received extensive discussion in the literature so a detailed discussion of them will not be given here, rather they will be briefly reviewed. There are four common implicit finite difference techniques: the Strelkoff and Preissmann techniques require linearisation prior to finding the new flow conditions at each new time during the solution whereas the Amein and Abbott schemes do not require linearisation of the friction and cross sectional parameters, but the latter two techniques require iterative solutions. Past numerical experiments have revealed that the weighted four point iterative non-linear scheme proposed by Amein (1968) is the most accurate and conservative of the implicit computational schemes. Numerical experiments with this solution process reveal that it is most accurate when the value of the weighting coefficient is approximately equal to 0.55 for flows in trapezoidal channels; the accuracy decreases but numerical stability improves as the weighting coefficient is increased toward unity.

Applying the Amein solution scheme to solve the finite difference form of Equations (3) and (4) requires the evaluation of many partial derivatives; numerical differentiation is expensive so an order of magnitude analysis coupled with analytic differentiation of Equations (3) and (4) permits the development of an efficient numerical model. The analytic differentiation and order of magnitude analysis shows that relatively simple and numerically efficient expressions can be obtained for the partial derivatives of the continuity, momentum and boundary condition equations. Results have shown that the proposed model, using the simplified derivative expressions, has the same convergence characteristics as a model using numerical differentiation to obtain the derivatives. In the complete solution process, at each time increment, it is necessary to estimate the new flow conditions then use the estimated flow conditions to calculate corrections to the estimated conditions. The flow condition corrections are found using the generalised Newton-Raphson process. In the Newton-Raphson process the estimated flow conditions are used to evaluate the necessary partial derivatives and the residuals or errors due to not satisfying the governing differential equations, then the partial differentials and residuals are solved to find the flow corrections. This process is repeated until the residuals are suitably minimised.

The matrix of partial derivative terms which is solved to give the flow corrections, to flow depth and velocity, to be applied in the iterative solution of Equations (3) and (4) has a special structure when doing calculations for single pipes. For single pipe problems the assembled matrix is pentadiagonal and can be solved using efficient banded matrix solution routines. When solving for flows in pipe networks the matrix looses its banded structure, due to the junction compatability conditions, and a sparse matrix solution scheme is then used to find the flow corrections. Details of the mathematical model and the solution process have been given previously by Joliffe (1982).

4. APPLICATION TO NETWORK PROBLEM

To demonstrate the application of this numerical model we will examine the flow in a network, previously used by Sevuk (1973), consisting of 7 pipes which are invert aligned. The pipe layout is shown in Figure 2. The boundary conditions applied to this problem are specified inflow hydrographs entering the pipes labelled C and a free overfall at the downstream end of pipe A. The continuous water surface boundary condition is applied at the interior pipe junctions within the network.

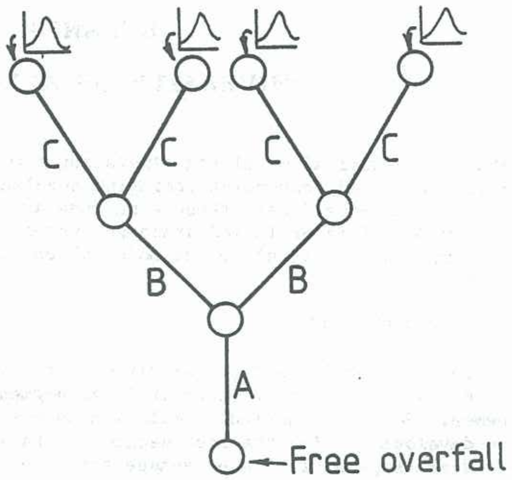

Figure 2. Pipe layout for hypothetical problem

The details of the pipes, forming the network, are given in Table 1.

TABLE 1. PIPE PROPERTIES

Pipe Code	Diameter (m)	Length (m)	Slope	Friction factor
A	1.219	610	0.001	0.02
B	0.915	427	0.001	0.02
C	0.763	305	0.001	0.02

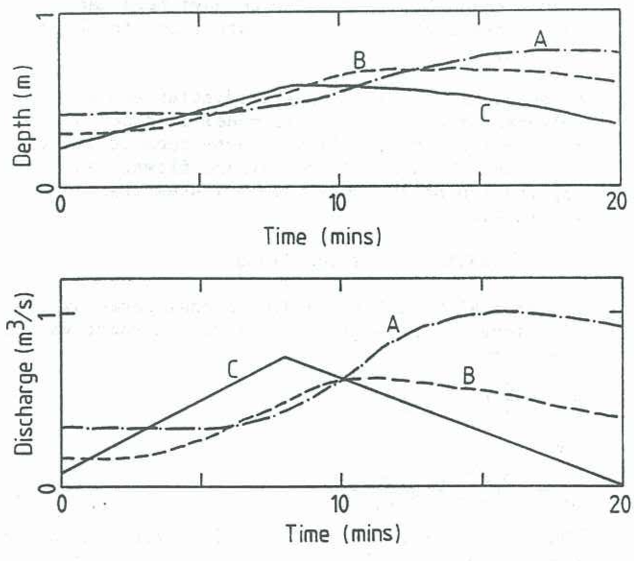

Figure 3. Results at pipe inlets for Sevuk's inflow hydrographs

Sevuk assumed that an assymetrical inflow hydrograph entered each pipe labelled C; the hydrograph had an initial flow of 0.085 m^3/sec, the flow then increased linearly to 0.4243 m^3/sec at a time of 8 minutes, then decreased linearly to 0.085 m^3/sec at 20 minutes and then the flow remained steady at 0.085 m^3/sec.

To perform the simulations a waterhammer wave celerity of 1100 m/sec was assumed; each pipe was assumed to contain 10 computational nodes, a time increment of 7.5 secs was used and the weighting coefficient was set equal to 0.75.

The results of the first simulation, identical to that of Sevuk, are given in Figure 3. The flows within the pipe network remain free surface during the entire simulation period of 20 minutes. The results indicate that the flow depths and discharges, within the network, gradually increase to a maximum and then decrease at all interior junction locations.

The inflow hydrographs, for the second simulation, were selected so that surcharging of the pipes would occur. The inflow hydrographs had the same shape and duration as previously but the peak inflow rate was 0.6365 m^3/s.

The results of this simulation, Figure 4, showed a pronounced surcharge effect. Figure 4 indicates that the surcharge first occurred at the inlet to pipes C and propagated downstream through the network until all pipes except A were completely pressurised. Also, when pipes B became pressurised there was a pressure wave transmitted back along pipes C causing a rapid increase in the pressure head at the inlet to pipes C. The maximum pressure at the inlets to pipes B and C occurred when pipes B and C were completely surcharged and if the surcharge hadn't been adequately simulated these peak pressures would have not been observed. The inflow hydrographs entering pipes B and A also show the effect of pipe pressurisation; there is a rapid increase in the flow into pipes B when pipes C become completely pressurised and there is a rapid increase in the flow entering pipe A when pipes B become completely pressurised.

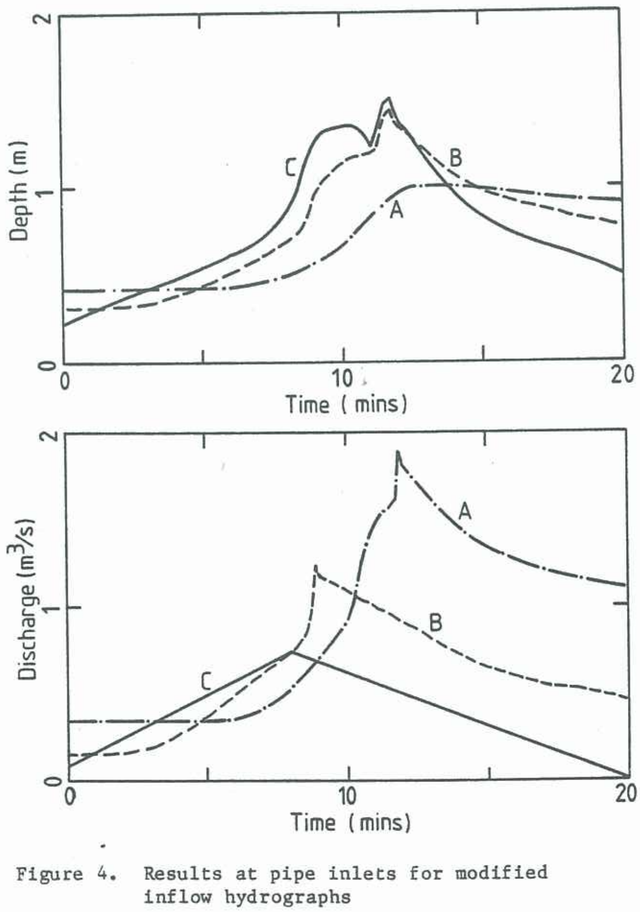

Figure 4. Results at pipe inlets for modified inflow hydrographs

A check of the flows and pressure heads throughout the pipe network reveal a close agreement between the results in Figures 3 and 4 and the flows anticipated by calculating the flow, at a particular instant, entering the network as inflow hydrographs into pipes C. The maximum error is 4.7% of the discharge at any location and this error is well within calculation accuracies which were set to 0.005m for the depth at any node during the interative improvement of the flow depths.

5. CONCLUSIONS

A mathematical model for simulating flows in urban drainage systems has been presented. The advantages of the proposed model, over existing models, are that it uses an efficient implicit finite difference solution process and it can reliably simulate both free surface and pressurised flows in the drainage system. An application of the model demonstrates how pressure transients, caused by pipes surcharging, can travel both upstream and downstream through the pipe network. This particular application shows the need for reliable simulation techniques since pressure transients rapidly propagate through the network and significantly change the pressures throughout the network.

6. REFERENCES

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