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SHOCK WAVE INSTABILITY AND SPONTANEOUS ACOUSTIC EMISSION FOR ARBITRARY DISTURBANCES IN REAL GASES

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SUMMARY A linear theory for the reflection of post-shock acoustic waves off the shock front is described, and shock wave instability criteria are derived from considerations of the behaviour of the reflection coefficient. The predictions compare well with observations of shock wave instability in carbon dioxide. However, extensions to the model seem necessary to give correlation in ionizing argon.

1. INTRODUCTION

In 1954, D'yakov considered the stability of plane shock waves by adopting an arbitrary equation of state for the shock medium, and was able to establish stability criteria in terms of the slopes of the Rankine-Hugoniot curve and the Rayleigh line. However, Griffith et al. (1976) found that unstable shock waves occur in a region far removed from that predicted. They suggested that D'yakov's linear analysis did not give an accurate representation of real shock tube flow, and proposed a less restrictive criterion for shock wave instability. This criterion corresponded to a condition where perturbations on the shock front were predicted to neither grow nor decay with time. This was later shown by Fowles (1981) to correspond to a criterion derived much earlier by Kontorovich (1957) for spontaneous emission of sound waves from the shock. However, the calculations made by Griffiths et al. of the stability parameters were not sufficiently accurate to allow definitive comparison with their experimental results. Furthermore, other authors have suggested instability criteria different from those proposed by D'yakov and Griffiths et al. Consequently, we have developed a more general analysis, that incorporates earlier work as special cases, and have made improved calculations of the stability parameters.

2. THE REFLECTION COEFFICIENT AND SHOCK INSTABILITY

We consider the reflection of post-shock acoustic waves off the shock front, illustrated in figure 1. The pressure, density, specific volume, velocity and sound speed of the shock medium are denoted by p, ρ, $v = 1/\rho$, $\underset{\sim}{u}$ and a, respectively. The perturbation of a quantity η is represented by $\overline{\eta}$, and it is assumed that $\overline{\eta} \ll \eta$. Unperturbed quantities in the pre-shock and post-shock regions for the conventional shock-fixed reference frame are identified by the subscripts 1 and 2, respectively. In the reference frame of figure 1, the unperturbed post-shock flow is stationary and therefore the pre-shock velocity is $u_2 - u_1$. The flow on either side of the shock is assumed to be in thermodynamic and chemical equilibrium. The incident and reflected acoustic disturbances are assumed to be plane waves of arbitrary shape, and are thus described by

$$\overline{p}_\theta = f(\omega_\theta[x\cos\theta/a_2 + y\sin\theta/a_2 - t]),$$
$$\overline{\underset{\sim}{u}}_\theta = \frac{\overline{p}_\theta}{\rho_2 a_2}(\cos\theta, \sin\theta). \tag{1}$$

The angle θ is the direction of propagation of the wave with respect to the x axis, ω_θ is the wave frequency, and f is an arbitrary function. For the incident wave $\theta = \alpha$, while for the reflected wave $\theta = \beta$. The distortion of the shock wave as a result of interaction with the acoustic disturbance is given by

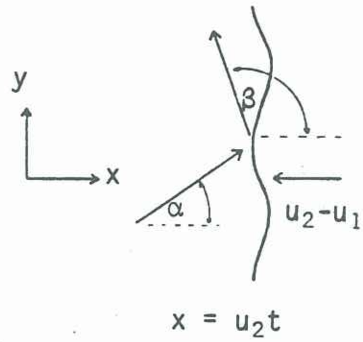

Figure 1. The reflection problem

$$\overline{x} = g(k_s y - \omega_s t), \tag{2}$$

where $\overline{x}$ is the displacement of the shock from the unperturbed position at $x = u_2 t$, with wave number k_s and frequency ω_s, and g is an arbitrary function. The consequent perturbation in the x component of the shock speed is

$$\overline{u}_s = \partial g/\partial t . \tag{3}$$

This variation in shock speed must modify the shock strength and thus produces an entropy perturbation $\overline{\eta}_e$ behind the shock. It is well understood that, while acoustic waves propagate along the characteristic paths, entropy disturbances travel along the particle paths. Consequently, the solution for $\overline{\eta}_e$ can be found by neglecting non-linear terms and integrating the mass and momentum equations along a particle path. If $\overline{\eta}_e$ is assumed to be an arbitrary plane wave, then this integration gives

$$\overline{p}_e = 0 ,$$
$$\overline{\underset{\sim}{u}}_e = h(\ell_e x - k_e y)(k_e, \ell_e) , \tag{4}$$

where ℓ_e and k_e are wave numbers and h is an arbitrary function. The perturbation quantities described by equations (1) to (4) must satisfy boundary conditions determined by the Rankine-Hugoniot relations for the flow variables at the shock wave, i.e. at $x = u_2 t$.

When non-linear terms are neglected, the full system of equations can be solved to give the reflection coefficient,

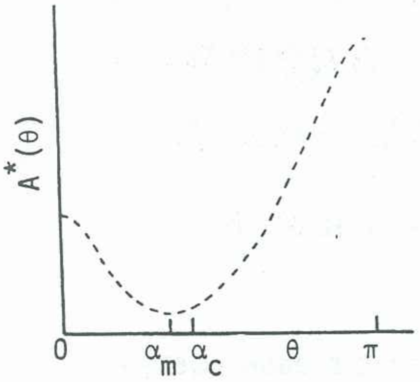

Figure 2. The function $A^*(\theta)$

$$R = \left.\frac{\overline{p}_\beta}{\overline{p}_\alpha}\right|_{x = u_2 t} = \frac{A - A^*(\alpha)}{A^*(\beta) - A} \tag{5}$$

where

$$A^*(\theta) = \frac{2M_2^2\, Q(\theta)\sin\theta - v_1 v_2^{-1} M_2^2\, Q^2(\theta) - 2M_2\cos\theta + 1}{v_1 v_2^{-1} M_2^2\, Q^2(\theta) + 1} \tag{6}$$

$$Q(\theta) = \sin\theta/(1 - M_2\cos\theta), \tag{7}$$

and

$$A = j^2\, dv/dp, \tag{8}$$

j being the slope of the Rayleigh line, dv/dp that of the Rankine-Hugoniot curve, and $M_2 = u_2/a_2$. The function $A^*(\theta)$ is illustrated in figure 2. If we restrict ourselves to positive values of α, then

$$0 < \alpha < \alpha_c = \cos^{-1} M_2 = \beta_c < \beta < \pi, \tag{9}$$

since acoustic waves with $\alpha > \alpha_c$ can never overtake the shock. Relevant values of $A^*(\theta)$ are

$$A^*(0) = 1 - 2M_2 , \tag{10}$$

$$A^*(\alpha_c) = \frac{1 - M_2^2(1 + v_1 v_2^{-1})}{1 - M_2^2(1 - v_1 v_2^{-1})} , \tag{11}$$

and

$$A^*(\pi) = 1 + 2M_2 . \tag{12}$$

By considering the values of $A^*(\theta)$ and $dA^*/d\theta$ at $\theta = 0$, α_c and π for different values of A, it is possible to develop a qualitative picture of the behaviour of R with α and A. It is found that R has the form shown in figure 3 and summarized in table 1. Of particular note is the fact that R will be infinite when $A = A^*(\beta)$. From figure 2, it is seen that this is only possible if

$$A^*(\alpha_c) < A \leqslant A^*(\pi) \tag{13}$$

These conditions suggest that it is possible to have a "pseudo-reflected" wave in the absence of an incident disturbance. The existance of such a pseudo-reflected wave implies that the same equilibrium state may be achieved by passing the flow through either a single shock or a stronger oblique shock followed by an expansion wave propagating at an angle $\beta = \beta_\infty$ to the x axis. Consequently, one may interpret the case of an infinite reflection coefficient as corresponding to a "wave splitting" instability.

From figure 3 and Table 1, we can identify the regions of shock wave instability considered by previous authors. The condition $A = 1 + 2M$ is D'yakov's (1954) upper limit for shock wave stability. Gardner (1963)

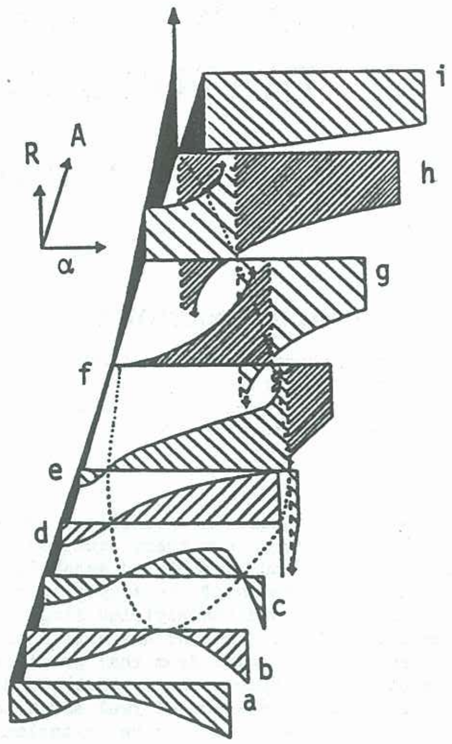

Figure 3. The reflection coefficient

TABLE 1

Case	Range of A	Range of R
a	$-1 < A < A^*(\alpha_m)$	$-1 \leqslant R < 0$
b	$A = A^*(\alpha_m)$	$-1 \leqslant R < 0,\ R(\alpha_m) = 0$
c	$A^*(\alpha_m) < A < A^*(\alpha_c)$	$-1 \leqslant R < 1$
d	$A = A^*(\alpha_c)$	$-1 \leqslant R < 1,\ R(\alpha_c) = \pm 1$
e	$A^*(\alpha_c) < A < A^*(0)$	$-\infty \leqslant R \leqslant \infty$
f	$A = A^*(0) = 1 - 2M_2$	$-\infty \leqslant R \leqslant \infty,\ R(0) = 0$
g	$A = 1$	$-\infty \leqslant R \leqslant \infty,\ R(0) = 1$
h	$A = A^*(\pi) = 1 + 2M_2$	$-\infty \leqslant R \leqslant -1, R(0) = \pm\infty$
i	$A^*(\pi) < A$	$-\infty \leqslant R \leqslant -1$

showed that, when this limit is exceeded, a shock will break up into a stronger shock travelling in the same direction and an expansion wave in the opposite direction. However, our approach is different from D'yakov's who considered only perturbations originating at the shock front itself (as in a diffraction problem). In the present analysis, the region $A > 1 + 2M$ (case i) corresponds to all incident compression waves being reflected as expansions with amplification, i.e. $R < -1$. Fowles (1976) investigated the problem of a step wave at normal incidence ($\alpha = 0$) reflecting off the shock front, and predicted amplification to occur when $A > 1$. Later, he (Fowles, 1981) extended his analysis to include waves of arbitrary incidence (i.e., $0 \leqslant \alpha \leqslant \alpha_c$) and predicted amplification to occur at other angles when $A > A^*(\alpha_c)$. From figure 3 and table 1, cases (g) to (i) with $\alpha = 0$ reproduce his earlier results; while (e) to (i) include his later findings. The region $A^*(\alpha_c) < A \leqslant 1 + 2M_2$, (cases e to h), includes the surface $\alpha = \alpha_\infty$, $R = \pm\infty$, which corresponding to the solution considered by Kontorovich (1957), based on D'yakov's

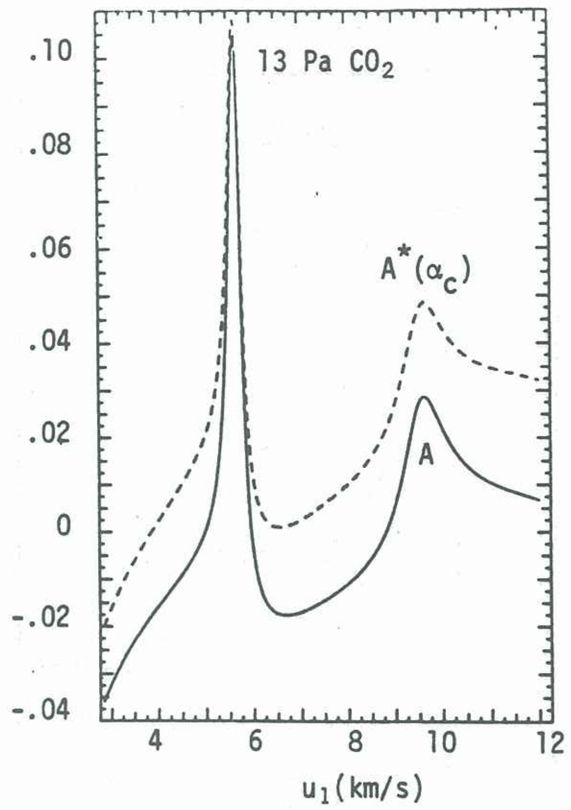

Figure 4. The stability parameters

analysis, for the spontaneous emission of sound waves from the shock. In view of the discussion above, this surface is now identified as the condition where a normal shock can split into a stronger oblique shock and an oblique expansion wave travelling at the angle β_∞ to the x axis. Consequently the wave splitting instability considered by Gardner (1963) is a special case corresponding to $\beta_\infty = \pi$.

Disturbances at all angles of incidence are attenuated in cases a to d, for which $A < A^*(\alpha_c)$. It is notable that for $A^*(\alpha_m) < A < A^*(\pi)$ (between b and h), a range of α occurs for which disturbances are not changed in sign on reflection ($R > 0$), that is compressions reflect as compressions and vice versa. This abnormal behaviour indicates a fundamental change in the acoustic impedance of the shock. Furthermore, R exceeds 1 for cases e to h and consequently these reflected waves are also amplified. The significance of $R > 0$ and $R > 1$ to the problem of shock wave instability is under further investigation. The limit $A = -1, M_2 < 1$ is D'yakov's lower bound to the region of shock wave stability. The region below this limit (i.e. below case a) has been identified by Fowles (1981) as that where a single shock splits into two normal shocks travelling in the same direction. This type of splitting instability is frequently observed when materials undergo shock-induced phase transformations (Duval & Graham, 1977).

3. EXPERIMENT

Griffiths, Sandeman and Hornung (1976) reported two main types of shock wave distortions for dissociating carbon dioxide and ionizing argon flows. One type was shown to result from disturbances which originated at the contact surface and destroyed the uniformity of the test sample. The other was shown to occur in the presence of a non-turbulent sample, suggesting that the shock was inherently unstable. In order to confirm these findings, we have performed further experiments in argon and carbon dioxide. The experimental arrangement was identical to that described by Griffiths et al., except that differential interferometry was used to detect the shock wave, while a new spectral line absorption method (Houwing & Sandeman, 1983) was used to detect the contact surface. This method enabled us to distinguish between cases of shock wave instability and contact surface instability. The cases of contact surface instability are discussed in more detail by Houwing and Sandeman (1983) and are not of direct importance to the present discussion.

The instability parameters A and $A^*(\alpha_c)$ were calculated for a wide range of experimental conditions and some results are illustrated in figure 4. The thermodynamic model for carbon dioxide was equivalent to the equilibrium model described by Simcox and Peterson (1965). The model for equilibrium argon took into account electronic excitation and ionization up to ARIII but neglected radiation losses and lowering of the ionization potential. For both gases, intermolecular forces were ignored. An instability region was predicted for carbon dioxide but not for argon. This region was arrived at by determining when $A \simeq A^*(\alpha_c)$. It is important to note that in none of the calculations was A found to exceed $A^*(\alpha_c)$. It is argued however that dissipative effects such as boundary layer entrainment and radiation losses will be responsible for increasing ρ_2 and consequently lowering $A^*(\alpha_c)$ for the experiments, so that the inequality may be satisfied. Experimental results for carbon dioxide, together with the theoretical Rankine-Hugoniot curves are plotted in figure 5. This shows that reasonable agreement is achieved within experimental error for carbon dioxide, although the instability region is predicted to occur at somewhat higher velocities. However, this agreement does not extend to the observations near 3.5 and 12 km/s. Disagreement also occurs for the argon case (figure 6), for which unstable shock waves are observed while only stable shocks are predicted. In order to overcome these discrepancies, a more sophisticated thermodynamic model to take into account intermolecular forces and lowering of the ionization potential is being considered.

4. DISCUSSION AND CONCLUSION

In solving the reflection problem, it was assumed that the flow was in thermodynamic and chemical equilibrium. In reality, however, the relaxation zone is of finite thickness, and it is therefore necessary to consider the interaction of acoustic disturbances with the non-equilibrium region. Extending the analysis to take into account this interaction may possibly explain some of the presently unexplained anomalies. For example, consideration of acoustic reflection off the electron cascade front in ionizing argon might predict a condition where such reflections become infinite, suggesting "spontaneous distortion of the electron cascade front". This might well explain the observations of Glass and Liu (1978). Furthermore, it has been pointed out that the change in sign of the reflection coefficient represents a fundamental change in the acoustic impedance of the shock. The significance of this change to the problem of shock wave instability is presently being investigated.

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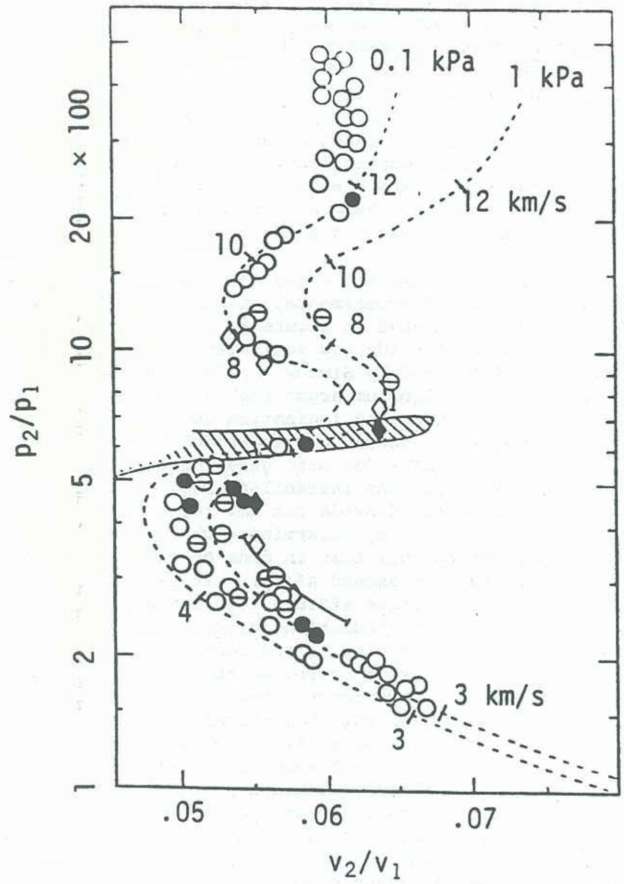

Figure 5. Rankine-Hugoniot curves and experimental data for CO_2.
○ - Griffiths et al.; ◇ - present investigation. Filled, horizontal line and open symbols correspond to cases classes as "unstable", "uncertain" and "stable" respectively. The region A ≈ A*(α_c) is shaded.

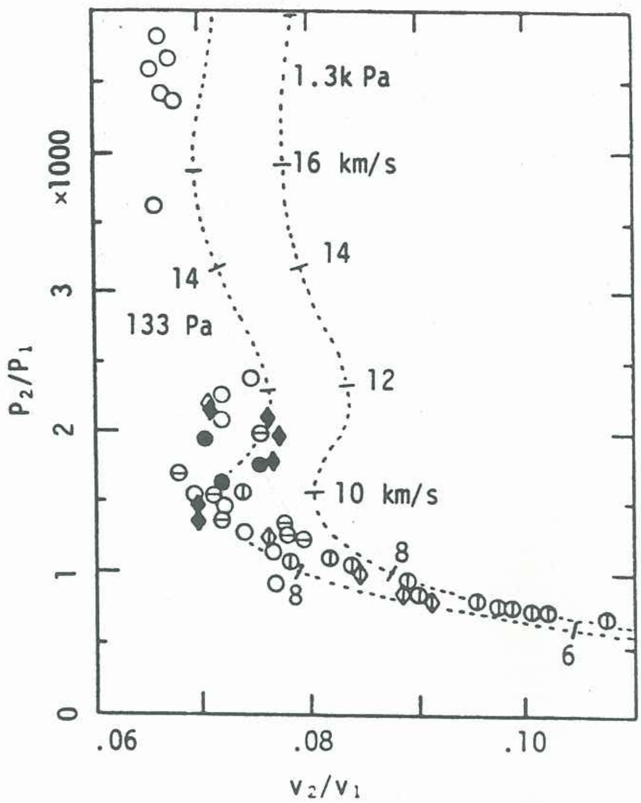

Figure 6. Rankine-Hugoniot curves and experimental results for argon. Classification is same as in figure 5, except that vertical lines indicate contact surface instability.

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