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ON THE SHAPE-CHANGE AND THE SHAPE-AT-DEPARTURE OF BUBBLES GROWING AT A WALL

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SUMMARY

The shape variation of bubbles growing at a wall in a boiling liquid is described for different growth conditions. The stresses at the bubble interface due to inertia, surface tension, gravity and liquid viscosity are evaluated and used in explaining the shape changes and to predict the onset of such variations.

NOTATION

g, g_e	gravity, at bubble, Earth
J_w	Jakob number $\frac{\rho_f}{\rho_g}\frac{C_f\,\Delta T}{h_{fg}}$
ℓ	characteristic length
P_v, P_∞	pressure, vapour, bulk
ΔP_i, ΔP_s, ΔP_g, ΔP_ν	stress, inertia, surface tension, gravity, viscosity
R, $\dot{R}$, $\ddot{R}$	bubble radius, velocity, acceleration
δs	thermal boundary layer thickness
T, T_w, T_{sat}	temperature, liquid, wall, saturation
t	time
t^*	variable $(\alpha_f/\ell^6)\,_o\!\int^t R^4 dt$
α_f	thermal diffusivity
δ_o, δ^+	microlayer thickness, initial, at time t
λ	variable $h_{fg}/c_f\,\Delta T$
μ_f	dynamic viscosity
ρ_f	density
σ	surface tension

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\left(\frac{dR^3}{dt}\right)}{3r^2}\frac{\partial}{\partial r}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \operatorname{Sin}\Theta}\left(\operatorname{Sin}\Theta\frac{\partial}{\partial \Theta}\right) + \frac{1}{r^2 \operatorname{Sin}^2\Theta}\frac{\partial^2}{\partial \Phi^2}$$

(r, Θ, Φ)	spherical polar coordinates

INTRODUCTION

When boiling begins in a liquid at a superheated wall, vapour is present, initially, in the form of individually nucleated bubbles. The dynamics of these bubbles has been found to determine the latent heat transport from the heating surface to the boiling liquid (3). The bubbles growing at a wall have always been observed to change their shape remarkably, depending on the relative influence of growth rate, gravity and surface tension (4). These shape variations have been explained qualitatively (2): for rapid growth rates, where the inertia stresses are dominant, the bubbles grow hemispherically; for slower rates where the surface tension stresses are significant, the bubble shape is closer to a sphere. The departure of bubbles with a "mushroom" shape is attributed to the buoyancy effect caused by the gravitational field.

An analysis is presented in this paper to explain the shape changes and the onset of these variations, for isolated bubbles growing at a wall in a non-uniform temperature field.

ANALYSIS

The equation of motion of a hemispherically growing bubble at a wall in an infinite mass of stagnant liquid is given by (5)

$$\rho_f\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) + \frac{2\sigma}{R} + 4\mu_f\frac{\dot{R}}{R} = P_v - P_\infty \qquad (1)$$

The pressure terms in equation (1) are recognised as follows:
Inertia pressure $\Delta P_i = \rho_f(R\ddot{R} + 3/2\,\dot{R}^2)$ which describes the inertia effect due to bubble expansion,

surface tension excess pressure $\Delta P_s = \frac{2\sigma}{R}$ and

excess pressure due to viscosity $\Delta P_\nu = 4\mu_f\,\dot{R}/R$.

For bubbles growing in a gravitational field, the gravity pressure term $\Delta P_g = \rho_f g R$ is also defined.

Among the pressure terms defined, only

$\rho_f(R\ddot{R} + 3/2\ \dot{R}^2)$ has a radial distribution owing to

the axisymmetric nature of the bubble interface expansion (5). The term $2\sigma/R$ remains unchanged along the hemispherical interface of the bubble, but very close to the wall, the microlayer causes the radius of curvature to be changed drastically. Thus ΔP_s becomes a non-radial field. Also the remaining pressure components are non-radial: $\rho_f gR$ is hydrostatic, $4\mu_f\dot{R}/R$ is significant only in the vicinity of the microlayer at the bubble base.

The magnitudes of these pressure terms are evaluated using the growth history of the bubble which is obtained by the analysis outlined below.

The heat diffusion problem associated with the bubble growth can be stated as follows:

$$\frac{DT}{Dt} = \alpha_f \nabla^2 T$$

$$T\ (r,\ \Theta,\ \Phi,\ 0) = f\ (r,\ \Theta,\ \Phi) \qquad (2)$$

$$T\ (R,\ \Theta,\ \Phi,\ t) = T_{sat}$$

and $T\ (r,\ \Theta,\ \Phi,\ t)$ is bounded as $\gamma \to \infty$

The transformation

$T^*\ (r,t) =$

$$\frac{1}{2\pi\ (T_w - T_{sat})}\ {}_0\!\int^{2\pi} d\Phi \int_0^{\pi/2} \left(T(r,\ \Theta,\ t) - T_{sat}\right) \mathrm{Sin}\ \Theta\ d\Theta \qquad (3)$$

is employed to transform the diffusion problem in equation (2) with a moving boundary to a case with a stationery boundary and the solution is obtained in the form

$$R^3 = \frac{3\alpha_f J_w}{\ell^3 \sqrt{\pi}}\ {}_0\!\int^t \frac{R^4}{\sqrt{t^*}} \left\{\int_0^\infty g(2\sqrt{\xi t^*})\ e^{-\xi} d\xi\right\} dt\ +$$

$$3J_w \int_0^R \lambda\ \delta_o(1-\delta+)\ rdr \qquad (4)$$

On the right hand side of the equation (4), the first term represents the evaporation from the interface of the bubble and the second term, the evaporation from the bubble base. The function $g(2\sqrt{\xi t^*})$ is obtained by transforming the initial temperature distribution $f(r,\ \Theta,\ \Phi)$. The detailed derivation of equation (4) is given in (1).

The growth history (R-t) of the bubble is obtained by solving equation (4) numerically for a given initial temperature profile.

EXPERIMENTS

High speed cine' photographic observations were made of individually nucleated vapour bubbles grown at a wall in n-hexane in a jacketed vessel which was mounted on a table (1). The table could be released to fall freely for a distance of about 0.3m simulating "zero" gravity ($g/g_e < 0.001$) at the boiling chamber. With the aid of a counter-weighted linkage arrangement, the motion of the table could be resisted and fractional gravity ($0.001 < g/ge < 0.035$) conditions could be achieved at the table. Prior to growing a bubble, a known temperature field (error function type) was set up in the boiling liquid by a transient heating method at the wall. Further details of experimental apparatus and techniques are given in (1).

RESULTS

On examining the high speed photographs, it was readily observed that the bubbles always started to grow hemispherically over a certain period before the shape changed (Figure 1). Subsequent shape variation was seen to be determined by the level of gravity: In "zero" gravity the bubble rounded off to a sphere at departure, in full gravity the hemispherical shape was retained until departure and in low gravity the bubble departed with an intermediate oblate shape (Figure 1). When bubbles grew rapidly (high wall superheats) the hemispherical growth phase was extended.

ANALYSIS OF RESULTS AND DISCUSSION

The shape change of bubbles is best explained by examining the significance of the pressure terms ΔP_i, ΔP_s, ΔP_g and ΔP_ν. Figure 2 illustrates the variation of these pressure components during the growth of a bubble for two typical test runs. Initially, ΔP_i predominates over ΔP_s or ΔP_g by a factor of about 100. Essentially being a radial pressure field, ΔP_i, therefore, causes the bubble to grow axisymmetrically as a hemisphere. However, ΔP_i falls rapidly with the growth and soon the condition $\Delta P_i \approx \Delta P_s$ (surface tension-affected) or $\Delta P_i \approx \Delta P_g$ (gravity-affected) is reached. The

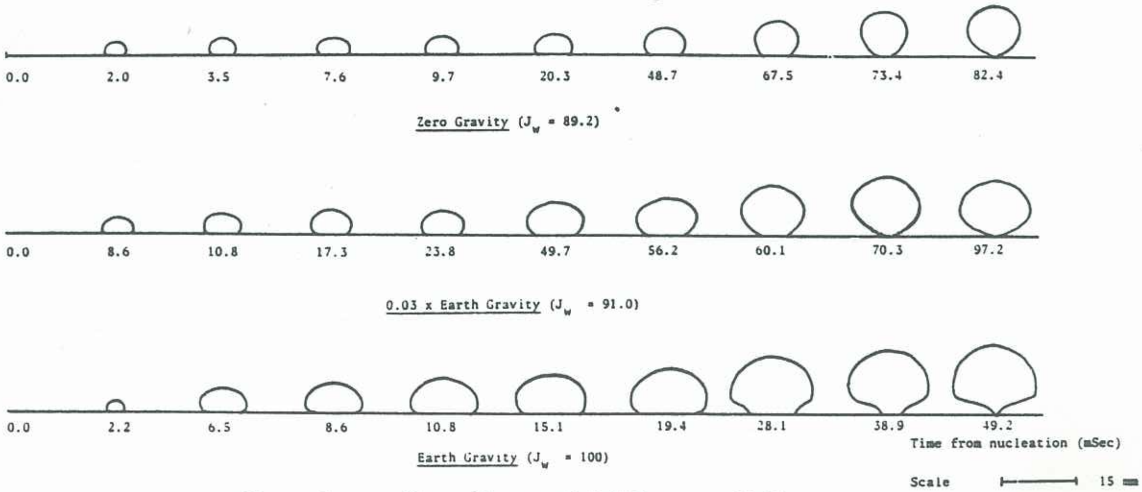
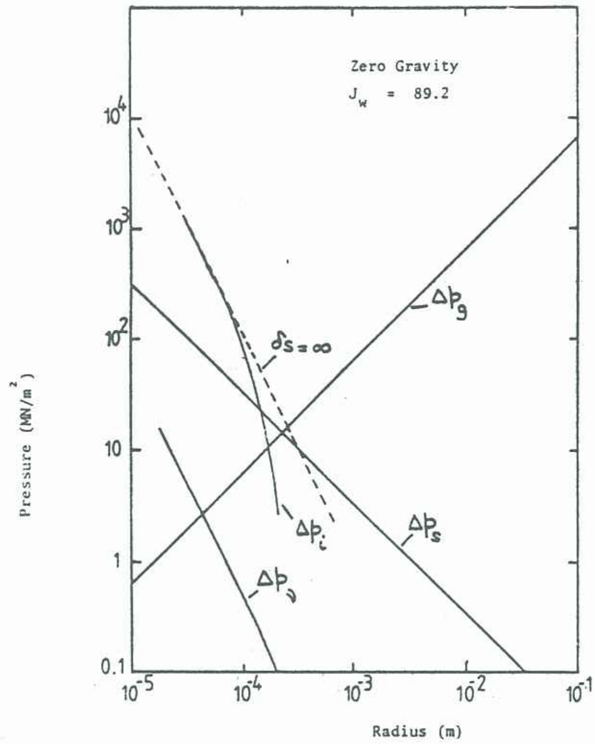
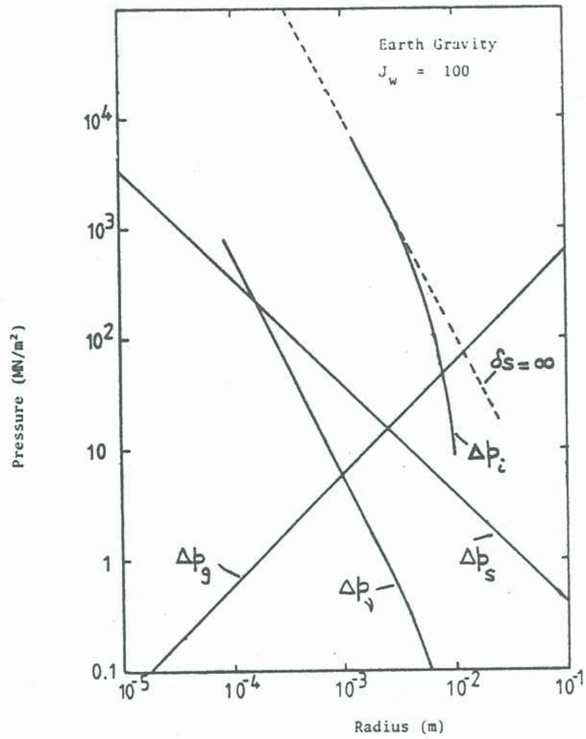


Figure 1 Shape Change of Bubbles at a Wall.



(a)



(b)

Figure 2 Pressure Variation During Bubble Growth.

prevailing pressure field, which characterises the axisymmetric growth, is then disturbed owing to the non-radial nature of ΔP_s or ΔP_g. Consequently the bubble ceases to grow hemispherically any longer and undergoes shape change.

The subsequent shape variation until departure, is governed by the relative magnitudes of ΔP_s and ΔP_g. When the initial phase of growth is checked by surface tension, i.e. $\Delta P_i \approx \Delta P_s > \Delta P_g$ (Figure 2(a)), the bubble rounds off to a sphere at departure in order to acquire the minimum surface area under the influence of surface tension. When the initial phase of growth is checked by gravity, i.e. $\Delta P_i \approx \Delta P_g > \Delta P_s$ (Figure 2(b)), the surface tension does not affect the shape, the bubble does not round off and the initial hemispherical shape is retained as long as the buoyant force causes detachment. For the growth condition $\Delta P_i \approx \Delta P_s \approx \Delta P_g$ the shape variation is determined by the interaction between both ΔP_s and ΔP_g and hence the bubble departs with an oblate shape. As evident from Figure (2), ΔP_ν is significantly smaller than both ΔP_s and ΔP_g and therefore, has very little influence on the shape variation.

At rapid growth rates (higher J_w) ΔP_i is higher in magnitude, is checked by ΔP_s or ΔP_g at a larger bubble radius and the hemispherical phase of growth is extended. Moreover, when the bubble is growing in a thicker thermal boundary layer, the growth rate is accelerated due to additional evaporation from the curved interface of the bubble, which gives rise to an extended hemispherical growth phase, as indicated by the dashed-line in Figure (2).

CONCLUSIONS

During bubble growth at a heated wall in a boiling liquid, the bubbles initially grow with a hemispherical shape since the growth is predominantly governed by the axisymmetric nature of the inertia pressure field at the liquid-vapour interface. Subsequently, either the influence of surface tension or gravity becomes significant for the growth and the bubbe ceases to grow hemipsherically. Following shape change until bubble detachment is determined by the magnitudes of the stresses due to surface tension and gravity at the end of the hemispherical phase of growth: If the influence of a surface tension is more significant the bubble gradually rounds off to a spherical shape at departure; If the influence of gravity is more significant, the bubble retains its initial hemispherical shape until departure. The viscosity has negligible influence on bubble shape. The hemispherical growth phase is extended when the bubble interface expands with a higher inertia stress resulting from a thicker thermal boundary layer or high wall superheat.

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