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ANALYSIS OF A RADIATING GAS IN THE THERMAL ENTRANCE REGION OF A DUCT

A. CAMPO, F. MALPICA AND A. JARRIN

DEPARTAMENTO DE TERMODINAMICA

UNIVERSIDAD SIMON BOLIVAR, CARACAS, VENEZUELA

SUMMARY A numerical procedure has been presented to investigate the interaction of thermal radiation with forced convection in thermally developing gas flow through a circular tube. The analysis is performed on a laminar flow of a gray gas bounded by constant temperature walls. The governing integro-differential equation of energy is modified in order to accomodate a four-flux model. This procedure provides five coupled partial differential equations which are solved numerically by the efficient TDMA algorithm. The results are shown to be highly accurate by comparison with other solutions.

1 NOTATION

A_r, A_z = parameters, equation (4)
B_r, B_z = parameters, equation (4)
c_p = specific heat at constant pressure
h_z = convective coefficient
H_r = B_r/A_r
H_z = B_z/A_z
k = thermal conductivity
k_a = total volumetric absorption coefficient
N = radiation-conduction parameter
Nu_C = convection Nusselt number
Nu_R = radiation Nusselt number
Nu_T = total Nusselt number
Pe = Peclet number
q_c = convective heat flux
q_R = radiative heat flux
q_r^+, q_r^- = radiative heat flux in the positive and negative radial direction
q_z^+, q_z^- = radiative heat flux in the positive and negative axial direction
r = radial position
R = pipe radius
T = absolute temperature
$\underline{u}$ = gas velocity
$\bar{u}$ = mean gas velocity
z = axial position
T_E = reference temperature
T_i = inlet gas temperature and upstream wall temperature
To = downstream wall temperature
Tiq = blackbody temperature at the inlet
Toq = blackbody temperature at the exit
ε_w = wall emissivity
η = dimensionless radial position, r/R
θ = dimensionless temperature, T/T_E
θ_b = dimensionless bulk temperature
θ_w = dimensionless wall temperature
ρ = density
τ_R = optical thickness
σ = Stefan-Boltzmann constant
ψ_o = cone angle

2 INTRODUCTION

The tendency in modern technology toward an increase of efficiencies in thermal systems necessitates high operating temperatures. From here, it is obvious that accurate methods of analysis are needed for the heat transfer calculations of such systems.

One of the applications of this general topic is connected with the flow of high temperature gases through circular ducts. Here, the presence of combined mechanisms: conduction, convection and radiation have to be taken into account. The importance of this class of problems is evident in the study of propulsion systems, combustion chambers, industrial furnaces and steam generation units, among others.

A survey of the literature reveals that the thermal analysis of the fluid flow through ducts is a very old problem. Thus, Graetz (1883, 1885) integrated the energy equation for fully developed velocity in order to obtain the temperature distribution in circular ducts.

An exhaustive review of the available literature on laminar forced convection in ducts appears in the excellent monograph written by Shah and London(1978).

The classical Graetz approach is unable to describe the thermal behavior of the laminar flow of radiant gases. For this situation, the energy equation has to be accompanied by additional terms accounting for thermal radiation effects.

The process of thermal radiation does not require a direct contact between the media, like in the counterparts of conduction and convection. For this peculiar reason, Hottel and Cohen(1958) defined it as a phenomenon with "action at a distance." Accordingly, a fluid element in a participating medium can exchange energy by radiation with any other element in the medium. This renders a tri-dimensional character to the heat exchange by thermal radiation.

Due to the impossibility of obtaining exact solutions for this class of coupled problems, which are highly nonlinear; numerical methods are more appropriate. Among these, one of the most widely used is the zone method, developed by Hottel and Cohen. It consists in a transformation of the integro-differential equation of energy into a system of algebraic equations. Correspondingly, the investigations of Viskanta(1963) and Einstein (1963) were performed within the framework of this solution method too.

Disregarding the influence of axial conduction, deSoto (1968) investigated the situation of a nongray gas flowing in a circular duct having black walls. Entrance effects caused by radiation were accounted for. His results include temperature distributions which were compared with those computed by the Graetz series. The solution method, via finite differences, was essentially iterative. First, a temperature distribution was guessed and a heat flux distribution was calculated. With this input inserted into the energy equation, a new temperature distribution was computed. This strategy provided a continuous process until an acceptable level of convergence was attained.

Echigo (1975) solved the same problem, but with the idealization of gray gas along a circular duct with a step change in wall temperature. His numerical results show mean bulk temperature distributions and local Nusselt number distributions too.

Spalding (1971) proposed the multiple flux model, which discretizes the radiation intensity with respect to the solid angle, such that the intensity remains as a constant value. Similarly, Richter and Quack (1974) constructed a four-flux model for the radiation intensity whose characteristics resemble those of Spalding's model. These approaches replace the descriptive integro-differential equation of energy by a system of partial differential equations.

Vistanka (1982) in a review paper examined the state-of-the-art of published research associated to combined mechanisms of conduction, convection and radiation.

In light of this bibliographic compilation, the influence of radiation on internal forced convection has been widely studied. Nevertheless, to the author's knowledge, the multiple flux model, an in particular, the four-flux model has not been applied to this problem before. The main goal of this investigation deals with the coupling of the four-flux model of Richter and Quack and the integro-differential equation of energy for laminar duct flow. As already mentioned, a direct by-product is an equivalent system of partial differential equations. This system is written in finite difference form following the methodology of Patankar (1980). Its solution is obtained by the efficient "Tri-Diagonal Matrix Algorithm", normally abbreviated as TDMA, recommended by Patankar.

3 PROBLEM FORMULATION

Consider a gas flowing through a circular duct as shown in Figure 1. Here, the physical model and its coordinate system is also illustrated.

The following assumptions apply to the formulation of this problem:
1) The flow is laminar and fully developed.
2) There exists of step change in wall temperature.
3) Thermal radiation in the gas is significant.
4) The gas is gray with constant properties.

Accordingly, the energy equation governing the temperature field of the gas may be written as follows

$$2\rho c p \bar{u}\left[1-(r^2/R^2)\right]\frac{\partial T}{\partial z} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) - \mathrm{div}\,\vec{q}^{R} \qquad (1)$$

where $\vec{q}^{R}$ designates the radiative heat flux at any point in the gaseous medium.

For the axi-symmetric case under consideration, $\vec{q}^{R}$ is given by

$$\mathrm{div}(\vec{q}^{R}) = \frac{1}{r}\frac{\partial}{\partial r}\left[r(q_r^+ - q_r^-)\right] + \frac{\partial}{\partial z}\left[q_z^+ - q_z^-\right] \qquad (2)$$

as depicted in Figure 2, where the physical significance of the symbols appear in the Notation.

Invoking the four-flux model developed by Richter and Quack, the equations for the relationship between heat fluxes and temperatures are

$$\frac{1}{r}\frac{\partial}{\partial r}(rq_r^+) = -k_aA_rq_r^+ + k_aB_r\sigma T^4 + \frac{q_r^-}{r} \qquad (3a)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rq_r^-) = k_aA_rq_r^- - k_aB_r\sigma T^4 + \frac{q_r^-}{r} \qquad (3b)$$

$$\frac{\partial q_z^+}{\partial z} = -k_aA_zq_z^+ + k_aB_z\sigma T^4 \qquad (3c)$$

$$\frac{\partial q_z^-}{\partial z} = k_aA_zq_z^- - k_aB_z\sigma T^4 \qquad (3d)$$

The parameters A_r, B_r, A_z and B_z in the preceding system of equations are related to the cone angle by

$$A_r = \frac{2\pi\cos\psi_o}{\pi-2\psi_o+\mathrm{sen}(2\psi_o)} \qquad (4a)$$

$$B_r = 2\cos\psi_o \qquad (4b)$$

$$A_z = \frac{2}{1+\cos\psi_o} \qquad (4c)$$

$$B_z = 2(1-\cos\psi_o) \qquad (4d)$$

Correspondingly, combining equations (1)-(3) gives rise to the energy equation written as follows

$$2\rho c p\bar{u}(1-r^2/R^2)\frac{\partial T}{\partial z} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + k_aA_r(q_r^+ + q_r^-) + k_aA_z(q_z^+ + q_z^-) - 2k_a\sigma T^4(B_r + B_z) \qquad (5)$$

At this point, it should be said that the adoption of the four-flux model transforms an integro-differential equation, that is, equation (1) into a system of five partial differential equations, given by equations (3) and (5) respectively.

This combined convective-radiative problem is subjected to a set of thermal boundary conditions. They are listed in two parts:

a) for temperature:

$$z=-\infty,\ T=T_i \qquad (6)$$

$$-\infty<z<\infty,\ r=0,\ \frac{\partial T}{\partial r}=0 \qquad (7)$$

$$z<0,\ r=R,\ T=T_i \qquad (8)$$

$$z>0,\ r=R,\ T=T_o \qquad (9)$$

b) for heat flux:

$$z=-\infty,\ q_z^+ = H_z\sigma T_{iq}^4,\ q_z^- = 0 \qquad (10)$$

$$z=+\infty,\ q_z^- = H_z\sigma T_{oq}^4,\ q_z^+ = 0 \qquad (11)$$

$$-\infty<z<\infty,\ r=0,\ q_r^+ = q_r^- \qquad (12)$$

$$z<0,\ r=R,\ q_r^- = H_r\varepsilon_w\sigma T_i^4 + (1-\varepsilon_w)\,q_r^+ \qquad (13)$$

$$z>0,\ r=R,\ q_r^- = H_r\varepsilon_w\sigma T_o^4 + (1-\varepsilon_w)q_r^+ \qquad (14)$$

4 NUMERICAL PROCEDURE

The system of partial differential equations, equations (3) and (5), along with the associated boundary conditions, equations (6)-(14), are solved by finite difference techniques. The methodology employed in this publication is based on the physically-oriented control volume approach proposed by Patankar. The resulting system of algebraic equations was solved by the "Tri-Diagonal Matrix Algorithm" (TDMA). This decision was supported by its high reliability and by the small CPU time required for convergence purposes. Further details of the methodology are given by Patankar, but Jarrín outlines specific aspects applied to this particular problem.

5 RESULTS

deSoto solved the applicable integro-differential equation of energy numerically. He employed an exponential band model for CO_2, which accounts for the dependence of both wavelength and temperature of the monochromatic absorption coefficient. Figure 3 illustrates by solid lines the local temperature distribution obtained by deSoto for three different axial positions. The dashed lines represent the corresponding results calculated by the present method for a cone angle of $\psi_o = 10°$. Although both methods characterize thermal radiation by totally different models, the results are in excellent agreement.

On the other hand, Echigo et al. analyzed the two - region-problem accounting for axial radiation effects. These authors used an iterative numerical scheme which employed Simpson's rule to compute the integral part of the energy equation. In Figure 4, bulk temperature distributions are compared for a typical case of strong radiation interaction: $\tau_R = 1$, $P_e = 1000$ and $N = 25$. It can be observed that both results are very close with a maximum discrepancy of 4%.

Total Nusselt numbers N_{uT} at local stations along the tube were calculated using the relation

$$N_{uT} = N_{uc} + N_{uR} = \frac{h_z D}{k} = \frac{D(q_c + q_R)|_{R=1}}{k(T_w - T_b)} \tag{15}$$

where convective Nusselt numbers N_{uc} are given by

$$N_{uc} = \frac{2\ \partial\theta|\partial\eta|_{\eta=1}}{\theta_w - \theta_b} \tag{16}$$

and radiation Nusselt numbers are given by

$$N_{uR} = \frac{2N}{\sigma T_E^4} \left(\frac{q_R|_{\eta=1}}{\theta_w - \theta_b}\right) \tag{17}$$

respectively.

Figure 5 shows the variation of the total Nusselt number along the pipe for the following conditions: $\tau_R = 1$, $P_e = 1000$ and $N = 2.5$. Comparison of the curves reveals reasonable good agreement with a maximum deviation of 25% at the origin. In general, the four-flux model tends to underpredict the total Nusselt number. However, such discrepancy is not surprising, because the origin displays a discontinuous change in wall temperature and also the temperature profile is distorted due to the upstream penetration of axial radiation.

The discrepancy in the calculation of the Nusselt number tends to decrease significantly as the radiation-conduction parameter N decreases. This was demonstrated by Jarrín (1983). He examined the effect of various convection and radiation parameters on bulk temperatures, radiative fluxes and Nusselt numbers.

As a concluding remark, it can be said that four flux models are of great utility for the engineering analysis of radiation components in forced convection problems.

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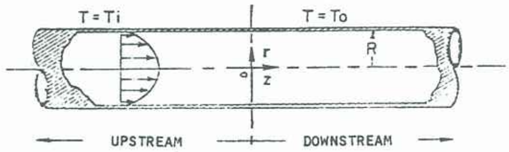

Figure 1 Physical system.

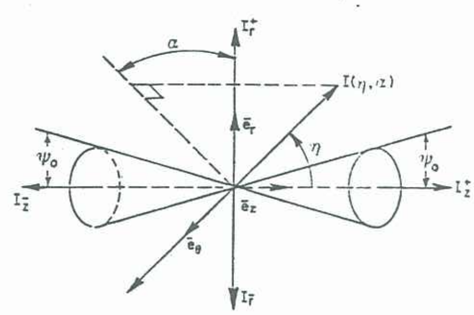

Figure 2 Four flux model.

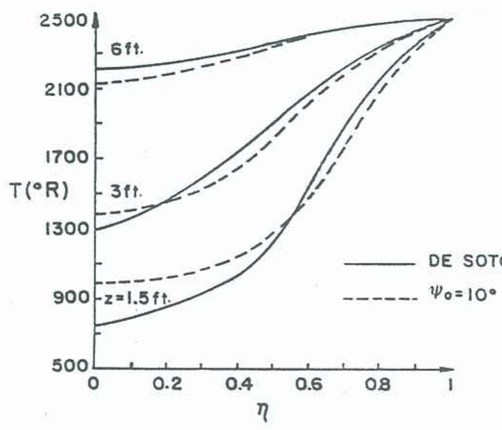

Figure 3 Radial temperatures.

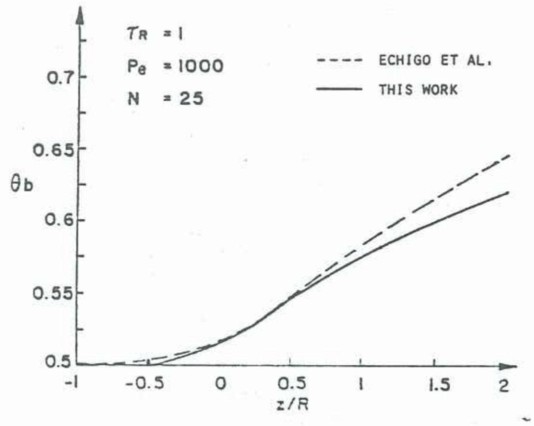

Figure 4 Mean bulk temperature.

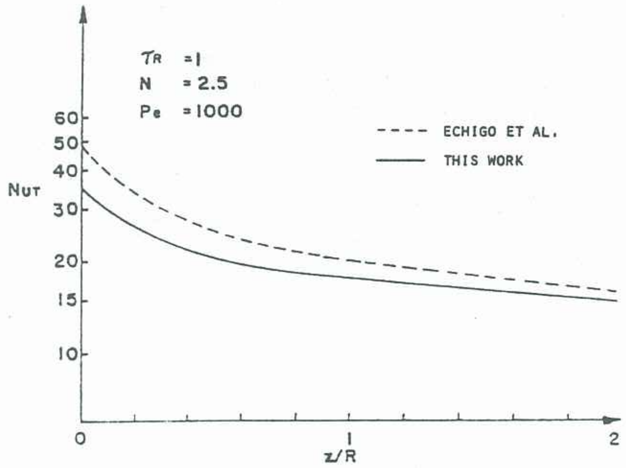

Figure 5 Total Nusselt number.