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The Validity of Backwater Models for Flood Routing Applications

C. ZOPPOU
Research Scholar, Department of Civil Engineering, The University of Melbourne
and
I.C. O'NEILL
Senior Lecturer, Department of Civil Engineering, The University of Melbourne

SUMMARY Backwater profile computer packages such as the U.S. Army Corps of Engineers' HEC-2 are being widely used in flood studies - in particular for simulating high water levels along river reaches. These "known-discharge" models consider the inflow hydrograph as a sequence of discrete discharge steps. Flows along the channel are frozen during each time interval and the water surface profile is determined by a backwater model. The equations used in these models can be considered as simplified versions of the complete one-dimensional Saint Venant equations, where the known-discharge assumption is used in lieu of the unsteady terms.

A case study is presented in this paper which shows that significant errors in simulated high water levels can result when a known-discharge model is used rather than a dynamic model based on the full Saint Venant equations.

1 INTRODUCTION

The last decade has seen increased use of one-dimensional models simulating unsteady open channel flows. Generally these models are based on the equations of continuity and motion which can be given in the form

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L \tag{1}$$

and

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial y}{\partial x} - S_o - S_f = 0 \tag{2}$$

where A = cross-section area,
Q = discharge,
q_L = lateral inflow per unit length,
S_f = friction slope,
S_o = bed slope,
t = time, and
x = distance along the channel.

These equations, commonly referred to as the Saint Venant equations, are first order partial differential equations of the hyperbolic type. Although there is no analytical solution for these equations they may be solved (together with appropriate initial and boundary conditions) using numerical techniques (Sevuk and Yen, 1973; Price, 1974; Liggett and Cunge, 1975).

The most general model for flood routing, the dynamic wave model, is obtained when all terms in the equation of motion (2) are retained. Two simpler but less general wave equations in wide-spread use follow when various terms are neglected in the equation of motion. The kinematic wave model neglects inertia and pressure gradient terms in (2) and results in a purely convective wave. The diffusion wave model follows when only the two inertia terms in (2) are neglected and results in a wave subject to convection and diffusion.

Henderson (1966) illustrated that the individual terms in (2) can differ in order of magnitude. Recent efforts, notably by Ponce et al (1977, 1978). Grijsen and Vreugdenhil (1976), have been made to establish criteria indicating flow and channel conditions necessary for the respective approximate methods to give acceptable representations of a flood wave.

Ponce's work was based on a linearised version of the Saint Venant equations and a subsequent study of the propagation characteristics of small sinusoidal disturbances according to kinematic diffusion and dynamic models.

Computer-based water surface profile models such as HEC-2 (1973) developed by the U.S. Army Corps of Engineers are being used increasingly in flood investigations - in particular to investigate maximum flood stages along a reach of river. This type of model is commonly referred to as a known-discharge model, since the method treats a continuous hydrograph as a sequence of discrete steady flow events. These discharges are considered to apply (with appropriate allowance for lateral inflow) along the entire river reach during successive computational intervals and a series of backwater profiles can be calculated. The known-discharge model follows from the Saint Venant equations, (1) and (2), if the unsteady terms in both equations are neglected. The implications of this assumption are examined in this paper.

2 CASE STUDY

2.1 Data Details

The reach chosen for this study was a well documented 72 km length of the Neuse River, North Carolina, from Goldsboro (upstream) to Kinstein (downstream). Gauging stations were located at each end of the reach and cross-sectional data were available for 14 intermediate sections equally spaced along the reach. For the 1954 flood, the inflow hydrographs, outflow hydrographs and mean rating curves at both Goldsboro and Kinstein were obtained from Chen (1973). Manning's roughness coefficient, n, was given as a function of flow depth for each end of the reach. The values were computed using Manning's equation together with the appropriate rating curve. The values of n at intermediate sections were obtained by linear interpolation.

It must be emphasised that these data were used in both models examined in this paper and no additional calibration was undertaken to reproduce the observed hydrographs. The channel bed slope, $S_o = 0.00133$, was assumed constant along the reach and was based on the bed elevations at Goldsboro and Kinstein. The lateral inflow was assumed constant along the reach and a value within the range 0.0005 to 0.002 $cumecs.m^{-1}$ suggested by the U.S. Geological Survey hydrologists was adopted.

The observed stage hydrographs at Goldsboro and Kinstein are shown in figure 1 and the single rating functions in Table 1.

2.2 Numerical Scheme

In this study the numerical scheme used for solving the full Saint Venant equations is the four point linear fully implicit scheme of Preissmann (Liggett and Cunge, 1975). Here the following finite difference quotients are used

$$f(x,t) \simeq \frac{\theta}{2}(f_{j+1}^{n+1} + f_j^{n+1}) + \frac{1-\theta}{2}(f_{j+1}^{n} + f_j^{n})$$

$$\frac{\delta f}{\delta x} \simeq \theta\,\frac{(f_{j+1}^{n+1} - f_j^{n+1})}{\Delta x} + (1-\theta)\,\frac{(f_{j+1}^{n} - f_j^{n})}{\Delta x}$$

$$\frac{\delta f}{\delta t} \simeq \frac{f_{j+1}^{n+1} - f_{j+1}^{n} + f_j^{n+1} - f_j^{n}}{2\Delta t}$$

where the subscript and superscript refer to distance and time respectively, and θ is a weighting coefficient. For the fully implicit scheme $\theta = 1$.

The resultant simultaneous equations are linearised and solved using the double sweep algorithm with appropriate boundary conditions.

2.3 Initial and Boundary Conditions

Before any computational scheme can proceed two initial and two boundary conditions must be specified in the implicit scheme. For the initial conditions, discharge, Q, and depth, y, are generally given at all computational points in the reach.

Boundary conditions are required to account for the influence of flow conditions both upstream and downstream of the reach concerned. At first glance several combinations of boundary conditions appear suitable and these include:

(a) y = F(t) at one boundary combined with Q(t) at the other boundary;
(b) Q(t) at both boundaries;
(c) y(t) at both boundaries; and
(d) y(t) or Q(t) at one boundary combined with a rating curve at the other boundary.

Unfortunately some combinations such as (b) and (c) can lead to difficulties as indicated by Cunge (1969).

From a practical point of view perhaps the most useful boundary conditions consist of an upstream stage hydrograph (reflecting storm runoff in the upper regions of a river basin) and a downstream rating curve. If a single-valued rating function corresponding to steady flow is used, then the computed results of the unsteady flow in the reach upstream of this condition are biased within the range of the backwater influence. Fortunately, this backwater influence does not propagate a large distance upstream and affect the results significantly. The problem can be further reduced by placing the unsatisfactory condition as far downstream as possible.

2.4 Dynamic Model

The solution of the Saint Venant equations using the linear implicit scheme is well documented. For the Neuse River the upstream boundary condition is a stage hydrograph and the downstream boundary condition a single-valued rating curve. The stage hydrograph at Kinstein was simulated using a computational time interval of two hours. Two simulations were made, one with a lateral inflow, $q_L = 0.0009$ $cumecs.m^{-1}$ and one with $q_L = 0.0$. The results are shown in figure 1. Excellent agreement between simulated and observed peak stage was obtained when lateral inflow was considered and poor agreement when it was neglected. The poor simulation of hydrograph shape could be caused by assuming a constant rate of lateral inflow over the routing period.

2.5 Known-Discharge Model

Known-discharge models such as HEC-2 are explicit, in that the computed stage depends exclusively on the previously calculated stage. However, in this study the unsteady model will be used and operated as a known-discharge model in simulating the peak (or high water) levels along the Neuse River reach. These levels will then be compared with those obtained using the complete unsteady (dynamic) form of the model.

The unsteady terms in the dynamic model must be eliminated if it is to be reduced to a known-discharge model. The unsteady terms vanish from the dynamic model if a constant stage is used as the upstream boundary condition and a single-valued rating curve used as the downstream boundary condition. In considering high water levels the sustained upstream stage is of course the peak stage from the observed stage hydrograph at Goldsboro.

It is necessary to have two boundary conditions in the implicit scheme irrespective of the model concerned. Its use is not inconsistent with the backwater profile computation approach used in traditional known-discharge models, since implicit schemes use all values within the reach, including both boundary conditions. A high water profile for the Neuse River is shown in figure 2, where the upstream stage corresponds to the peak inflow stage.

The computations were also performed in the upstream direction from Kinstein using the calculated peak stage at Kinstein (obtained from the simulated profile, A, shown in figure 2) as the downstream boundary condition. The two profiles were

TABLE 1

RATING CURVES AT GOLDSBORO AND KINGSTEIN

Depth y in m	Goldsboro Q in cumecs	Kinstein Q in cumecs	Depth y in m	Goldsboro Q in cumecs	Kinstein Q in cumecs
0.5	6.3	7.0	5.0	242.4	326.1
1.0	21.8	28.2	5.5	284.6	439.1
1.5	41.5	39.0	6.0	337.6	560.9
2.0	60.2	64.3	6.5	409.6	687.5
2.5	82.6	94.7	7.0	497.1	833.5
3.0	110.7	124.4	7.5	634.8	993.7
3.5	140.7	157.9	8.0	789.0	1153.9
4.0	172.1	196.7	8.5	951.6	1314.2
4.5	205.0	248.8			

almost identical, verifying the point that the use of the implicit scheme to solve the known-discharge model is independent of the direction of computation. However the direction of computation is important whenever the traditional explicit formulation of the known-discharge model is used. Here, for subcritical flows, computation should proceed upstream, since any errors introduced in the downstream boundary condition (control point) will not significantly affect the results. This aspect is illustrated in figure 3 where three high water profiles have been calculated using the implicit scheme, with peak stages upstream and downstream as boundary conditions. Profile C is the water surface profile obtained using the measured peak stage values at Goldsboro and Kinstein. Profiles A and B were obtained after introducing an error of 0.3 m in the respective peak stages at Goldsboro and Kinstein. The error introduced at the downstream boundary can be seen to have little significance on the calculated profile which eventually converges to the correct solution. However the errors introduced at the upstream boundary can be seen to propagate throughout the reach producing an erroneous profile.

3 COMPARISON OF MODELS

In figure 2, the high water level profile was obtained using the dynamic model and indicated close agreement with the observed peak stage downstream at Kinstein. A comparison of profiles A and C indicates that the use of the known-discharge model approach can lead to significant errors when used to simulate high water levels along a river reach. The errors can be attributed to the model formulation and are

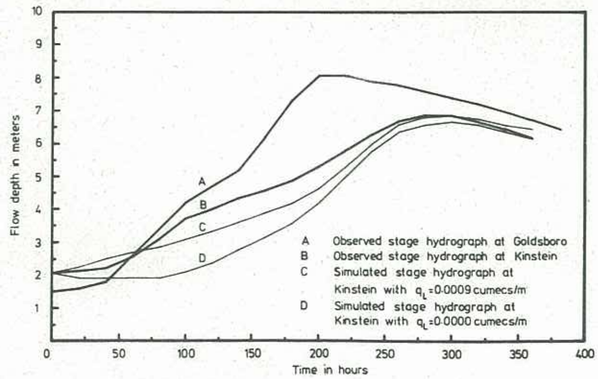

Figure 1 Simulated stage hydrographs for the 1954 flood at Kinstein.

a consequence of introducing the known-discharge assumption which neglects the unsteady terms in the Saint Venant equations. The implicit scheme was used here for the known-discharge model to enable direct comparison with the dynamic model. This avoids introducing uncertainties associated with traditional backwater model packages (Motayed and Dawdy, 1975) - a topic outside the scope of the present paper.

It would be useful to establish guidelines indicating when the known-discharge model is likely to give unsatisfactory simulations of high water levels. Unfortunately this model does not lend itself to the type of analytical approach used by Ponce in establishing limits of application for diffusion and kinematic models. However, the same parameters are likely to be relevant and include the flood hydrograph duration, the channel bed slope and the flow depth.

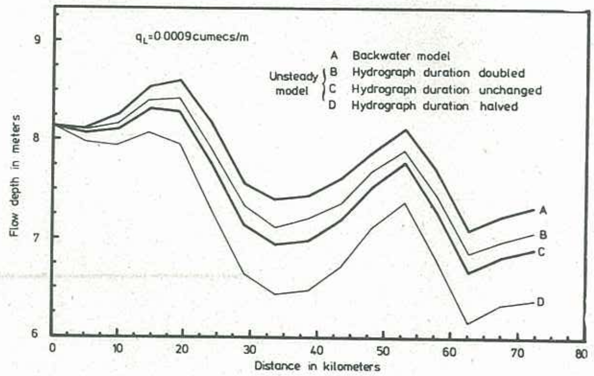

Figure 2 Simulated high water levels for the Neuse River.

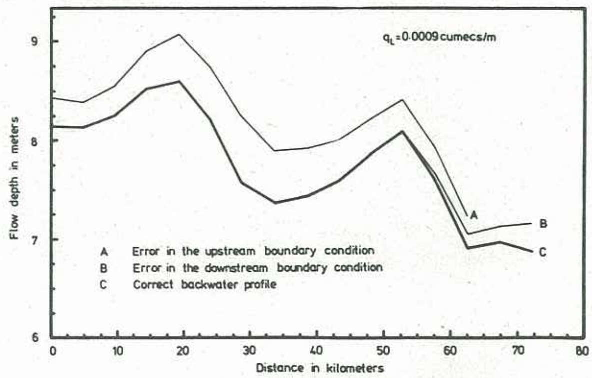

Figure 3 Propagation of errors in the backwater profile.

An indication of the sensitivity of the high water level simulation to changes in the flood hydrograph duration, T, is illustrated in figure 2, where profiles B and D indicate the effects of hypothetical changes in T. Increasing T moves the profile closer to the known-discharge result as anticipated. Reducing T leads to an increase in the importance of dynamic effects which is accompanied by an associated decrease in simulated flow depths and an increase in simulated discharge.

4 CONCLUSIONS

By neglecting dynamic effects in the Saint Venant equations, known-discharge (or backwater) models, widely used in flood studies, can lead to significant errors in simulated high water levels. Further studies are needed before guidelines for estimating these errors in particular situations can be established.

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