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A Predictive Model of Tropical Cyclone Wind-Waves

I.R. YOUNG
Student, Department of Civil and Systems Engineering, James Cook University of North Queensland
and
R.J. SOBEY
Senior Lecturer, Department of Civil and Systems Engineering, James Cook University of North Queensland

SUMMARY A numerical model describing the generation, propagation, interaction and decay of wind-waves forced by a tropical cyclone is presented. It is based on air-sea interaction theories and a field solution of the Radiative Transfer Equation. The nodal variable is the directional energy spectrum and the model is applicable in shallow as well as deep water. Results for a deep water prediction are included.

1 INTRODUCTION

Recently, increased construction activity in both coastal and offshore regions has highlighted the potentially destructive and almost explosive nature of the marine environment. The sea response to extreme meteorological events is itself extreme, and no more so than the surface gravity wave response to tropical cyclones (or hurricanes or typhoons). The forced gravity wave spectrum is largely concentrated in two bands, corresponding to storm surge and wind waves respectively. Storm surge is more intense in nearshore waters but wind wavesof potentially destructive magnitudes are generated in both offshore and coastal waters.

Offshore mining activity is frequently located in notorious tropical cyclone regions, such as the Gulf of Mexico or Australia's North West Shelf, and associated offshore constructions and operations must contend with tropical cyclone wind waves. Nearshore and coastal facilities as well as numerous coastal settlements in tropical waters must withstand the combined effects of tropical cyclone storm surge and the superimposed wind waves, the storm surge providing the vehicle to intensify wave damage in coastal regions through the sustained super-elevation of the mean water level. The human and material devastation during Hurricane Camille in 1969 along the U.S. Gulf Coast and in 1971 in Bangladesh bear witness to the destructive potential.

Common practice in the prediction of tropical cyclone wind waves is largely empirical, being based on the well known forecasting curves of Sverdrup, Munk and Bretschneider . In the long term however, an approach based on wave energy conservation and air-sea interaction theories is the only real alternative. A predictive numerical model along such lines is described in this paper. It is called SPECT.

2 RADIATIVE TRANSFER EQUATION

An appropriate representation of complex ocean wave conditions is the directional wave energy density spectrum,

$$E = E\,(f,\theta;x,y,t) \qquad (1)$$

which is assumed to be a slowly-varying function of position and time. At each position and time, E represents the superposition of free linear wave components of all frequencies and from all directions. The wave energy conservation may be written

$$\begin{aligned} \frac{dE}{dt}(f,\theta;x,y,t) &= \frac{\partial E}{\partial t} && \text{(temporal accumulation)} \\ &+ \frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} && \text{(propagation)} \\ &+ \frac{\partial E}{\partial \theta}\frac{d\theta}{dt} && \text{(refraction/shoaling)} \quad (2) \\ &= S \end{aligned}$$

(2), known as the Radiative Transfer Equation (Hasselmann, 1968) formally summarises all the various physical processes that contribute to the evolution of the directional energy density spectrum. It is essentially a Lagrangian equation describing the rate of change of the energy density relative to a wave group moving along wave orthogonals at the group velocity. The so-called characteristic equations defining the wave orthogonals are

$$\frac{dx}{dt} = C_g \cos\phi \qquad (3a)$$

$$\frac{dy}{dt} = C_g \sin\phi \qquad (3b)$$

$$\frac{d\theta}{dt} = \frac{C_g}{C}\left[\frac{\partial C}{\partial x}\sin\phi - \frac{\partial C}{\partial y}\cos\phi\right] \qquad (3c)$$

In deep water the x and y derivitives of the wave celerity are zero so that (3c) is identically zero and the wave orthogonals are straight lines, but this is not generally the case in shallow water where the bathymetry bends or refracts the orthogonals. The final term in (2) is the source term representing all processes which transfer energy to (or from) the spectrum. Should this term be zero, each discrete component of the spectrum will propagate along its appropriate orthogonal without change. A numerical solution of (2) would completely describe the evolution of the wave field in both space and time.

3 ENERGY SOURCE TERMS

Following Hasselmann (1968), the energy source involves the superposition of a number of individual influences. The dominant terms are listed in Table 1 and represent the commonly identified physical processes contributing to the growth, decay and interaction of the separate spectral components.

TABLE 1

ENERGY SOURCE TERMS

Source Term	Description
S_1	Constant energy transfer to wave field through turbulent atmospheric pressure fluctuat-ions
S_2	Increasing energy transfer to wave field through instability in coupling between waves and the mean atmospheric boundary layer flow
S_5	Energy transfer within wave field through weakly non-linear wave-wave interactions
S_6	Energy dissipation in shallow water through turbulent bottom friction
S_7	Energy dissipation through wave breaking or white capping

A comprehensive review and evaluation of these five source terms is given elsewhere (Young and Sobey, 1980) but the general character of these terms is illustrated through Figures 1 to 4. S_1 and S_2 are frequency-selective growth mechanisms. The Phillips mechanism S_1 is a resonance effect, peaking at spectral components with a wave celerity approximately equal to the near-surface wind velocity as indicated in Figure 1. It is a linear growth term and is largely responsible for the initial growth of the spectrum. The Miles term S_2 is an instability mechanism leading to a continuing exponential growth. S_5 represents the interaction among spectral components. As indicated in Figure 3, it results in a redistribution of wave energy from the peak, to both lower and higher frequencies. The term S_6 represents wave energy dissipation to turbulent bottom friction. This term is important only in shallow water and is most intense for the shorter frequency spectral components, as shown in Figure 4. The final term, S_7 introduces white capping or wave breaking through the concept of the saturation spectrum. White capping is the limiting process controlling otherwise unbounded wave growth represented by the S_1 and S_2 mechanisms.

Just what is the correct representation for each of these terms is still the subject of some controversy. A number of rather empirical representations have been proposed in the literature and our present choice of terms is based on as objective an analysis as is presently possible. The details are rather lengthy and are listed elsewhere (Young and Sobey, 1980). No claim is made that our choice or indeed any choice from the presently available alternatives is entirely satisfactory. There are particular difficulties in representing wave growth at high wind speeds, wave decay to opposing winds and the steep spatial gradients of tropical cyclone forcing. These aspects are the basis of our continuing research.

The meteorological forcing is directly related to the near-surface wind field and the NHRP model describing the space and time variable wind field has been adopted. The details of this model have been documented by Sobey, Harper and Stark (1977), in the context of a parallel predictive model of tropical cyclone storm surge.

4 NUMERICAL SOLUTION OF RADIATIVE TRANSFER EQUATION

An appropriate numerical solution algorithm for (2) must accommodate a number of conflicting demands. It is essentially a convective transport problem but wave energy is convected along ray paths that are straight lines only in deep water. In addition any field solution of the radiative transfer equation that allows a reasonable resolution of the directional energy spectrum at each node imposes massive demands on computer system resources. Discrete representations of convective transport problems lead to often severe numerical dispersion and associated solution oscillations. The remedy is higher order representations but this leads to even heavier demands on computer system resources, just what must be avoided. Some compromise is essential. Our solution is a combination of "bit packing" of the spectral components at each node and a fractional step algorithm combining the method of characteristics for a propagation step with an analytical solution for a forcing step.

Following Richtmyer and Morton (1967), Equation 2 may be represented as

$$\frac{\partial E}{\partial t} = (A_1 + A_2)\,E \tag{4a}$$

where the A operators are respectively

$$A_1 E = \frac{\partial E}{\partial x}\frac{dx}{dt} - \frac{\partial E}{\partial y}\frac{dy}{dt} - \frac{\partial E}{\partial f}\frac{df}{dt} - \frac{\partial E}{\partial \theta}\frac{d\theta}{dt} \tag{4b}$$

$$\text{and}\quad A_2 E = S_1 + S_2 + S_5 + S_6 + S_7 \tag{4c}$$

The commonly accepted form of all five forcing terms can be represented generally as a + bE, whence Equation 4c becomes

$$A_2 E = a + bE \tag{4d}$$

In the fractional step algorithm, the two operations represented by A_1 (propagation, refraction and shoaling) and A_2 (forcing) are given separate and consecutive consideration over each time step Δt. The *propagation step* advances the solution from E^n to $E^{n+\frac{1}{2}}$ through solution of the partial differential equation

$$\frac{\partial E}{\partial t} = 2A_1\,E \tag{5a}$$

The *forcing step* advances the solution from $E^{n+\frac{1}{2}}$ to E^{n+1} through solution of the partial differential equation

$$\frac{\partial E}{\partial t} = 2\,(a + bE) \tag{5b}$$

The propagation step, Equation 5a, describes propagation of wave energy at the group velocity and without loss along appropriate wave rays, which are defined by the characteristic Equations (3a) to (3c). Thus it is necessary to construct a wave ray for each frequency, direction band at each nodal point in the computational field. In a simulation involving 10 frequency and 16 directional bands at each of 225 nodes, 36000 ray diagrams must be constructed. Since current refraction is not presently considered, the ray diagrams need only be computed once. Utilising then the method of characteristics Equation 5a becomes

$$\frac{dE}{dt} = 0 \tag{5c}$$

along the respective wave rays. Linear interpolation is presently utilised at time $n\Delta t$ because of the extensive demands on computer system resources. Higher order interpolation is readily accommodated within SPECT but at a severe computational penalty.

The forcing step achieves maximum computational efficiency through a local analytical solution to Equation 5b,

$$E^{n+1} = E^{n+\frac{1}{2}} e^{b\Delta t} + a \left(\frac{e^{b\Delta t} - 1}{b}\right) \qquad (5d)$$

The term in brackets is evaluated through a truncated power series expansion to avoid difficulties when b is near zero.

The numerical model, SPECT, has been programmed for the James Cook University's DEC System-10 computer (KL10-E processor). The model is large and computationally expensive, requiring 60,000 thirty-six bit words of storage and approximately thirty minutes for twelve hours of real time. The program

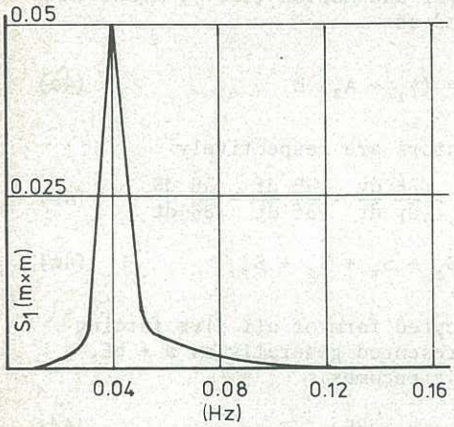

Figure 1 Phillips Mechanism, S_1

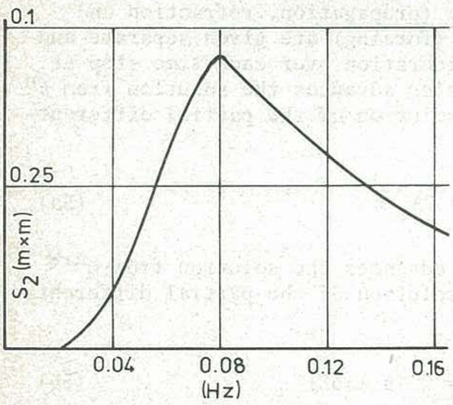

Figure 2 Miles Term, S_2

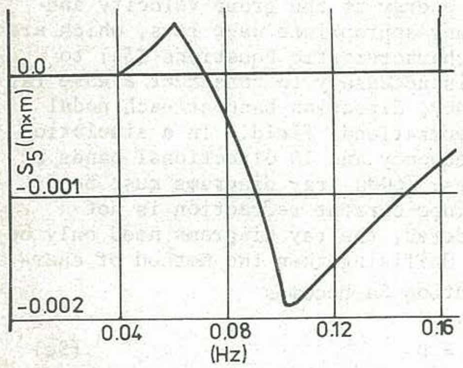

Figure 3 Wave-wave Interactions, S_5

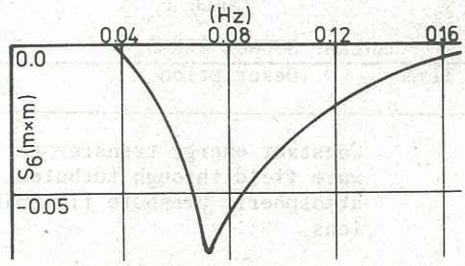

Figure 4 Turbulent Bottom Friction, S_6

has been written in a general fashion, allowing the representation of any coastal, offshore or undersea features required for a particular application.

As in any propagation or initial value problem, it is necessary to specify both boundary and initial conditions. In SPECT, the sea is assumed to be perfectly calm initially and the central pressure of the tropical cyclone is then gradually decreased to initialise the sea conditions prior to active simulation. Current boundary conditions are of two types: radiation boundaries, at which energy can propagate out of but not into the computational field and active boundaries, where the energy spectrum is represented in parametric form (Hasselmann et. al., 1976). Land boundaries are represented as points with zero energy.

5 A DEEP WATER HINDCAST

For the purposes of illustration aspects of a deep water wave prediction are presented. The tropical cyclone has a central pressure of 950 mb and a forward speed of 30 km/hr.

The distribution of significant wave heights is shown in Figure 5, where the superimposed vectors

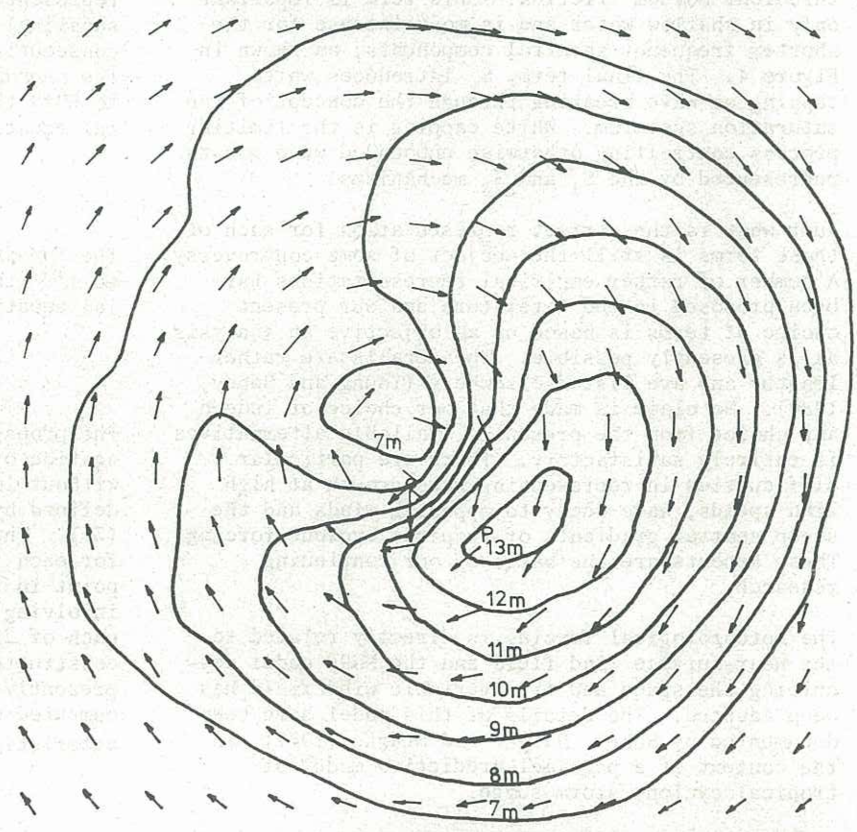

Figure 5 Distribution of Significant Wave Height

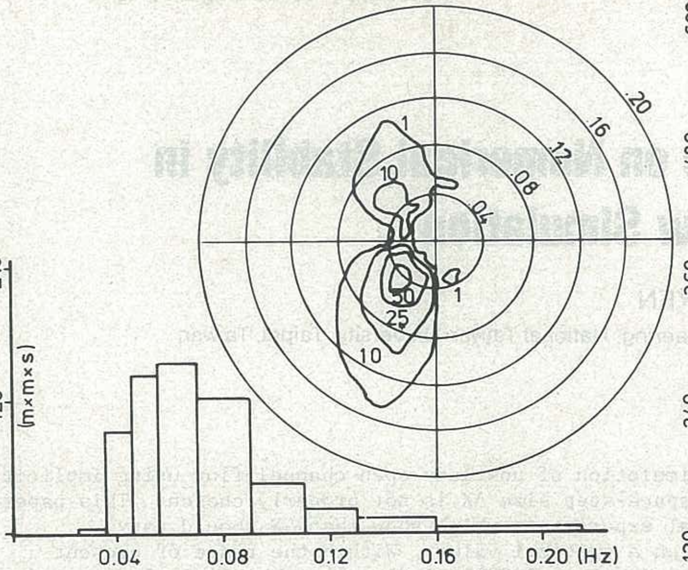

Figure 6 One and Two Dimensional Spectra in Eye of Tropical Cyclone

indicate the mean direction of propagation of the waves. Wave conditions are most severe in the crescent shaped area in the left front quadrant of the storm, consistent with the very limited field results that are available. It is almost certain that the peak magnitudes are too high and this is related to the present lack of data at the high wind speeds experienced within a tropical cyclone, as already mentioned in Section 3.

The dominant directions are closely related to the directional characteristics of the wind field as one might expect, at least initially. However recent measurements by King and Shemdin (1978) have cast doubt on such a prediction. These measurements were obtained from airborne synthetic aperture radar in the Gulf of Mexico and are the first really comprehensive pictures of the directional field characteristics of tropical cyclone wind waves. The striking feature of the results is the lack of correlation between the local wind direction and the direction of dominant wave propagation. They are contradictory to the predictions of this and all other published numerical models, although Shemdin's results do not as yet encompass a fully mature hurricane. If his results are subsequently confirmed, the energy source terms may require extensive revision. This aspect is a topic of current research.

In addition to the significant wave height distribution over the computational grid, detailed spectral predictions at specified points are also available from SPECT. Typical results of both one and two dimensional spectra are shown in Figures 6 and 7. Figure 6 is for a point within the eye of the storm whereas Figure 7 is for the point of maximum significant wave height. Although the one-dimensional spectra are similar for these two points the two-dimensional spectra are radically different. For the point within the eye the two-dimensional spectrum indicates a very confused sea condition with waves propagating in a number of directions. This relationship is perhaps more consistent with Shemdin's results. Further from the eye, however, there is more order with the direction of propagation being distributed about the wind direction.

7 CONCLUSIONS

Numerical models such as SPECT represent the first real attempts at developing a wave prediction technique with a sound theoretical base. However, the adequacy of such models remains difficult to determine as suitable field data in tropical cyclones is very limited.

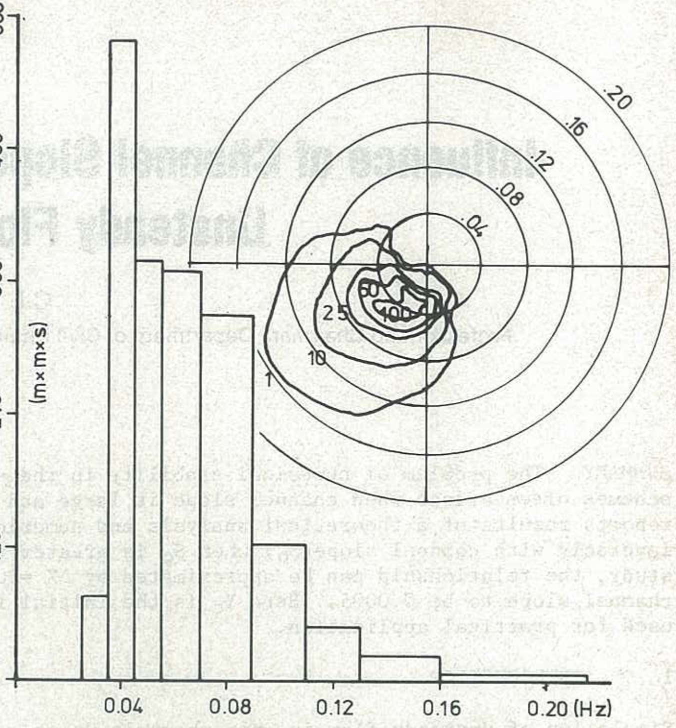

Figure 7 Spectra at Point P (Figure 5)

Almost all of the data that is available is in the form of one-dimensional spectra at isolated points (eg. the ODGP data during Camille and other hurricanes in the Gulf of Mexico). Comprehensive data sets comprising directional characteristics as well as magnitude are necessary before the present uncertainties may be answered with any confidence. SPECT has been designed for continuing research in tropical cyclone wind waves but nonetheless remains a comprehensive and flexible state-of-the-art predictive model.

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