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Comparison of Immiscible and Miscible Fluid Models for Sea Water Intrusion in Aquifers

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SUMMARY Two methods are outlined for predicting the location of the sea water interface in an aquifer discharging to the sea. One method assumes an abrupt interface when no mixing of the salt and fresh water occurs and the other accounts for dispersion which produces a mixing zone where the two waters meet. For the immiscible fluid case, boundary integral and finite element methods are used to determine the interface location while for the miscible fluid case the relevant equations are solved by finite difference techniques. The effects of different dispersion coefficients are evaluated with the miscible fluid model and the locations and shapes of the interface zones are compared with results from the abrupt interface assumption.

1 INTRODUCTION

When a coastal aquifer discharges fresh groundwater to the sea a mixing zone or interface occurs where fresh water meets the salt water. The location, shape and thickness of this zone depends on the groundwater discharge rate, the tidal range and the hydrogeologic properties of the aquifer. One method of determining the interface location is to assume that salt and fresh water are immiscible and to solve the relevant flow equation in each fluid separately. The interface location is determined so that the boundary conditions for both fluids are satisfied. The immiscible fluid assumption was used by Henry (1959) in obtaining solutions by the hodograph method. Cantatore and Volker (1974) presented a finite element formulation for the problem which allowed anisotropy and nonhomogeneity to be catered for as well as the presence of pumping or recharging wells in the vicinity of the interface. Volker and Young (1979) compared the finite element and boundary integral solution techniques for this problem.

In a real aquifer there will always be some dispersion resulting in a finite thickness of interface and hence the abrupt interface can only be expected to give a reasonable indication of the salt water position when dispersion is small. If mixing of the two waters is significant the salt distribution can be predicted by combining a flow equation with the convective-dispersion equation for mass transport of the salt. For real aquifer conditions these equations are usually solved by numerical methods such as finite elements (Lee and Cheng, 1974; Huyakorn and Taylor, 1976) or finite differences (Volker, 1979).

This paper compares results obtained from both the immiscible and miscible fluid assumptions.

2 MATHEMATICAL FORMULATION

2.1 Immiscible Fluid Assumption

When a line interface between fresh and salt water is assumed then, for either fluid, the concentration and density are constant and Darcy's law can be combined with the mass conservation equation to yield the flow equation to be solved. The general form of this equation is

$$\frac{\partial}{\partial x_i}\left(K_{ij}\frac{\partial h}{\partial x_j}\right) = 0 \qquad (1)$$

where h is the piezometric or hydraulic head [L]; K_{ij} is hydraulic conductivity of the aquifer medium [LT^{-1}]; and x_i, x_j are Cartesian coordinates [L].

For a two-dimensional section of a homogeneous isotropic aquifer (1) reduces to the Laplace equation (Cantatore and Volker, 1974):

$$\partial^2 h/\partial x_1{}^2 + \partial^2 h/\partial x_2{}^2 = 0 \qquad (2)$$

To describe the boundary conditions, consider Figure 1 which represents schematically a vertical section through the seaward end of a coastal confined aquifer.

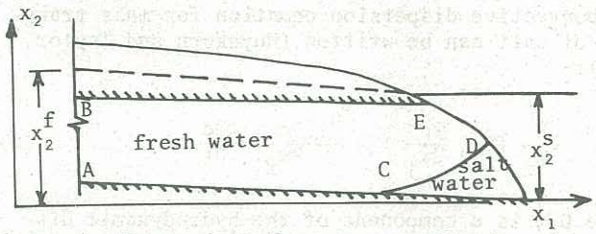

Figure 1 Abrupt interface in a confined aquifer

For the steady state position of the interface CD there will be no flow in the salt water region and CD can therefore be considered as an impervious boundary section for the fresh water flow field. The upper and lower confining strata BE and AC are also impervious boundaries for steady flow. This condition is represented by

$$K_{ij}\frac{\partial h}{\partial x_j} n_i = 0 \qquad (3)$$

where n_i is the component in the x_i direction of the unit outward normal to the boundary.

The interface CD must have equal pressures in the fresh and salt water for equilibrium which leads to (Cantatore and Volker, 1974):

$$h = x_2 + \frac{\rho_s}{\rho_f}(x_2^s - x_2) \qquad (4)$$

where ρ_s and ρ_f are the mass densities of salt and fresh water respectively; and h is head at a point on the interface whose vertical coordinate is x_2.

On AB a constant head or constant flux is specified while on ED the head distribution is also given by (4).

2.2 Miscible Fluid Assumption

The miscible fluid model requires the solution of two equations, the flow equation and the convective-dispersion equation, to describe the contaminant movement. Huyakorn and Taylor (1976) defined hydraulic head h as:

$$h = p/(\rho_f g) + x_2 \tag{5}$$

where p is pressure $[ML^{-1}T^{-2}]$; g is acceleration due to gravity $[LT^{-2}]$.

By combining Darcy's law with the continuity equation, the ground water flow equation can be derived as:

$$\frac{\partial}{\partial x_i}\left[K^f_{ij}\left\{\frac{\partial(h-x_2)}{\partial x_j} + \frac{\rho g_j}{\rho_f g}\right\}\right] = 0 \tag{6}$$

where $K^f_{ij} = k_{ij}\rho_f g/\mu$ and represents hydraulic conductivity of the aquifer medium to fresh water $[LT^{-1}]$; k_{ij} is intrinsic permeability $[L^2]$; and μ is viscosity $[ML^{-1}T^{-1}]$

The density ρ is expressed in terms of the concentration c using:

$$\rho/\rho_s = 1 + \varepsilon c' \tag{7}$$

where $\varepsilon = (\rho_s - \rho_f)/\rho_f$; and $c' = c/c_s$, where c_s is salt concentration in sea water.

Density ρ can thus be eliminated from (6) leaving the equation in terms of h and c as the only unknowns.

The convective dispersion equation for mass transport of salt can be written (Huyakorn and Taylor, 1976):

$$\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial c}{\partial x_j}\right) - \frac{u_i}{\theta}\frac{\partial c}{\partial x_i} = \frac{\partial c}{\partial t} \tag{8}$$

where D_{ij} is a component of the hydrodynamic dispersion coefficient tensor $[L^2T^{-1}]$; θ is porosity of aquifer material; u_i is a Darcy velocity component $[LT^{-1}]$; c is salt concentration $[ML^{-3}]$; and t is time.

Neglecting molecular diffusion, the dispersion coefficient is usually assumed to be a tensor with components given by:

$$D_{ij} = D_T\delta_{ij} + (D_L - D_T)\frac{v_i v_j}{v^2} \tag{9}$$

where v_i, v_j are pore velocity components (u_i/θ and u_j/θ respectively); v is resultant pore velocity magnitude; δ_{ij} is Kronecker delta; and D_L, D_T are effective longitudinal and lateral dispersion coefficients respectively, given by:

$$D_L = d_1 v \tag{10a}$$

$$D_T = d_2 v \tag{10b}$$

where d_1, d_2 are dispersivity components [L].

Where the width of the dispersion zone is influenced by local nonhomogeneities and other factors such as tidal variations it may be more convenient to use a scalar dispersion coefficient of sufficient magnitude to account for these factors (Lee and Cheng, 1974; Huyakorn and Taylor, 1976). Solutions are presented in this paper for both cases for comparison purposes.

Equations (6) and (8) have to be solved in conjunction to determine the salt distribution in the interface zone. The boundary conditions applicable for a confined aquifer are shown in Figure 2.

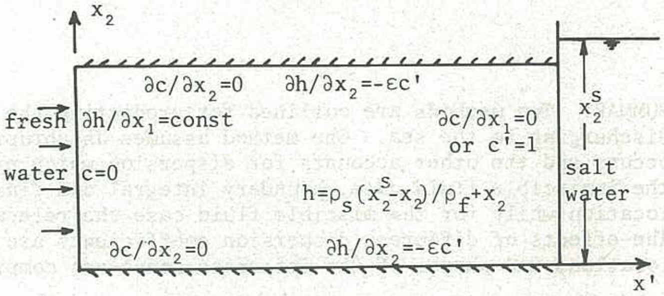

Figure 2 Boundary conditions for miscible fluid model

On the upper and lower confining strata the boundary conditions preclude flow of water or salt across the boundaries. On the sea water boundary a zero concentration gradient is set where water flows out of the aquifer and for the remainder the concentration is that of sea water. Further details are given by Volker (1979).

3 SOLUTION TECHNIQUES

3.1 Boundary Integral and Finite Element Methods

For the abrupt interface approximation the governing equation in the fresh water flow field can be solved by the finite element technique. The Galerkin weighted residual method is used and Green's theorem is applied to reduce second order differentials to first order ones. The general procedure is outlined by Zienkiewicz (1977) and the particular application to seepage and salt water intrusion problems is explained in more detail by Volker and Young (1979).

The boundary integral method presented by Liggett (1977) can also be used for this problem. This procedure involves a discretization of only the boundary of the flow field and hence has some advantages in solving problems where the location of at least one boundary is unknown because it is easier to relocate the boundary elements in the solution process than it is to readjust a mesh of elements covering the whole flow domain. Details of the method are given by Liggett (1977) and examples of its application to salt water intrusion problems are presented by Volker and Young (1977).

3.2 Finite Difference Method

For the miscible fluid model solutions have been obtained by a finite difference method. With high velocities or low dispersion coefficients, the solution of the convective dispersion equation can lead to numerical difficulties usually referred to as numerical diffusion and dispersion. In order to overcome these problems a method modified from that suggested by Spalding (1972) has been used. Further details are given by Volker (1979) where it is also shown that sufficient accuracy in the solutions can be obtained by using an appropriate finite difference mesh size.

4 RESULTS

The first problem studied was that of a homogeneous isotropic confined aquifer 50 m thick with a constant scalar dispersion coefficient. A vertical outflow face was assumed and a 100 m long horizontal section was adopted for the finite difference mesh. This is a similar geometric configuration to that used by Huyakorn and Taylor (1976) and other investigators before them. Other relevant parameters are: K^f (a scalar in this case) = .001 ms^{-1}; $D = 1.10 \times 10^{-4}\ m^2s^{-1}$ ($9.47\ m^2day^{-1}$); $\theta = .30$; $\varepsilon = .025$; upstream freshwater head gradient = -.006575. The results in terms of dimensionless concentration contours are shown in Figure 3. Also shown in Figure 3 is the abrupt interface determined in this case by the boundary integral method; the finite element solution gave an almost identical result. It can be seen that the abrupt interface location is very different from that of the mean isochlor ($c' = .5$) line. This is because there is actually a circulation set up within the salt water region to replenish salt that is "skimmed" off by fresh water in the upper region of the interface. This effect is not accounted for by the immiscible fluid model which therefore predicts a greater intrusion length than the miscible fluid model.

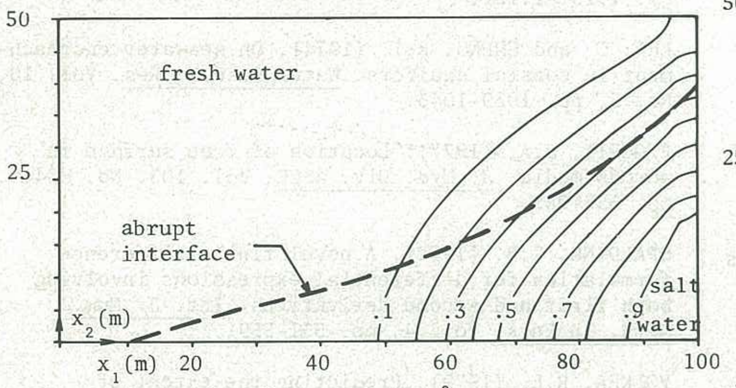

Figure 3 Results for $D = 9.47\ m^2$ per day

The effect of decreasing the dispersion coefficient was then investigated. Figure 4 shows dimensionless concentration contours for the same conditions as Figure 3 except that $D = 1.67 \times 10^{-5}\ m^2s^{-1}$ ($1.44\ m^2\ day^{-1}$). Figure 5 shows a further decrease in dispersion coefficient to $D = 3.35 \times 10^{-6}\ m^2s^{-1}$ ($0.29\ m^2day^{-1}$). The immiscible fluid (abrupt interface) result is also reproduced on these figures and it is observed that as expected, the two solutions converge as D decreases. For the smallest dispersion coefficient used (Figure 5) there is good correspondence between the solutions.

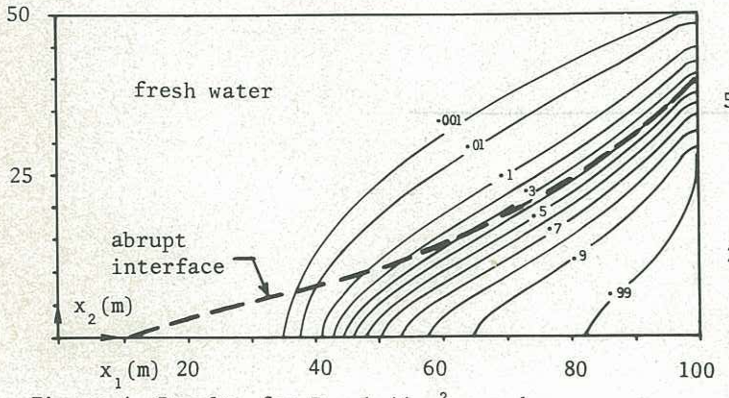

Figure 4 Results for $D = 1.44\ m^2$ per day

It should be noted that the abrupt interface solution is far less expensive on computer resources than are the miscible fluid results shown in Figures 3 to 5 and it also requires less information on aquifer properties. There is therefore an incentive to use the immiscible fluid model if it is sufficiently accurate. Results similar to those in Figures 3 to 5 would provide the data from which to assess the applicability of the abrupt interface result for a particular aquifer.

The influence of anisotropic dispersion was studied by introducing the dispersion coefficient tensor with components defined by (9). Results are given in Figure 6 for the same aquifer properties as in Figures 3 to 5 except that the tensor dispersion coefficient was used with $d_1 = 25$ m and $d_2 = 8.33$ m {see (10a) and (10b)}. The abrupt interface location is also shown in Figure 6 for comparison purposes.

The effect of decreasing the dispersivities is shown in Figure 7 where the values of d_1 and d_2 are 5 m and 1.67 m respectively. Although the shapes of the interface zones are different from the scalar dispersion coefficient case, the change produced by decreasing the dispersivities is similar to that produced by using a small scalar coefficient.

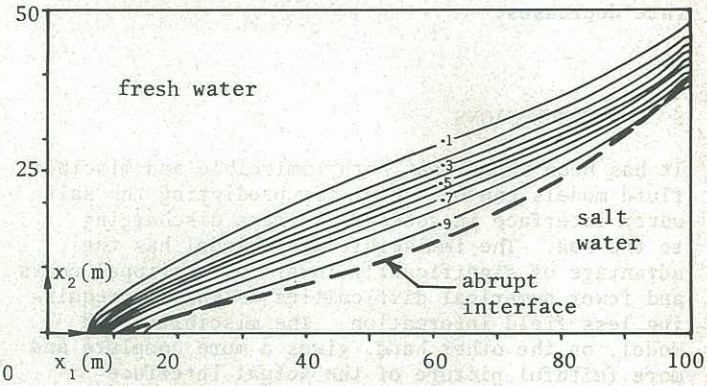

Figure 5 Results for $D = 0.29\ m^2$ per day

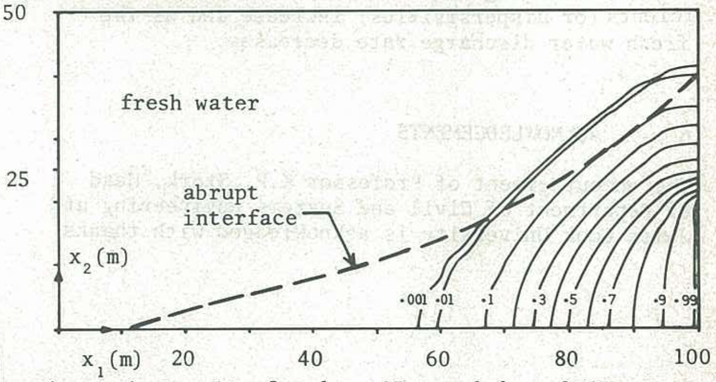

Figure 6 Results for $d_1 = 25$ m and $d_2 = 8.33$ m

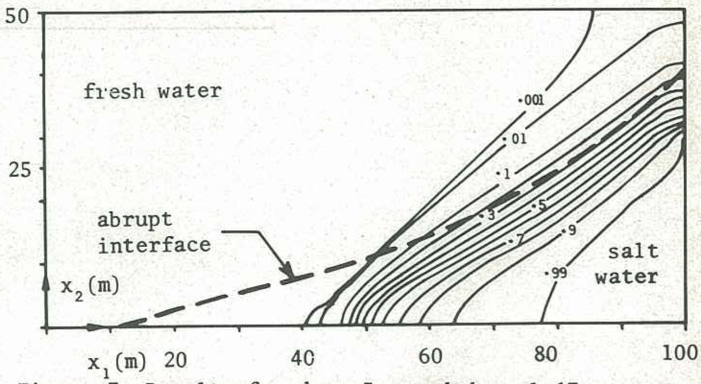

Figure 7 Results for $d_1 = 5$ m and $d_2 = 1.67$ m

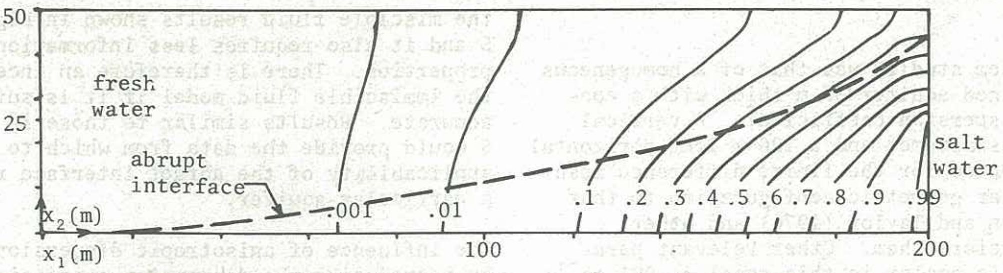

Figure 8 The effect of a reduction in freshwater discharge

Figure 8 shows the equilibrium interfaces calculated for an aquifer similar to that of Figure 3 except that the fresh water flow rate has been reduced by 50 percent. An aquifer section of 200 m in the horizontal direction has been used in this case because of the larger ingress of salt water as a result of the smaller fresh water discharge. The smaller freshwater velocities also mean that dispersion is relatively more important and hence the large discrepancy in the locations of the abrupt interface and the c' = 0.5 concentration contour. In general, the abrupt interface will give a progressively less accurate approximation for the actual mean interface as the dispersion coefficient increases or the fresh water discharge rate decreases.

5 CONCLUSIONS

It has been shown that both immiscible and miscible fluid models can be useful for predicting the salt water interface in coastal aquifers discharging to the sea. The immiscible fluid model has the advantage of significantly lower computational costs and fewer numerical difficulties as well as requiring less field information. The miscible fluid model, on the other hand, gives a more complete and more faithful picture of the actual interface location and salt distribution. This feature increases in significance as the dispersion coefficients (or dispersivities) increase and as the fresh water discharge rate decreases.

6 ACKNOWLEDGEMENTS

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