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The Effect of Viscosity on the Transition to Mach Reflexion in Pseudosteady Flow

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SUMMARY To investigate the effect of viscosity on the transition to Mach reflexion in pseudosteady flow, the Reynolds number of the flow behind the reflected shock was varied by changing the shock tube initial pressure. A model developed to describe the viscous effect on transition predicted a linear dependence of the observed transition angle on $Re^{-\frac{1}{2}}$. Experiments at a shock Mach number of 5.4 demonstrated this behaviour.

1 INTRODUCTION

The conditions for the transition to Mach reflexion of shock waves may be analytically calculated for a perfect gas (von Neumann, 1943; Bleakney and Taub, 1949), however it has been found experimentally that in pseudosteady flow regular shock reflexion persists beyond the calculated boundary (Bleakney and Taub, 1949; Henderson and Lozzi, 1975). It has been proposed that this persistence may be due to the boundary layer which grows behind the reflexion (Hornung, Oertel and Sandeman, 1979) although experiments were not carried out to verify this suggestion. This paper reports experiments which were conducted with this aim.

Before describing the experiments and their results (Section 5,6) a discussion of shock wave reflexion in pseudosteady flow (Section 2), the conditions which determine transition (Section 3) and the proposed effect of viscosity on transition (Section 4) are presented.

2 REFLEXION PROCESSES

When a plane travelling shock wave, like that produced in a shock tube strikes a wedge it is reflected either as a simple, two-shock, regular reflexion or as a more complex, three-shock, Mach reflexion, depending primarily on the shock Mach number and angle of incidence of the shock on the wedge.

Shock reflexion from a plane wedge in a shock tube may be described as a two-dimensional unsteady flow using three co-ordinates (x,y,t). The flow also has the special property of self-similarity, that is where there is a length scale associated with the flow, e.g. the Mach stem length, this grows linearly with distance and time from a centre of similitude, in this case the wedge-tip. The flow may be described by two reduced co-ordinates x/t, y/t (Jones, Martin, Moira and Thornhill, 1951).

The regular reflexion case is somewhat simpler, since, as the flow is supersonic behind the reflected shock the reflexion cannot be influenced by the wedge tip, the flow has no associated length scale, and may be transformed into a stationary reference frame in which the gas travels into the incident shock with a velocity of $v_1 = u_s/\sin\alpha$ (see fig. 2) where u is the shock speed and α the shock incidence angle. In this co-ordinate system the flow strikes the first shock (I) with velocity v_1 (state 1) and is deflected towards the wall through an angle δ_1 by the shock (state 2). The reflected shock then turns the flow parallel to the wall (state 3) satisfying the inviscid boundary condition $\delta_1+\delta_2=0$.

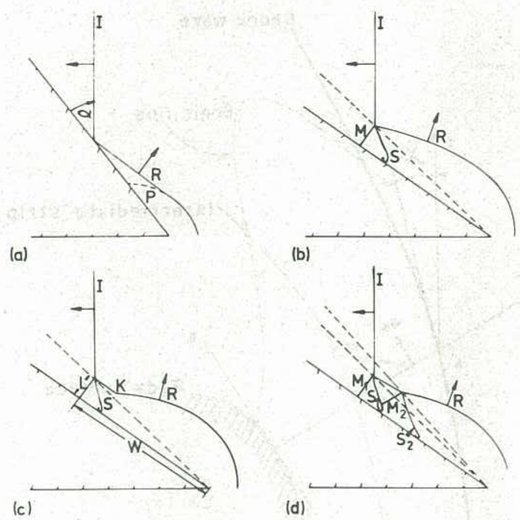

Figure 1 Types of shock reflexion in pseudosteady flow. (a) Regular Reflexion, I - incident shock, R - reflected shock, α - shock incidence angle, P - pressure signal from wedge tip. (b) Single Mach reflexion, M - Mach stem, S - slipstream. (c) Complex Mach reflexion, K - kink in reflected shock. L/w is the dimensionless Mach stem length. (d) Double Mach reflexion.

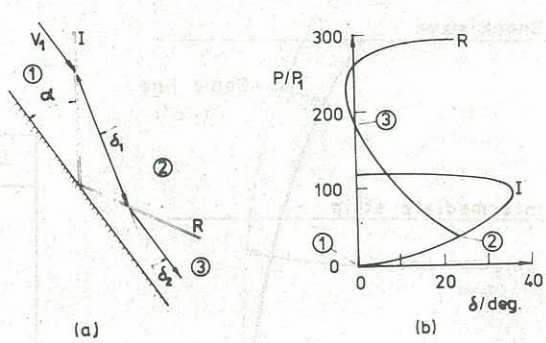

Figure 2 (a) Regular reflexion in a co-ordinate system fixed at the shock intersection. (b) Pressure deflection map for regular reflexion. $\gamma = 1.67$, $M_s = 5.4$, $\alpha = 34.0°$. P_1 is the shock tube initial pressure.

The reflexion may be mapped onto a pressure-deflexion diagram (fig. 2b). The incident shock locus (I) represents the possible states which may occur behind an oblique shock. For a given incidence angle one point on the locus is fixed and corresponds to state (2). The states which may then occur behind the reflected shock lie on the reflected shock locus (R). There are two possible solutions which satisfy the boundary condition $\delta_1+\delta_2 = 0$, indicated by the two pressure ratios where the flow deflexion (δ) is equal to 0. Of these the lower-pressure solution lying on the supersonic branch of the shock locus is closest to the physically occurring solution.

If the shock incidence angle α is now increased (or wedge angle decreased) while the shock Mach number remains constant, the pressure ratio across the incident shock stays constant while the flow deflection is increased. At some point the reflected shock locus no longer intersects the zero deflection axis, regular reflexion may not occur and the shock is reflected as a Mach reflexion. The Mach stem is represented by the segment of the incident shock polar between the zero deflection axis and the intersection of the reflected shock locus (fig. 3). The slipstream (S) represents a

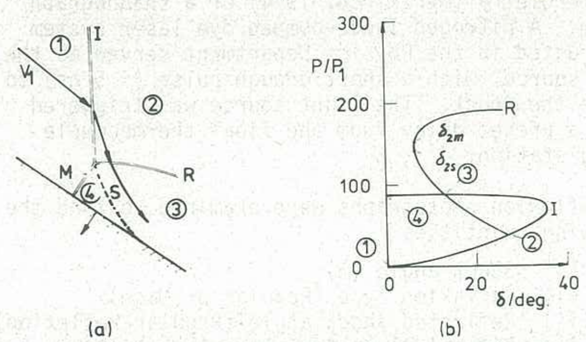

Figure 3 (a) Mach reflexion in a co-ordinate system fixed at the triple point. (b) Pressure deflection map for Mach reflexion $\gamma = 1.67$, $M_s = 5.4$, $\alpha = 40°$.

velocity and density discontinuity between the gas which has passed below the triple point, and thus through only one shock (the Mach stem) and that which has passed through the incident and reflected shocks. The pressure is continuous across S.

The reflexion is further complicated by the nature of the flow in region (3) which may be either subsonic with respect to the triple-point (lies above δ_{2s} on the reflected shock locus) or supersonic. When state (3) lies on the supersonic branch of the reflected shock locus the flow can only turn parallel to the wedge through a series of compression waves which are focussed to cause the kink (K) in the reflected shock. These compression waves may coalesce into a shock giving rise to a double Mach reflexion (fig. 1d).

3 CONDITIONS FOR TRANSITION TO MACH REFLEXION

Two conditions have been suggested to define the boundary for the existence of regular reflexion.

When the reflected shock locus becomes tangent to the zero deflexion axis there is only one possible regular reflexion solution and the deflexion through the second shock is equal to the maximum possible deflection (δ_{2m}). Any increase in incidence angle means that the boundary condition may not be maintained. This boundary was first proposed by von Neumann (1943) and has become known as the "detachment" condition in analogy to supersonic steady flow where the bow shock may be considered to be detached when the body angle exceeds the maximum flow deflection.

A more general condition for the termination of regular reflexion, applicable to both steady and pseudosteady flow (Hornung, Oertel and Sandeman, 1979) is that transition occurs when conditions change behind the reflected shock such that a length scale may be communicated to the reflexion point. In pseudosteady flow such a change occurs when the flow behind the reflected shock is just sonic with respect to the reflexion point. At this condition the pressure signal from the tip keeps pace with the reflexion, and so a length scale, the tip to Mach stem distance is communicated to the flow. This length is indeed found to be important to the reflexion since it scales the Mach stem size, as was mentioned previously.

These two conditions are found to lie so close to one another, since the point of maximum deflexion δ_{2m}, and the sonic point δ_{2s} are only slightly separated on the shock locus, that experimentally the difference between them may not be resolved.

Measurements of the transition angle have been made by several authors (e.g. Bleakney and Taub, 1949; Kawamura and Saito, 1956; Smith, 1959; Henderson and Lozzi, 1975). Bleakney and Taub reported transition at an angle approximately 2° greater than the detachment condition, Kawamura and Saito measured an excess angle of 2.5° at $M_s = 1.7$ and 2° at $M_s = 1.1$. Smith performed experiments in which the wedge surface was replaced by a plane of symmetry, so that the reflection occurred between two colliding shocks thus removing the boundary layer from the problem. His results agreed with the theoretical transition conditions in this geometry. No association was made between the removal of the boundary layer and the agreement with theory since his experiments on reflexion from a wedge produced the same agreement. No Reynolds numbers for his experiments were reported so that it is not possible to assess the likely effect of viscosity on his results.

Henderson and Lozzi performed experiments in both geometries, obtaining an excess angle of 2° above theory in reflexion from a wedge, and an excess of -0.5° in the colliding shocks case (experimental error ± 0.5°). They also did not associate the disagreement with the influence of viscosity.

4 THE EFFECT OF VISCOSITY ON TRANSITION

The inviscid theory to describe transition to Mach reflexion seems, in the light of previous experimental results, to be inadequate to describe transition even in the case where the gas may be considered ideal. To give better agreement with the experimental results a relaxation of the boundary condition $\delta_1+\delta_2 = 0$ has been proposed (Skews, 1972). The relaxed boundary condition would be $\delta_1+\delta_2 = \varepsilon$, where ε is a small angle which the flow may be allowed to have towards the wall after the reflected shock (see fig. 4). Now, instead of transition occurring when the reflected shock locus becomes tangent to the pressure axis on a p-δ diagram, it continues to occur until the possible relaxation angle ε is exceeded.

One physical mechanism through which such a relaxation might occur is the growth of the boundary layer behind the reflexion point. This effect is most easily visualised in shock-stationary co-ordinates (fig. 4). The velocity of the flow behind the reflected shock is less than the velocity of the wall v_1, so that a shear region exists in the flow. The region affected by this shear, the boundary layer, grows due to the action of viscosity, and since the wall moves faster than the flow, the displacement thickness is negative.

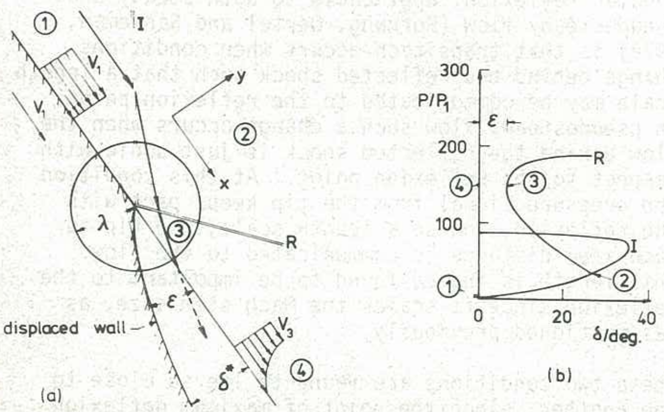

Figure 4 (a) Boundary layer growth behind the shock intersection point. δ^* is the displacement thickness. (b) Displacement effect on regular reflexion ε is on P-δ diagram flow relaxation angle.

From examination of the boundary layer equations of motion the boundary layer thickness must be of the order of magnitude

$$\delta^* \simeq (x\mu/\rho v)^{\frac{1}{2}} \qquad \ldots(1)$$

where ρ, v, μ, x are a typical density, velocity, viscosity and length for the boundary layer. Rearranging this expression gives the proportionality of dimensionless boundary layer thickness to inverse square root of Reynolds number

$$\delta^*/x = C_1/(Re_x)^{\frac{1}{2}}. \qquad \ldots(2)$$

Differentiating this expression to find the rate of growth of the boundary layer gives

$$\frac{d\delta^*}{dx} = C_1/2(Re_x)^{\frac{1}{2}} \qquad \ldots(3)$$

In determining the transition angle the important length scale will be the observer's smallest resolvable length λ, since only when the Mach stem length exceeds λ will the reflexion be resolvable as a Mach reflexion.

The slope of the displaced wall at $x = \lambda$ will be equal to the relaxation angle ε.

$$\varepsilon \simeq \left(\frac{d\delta^*}{dx}\right)_{x=\lambda}. \qquad \ldots(4)$$

Assume that the change in transition angle α is small enough so that it may be taken to be linear, $d\alpha/d\varepsilon = C_2$. Using (3) it is found that the observed transition angle will be changed due to viscosity by an amount which, to a first approximation, is proportional to the inverse square root of the Reynolds number

$$\alpha_{trans} = (\alpha_{trans})_{Re=\infty} + C/(Re_\lambda)^{\frac{1}{2}} \qquad \ldots(5)$$

5 EXPERIMENT

The nature of equation (5) suggests that, to investigate the viscous effect on transition, the transition angle could be measured at a constant shock Mach number with several different Reynolds numbers behind the reflected shock. The Reynolds number may be conveniently varied by adjusting the shock tube filling pressure, since, if the shock speed is maintained constant with a given ideal gas, the Reynolds number behind the shock varies linearly with the initial shock tube pressure.

It was hoped that the experiments would unambiguously test for a viscous effect on transition so that they needed to be conducted at a condition where ideal gas analysis could be validly applied to the test gas, and real gas effects (e.g. dissociation, ionization) would not be significant. Because of their vibrational energy states, molecular gases deviate from ideal behaviour at much lower Mach numbers than atomic gases (Ben-Dor and Glass, 1979). Argon was selected as test gas because of its monatomic nature and high viscosity, real gas effects only becoming important above M_S=6.

6 EXPERIMENTAL EQUIPMENT AND METHODS

Experiments were conducted in the large free piston facility T3 at ANU (Stalker, 1972). When operating in a shock tube configuration, the shock tube opens into a test section with remote sidewalls. The wedge model is placed immediately at the end of the shock tube and the shock is photographed as it leaves the shock tube and reflects from the wedge.

The shock was visualised using either schlieren interferometry (Merzkirch, 1974) or a shadowgraph system. A nitrogen laser-pumped dye laser system constructed in the Physics Department served as the light source, with a short enough pulse ($\simeq$ 5 ns) to freeze the shock. The light source was triggered after a preset delay from the final thermocouple timing station.

The reflexion photographs were examined to find the following quantities

(i) Shock angle (α)
(ii) Reflexion type (Regular or Mach)
(iii) Reflected shock angle (Regular reflexion)
(iv) Dimensionless Mach stem length (L/w)

To determine the transition angle, graphs were drawn of the dimensionless Mach stem length against shock angle. Extrapolation of these plots to zero Mach stem length fixed the transition angle to within ±1°.

7 RESULTS AND DISCUSSION

Experiments were conducted at a shock speed of 1.74 (± 0.05)$\times 10^3$ ms^{-1}. This corresponds to a shock Mach number of 5.4 ± 0.2, which was low enough to allow real gas effects to be neglected and a perfect gas analysis of the results applied. This assumption was checked by numerical calculations of the inviscid transition angle considering the gas to come to thermal equilibrium after each shock. At the limit of the Mach number range (M_S = 5.6) the calculated increase in transition angle due to ionisation was approximately 0.2°, which, considering the error in determining the transition angle (±1°) is negligible.

Graphs of dimensionless Mach stem length versus shock angle (fig. 5) for this condition showed a systematic increase in dimensionless Mach stem length for a constant shock angle as the initial shock tube pressure increased. This effect cannot be accounted for by the ideal gas analysis which should be valid to describe this case. It is thought to be due to the effect of viscosity. To investigate this more carefully, the transition angle was plotted against the calculated (Reynolds number)$^{-\frac{1}{2}}$ at the conditions behind the reflected shock, for a shock Mach number of 5.4 and shock angle of 35.4° (see fig. 6).

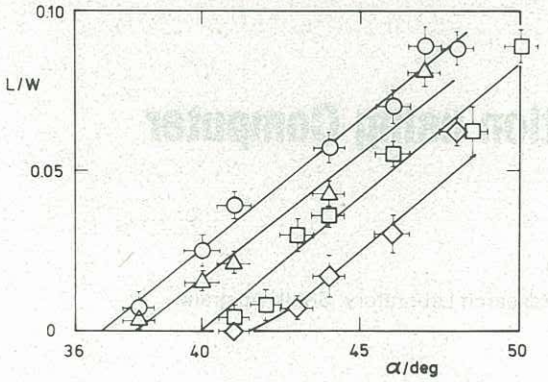

Figure 5 Plots of dimensionless Mach stem-length, L/w as a function of shock incidence angle α, shock speed $u_s = 1.74\times10^3\ \mathrm{ms}^{-1}$.
◇ $p_1 = 6.66\times10^2$ Pa □ $p_1 = 1.33\times10^3$ Pa
△ $p_1 = 3.33\times10^3$ Pa ○ $p_1 = 9.33\times10^3$ Pa
The lines extrapolate the data to zero L to determine the transition angle.

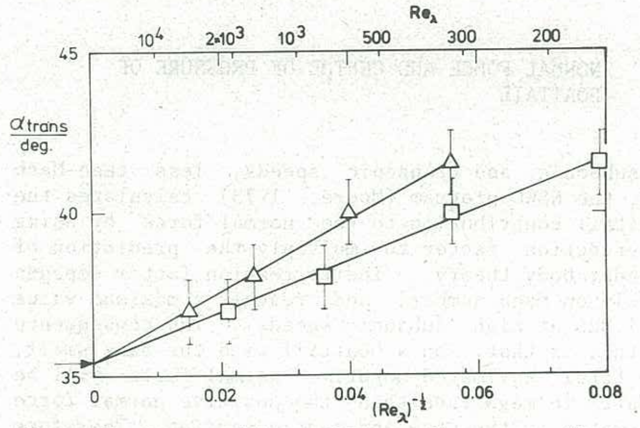

Figure 6 Measured transition angle versus (Reynolds number)$^{-\frac{1}{2}}$ $u_s = 1.74\times10^3\ \mathrm{ms}^{-1}$. △, $\lambda = 0.5\times10^{-3}$m. □, $\lambda = 0.25\times10^{-3}$ m. The ideal gas inviscid transition angle is 35.4° (shown by arrow). Slopes are 1.9 and 1.4 respectively.

Within the error limits, this plot was linear as predicted by equation (5), with extrapolation to $p_1 = \infty$ or $Re = \infty$ putting transition at the inviscid ideal gas transition angle.

The length scales taken for the Reynolds number calculations were 0.25×10^{-3} and 0.5×10^{-3} m. These mark the range of the error bars on the plots of dimensionless Mach stem length which were a measurement of the resolution limit of the experiment.

The constant C may be calculated as follows. Using the calculations of Mirels (1971) (Mirels' figure 3a) for this type of boundary layer in argon, the constant C_1 may be obtained as 3.7 for the conditions of the present experiment. The constant C_2 may be found by drawing pressure-deflection diagrams over a range of incident shock angles and measuring $d\varepsilon/d\alpha$ from these. Such calculations showed that, over the range of interest C_2 is indeed constant and equal to 0.7. The constant $C = C_1C_2/2$ therefore becomes $C = 1.3$ which is in good agreement with the experimental result of 1.4 for $\lambda = 0.25$ mm.

More evidence supporting viscosity as the cause of the change in transition angle was found by measuring the reflected shock angles in regular reflexion. These were found to be significantly below the theoretical values which is as predicted for the relaxation of the boundary condition due to the effect of viscosity.

8 CONCLUSIONS

Measurements of the angle for the transition to Mach reflexion showed that there is an effect on the transition angle with variation of the shock tube initial pressure which cannot be accounted for by ideal gas theory. The model for the effect of viscosity on the transition angle developed in this paper predicts an effect which gives good agreement with experimental observations. This evidence, along with that of earlier workers (Henderson and Lozzi, 1975) which showed that transition to Mach reflexion occurred at the inviscid condition when the boundary layer was eliminated by replacing the wall by a plane of symmetry, suggests that the boundary layer is indeed responsible for the excess observed by many other experimenters, of the measured transition angle over the calculated value. The experiments described were repeated at higher shock Mach number. However, the significant effects of ionization coupled with the boundary layer effect produce complex results which are not readily explained.

9 ACKNOWLEDGEMENTS

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