Stress at a Sheared Finger Interface

B.R. RUDDICK

Research Fellow, Research School of Earth Sciences, The Australian National University, A.C.T.

1   INTRODUCTION

When water is set in a tank, stratified so that its composition varies from warm and salty near the top to cooler and fresher (but slightly denser) near the bottom, an instability can develop, which derives its nature from the different rates of molecular diffusion of the density-changing constituents, salt and heat. The instability takes the form of long, thin vertical fingers (see Figure 1), alternating in vertical velocity. Downgoing, relatively warm and salty fingers continually diffuse heat out to adjacent upgoing, relatively cool and fresh fingers, but retain most of their salt due to the much smaller diffusion rate of salt than heat. This heat exchange between adjacent fingers causes the downgoing fingers to continually get heavier, and the upgoing fingers to continually get lighter, thereby driving the instability. The net result, on the large scale, is a rapid downward convective transport of salt and heat, and a net release of potential energy from the stratification.
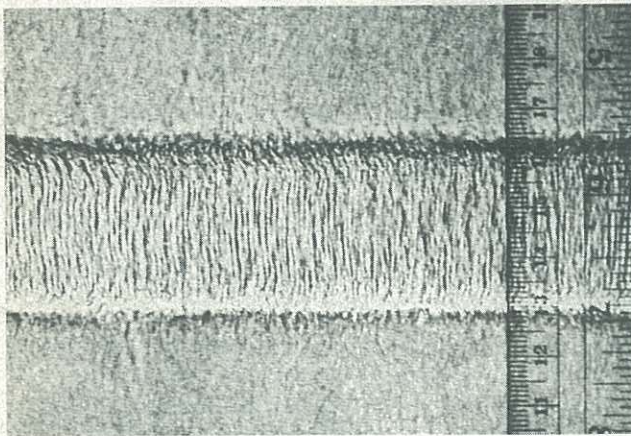

Figure 1   Close-up view of a sugar-salt finger interface with $R_\rho = 2.0$. The scale at right is in cm. Well-mixed convecting layers are visible above and below the interface. (The primary measure of finger activity is $R_\rho = 1 + \Delta\rho/\beta\Delta S$ where $\Delta\rho$ is the density difference between layers and $\beta\Delta S$ the sugar contrast in density units. $R_\rho \sim 1$ is associated with very active convection, and fingering continues, weakening, as R_ρ asymptotically approaches 3.)

This so-called "double-diffusive" instability is gaining recognition by oceanographers as one of the important mixing mechanisms in the deep ocean, and work is continuing towards quantifying the many roles of that mixing.

When a shear is imposed on a fingering interface, the turbulent convection within the layers and the convective mass exchange between the layers may affect the transport of the momentum associated with the shear. We describe below experiments in which we estimate the frictional effects of a sheared finger interface.

2   METHOD

The ideal method for measuring the stress across a finger interface would allow a steady shear to be applied and a direct measure of the stress to be made. Such an experiment was attempted in a long tank by using a pump to transfer water in the upper layer of a fingering system from one end to the other, thereby applying a steady transport in the upper layer. The pressure gradient in the lower layer was to have been inferred from the tilt and density jump of the interface, but entrainment caused a progressive downstream sharpening of the interface, leading to erroneous tilt measurements. Conceivably this experiment could be done in an annular tank with apparatus designed to produce shear via a measured stress.

An alternative method, first used by A. McEwan (personal communication, 1979) was adopted, in which a standing internal wave was excited on the interface, and allowed to decay. The decay rate was measured and corrected for friction at the solid boundaries. Changes were attributed to the fingering activity. This method has the disadvantage that the timescale of the applied shear is comparable to or shorter than the timescale of individual fingers, whereas a relatively steady shear is preferable.

A glass-walled tank was fitted with a flat paddle at one end, hinged about the middle and driven by a small d.c. motor and gearbox. The cavity in which the wave oscillated was 44.4 cm long, 28 cm deep, and 20.1 cm in breadth. The tank was filled with salt solution in the lower half, and sugar solution (the more slowly diffusing component) in the upper half, each with a specific gravity of about 1.1, the sugar solution being slightly less dense. "Sugar" fingers rapidly formed on the interface, causing a downward flux of sugar and an upward flux of salt. Sugar-salt fingers were used in the experiment rather than heat-salt because the rundown time scale was longer, and so more convenient. Although the two types of fingers are qualitatively the same, the fluxes are quantitatively different, and a similar set of experiments will have to be done using heat-salt fingers before the results can be extrapolated to the ocean.

Throughout the experiment, water samples were withdrawn from each layer and later analysed for conductivity and density (hence sugar and salt concentrations) using the instruments and methods

of Ruddick and Shirtcliffe (1979). These quantities were then interpolated (see Figure 2) to determine the layer properties at any given time. The estimates of sugar and salt concentrations were used together with the measured temperature to compute the viscosity ν of the fluid in each layer, using a combination of the equations given by Barber (1966), and Chen and Sandford (1977).
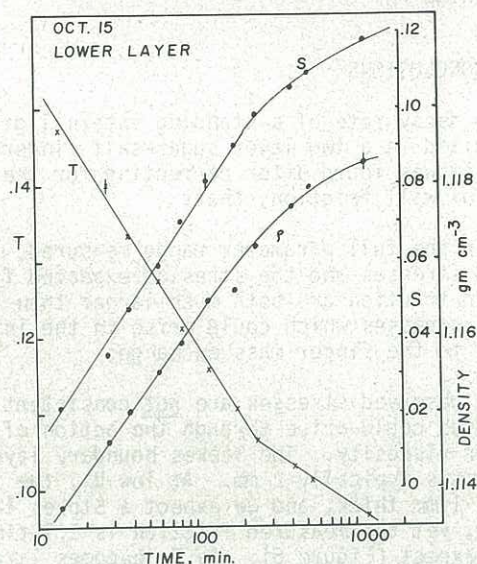
Figure 2 Plot showing variation with time, on a logarithmic axis, of the salt concentration (T), sugar concentration (S), both in g g^{-1} solution, and the density of samples withdrawn from the lower layer of the tank during the course of an experiment. The error bars of measurement and the smoothed lines used for interpolation of the data with time are also shown.

As often as possible, a standing wave was excited on the interface by cycling the paddle near the resonant frequency (period typically 10-15 s) of the first horizontal mode for several cycles, stopping the paddle, and measuring the rate of decay of interface displacement. The interface position was monitored using a small neutrally buoyant float, painted flat black except for a thin concentric shiny line near its base. The float was constrained to move vertically by a human hair, so that when illuminated and viewed by a slowly panning camera, it left a time record of its vertical motions (see Figure 3).
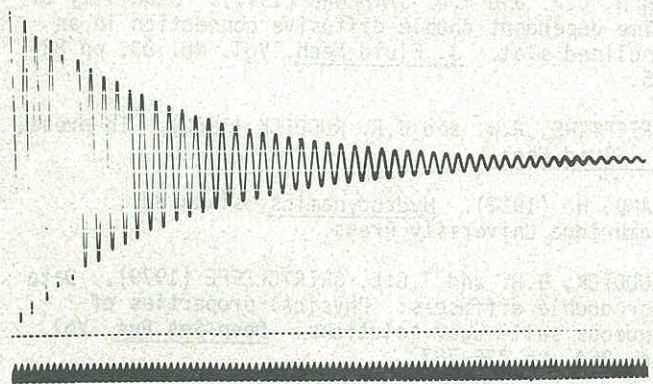
Figure 3 A typical tracing of interface position, obtained photographically as described in the text, showing the decay of wave amplitude with time. The regularly spaced marks at the bottom of the figure indicate elapsed time in five second increments.

The peak-to-peak amplitudes of successive wave cycles were measured from the film, and a weighted least squares fit was done to estimate the exponential decay rate of the wave, as in Figure 4 below.
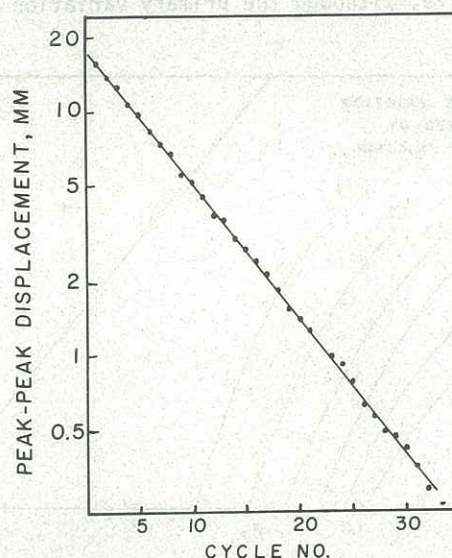
Figure 4 Plot of log (amplitude) versus cycle number of a typical wave, with a line indicating the amplitude-weighted least-squares fit to an exponential decay.

3   RESULTS

3.1   Stokes Boundary Layer at a Sharp Interface

Assuming that a double Stokes boundary layer were to occur at the interface, the expected decay rate, q, can be computed as the sum of wall friction and interface friction terms (A1). As described in A3, the "friction parameter", F, is defined as the observed decay rate minus the wall friction term, divided by the interface term. This parameter may be interpreted as the decay rate which would occur in the absence of wall friction, normalised by the decay rate expected from the interfacial Stokes boundary layer. In Figure 5 are plotted the observed decay rates normalised in this manner, as a function of the density ratio, R_ρ. A systematic variation of the interface friction is evident, from F = 1.7 at low R_ρ (strong convection) to about 1 at high R_ρ (weak convection).
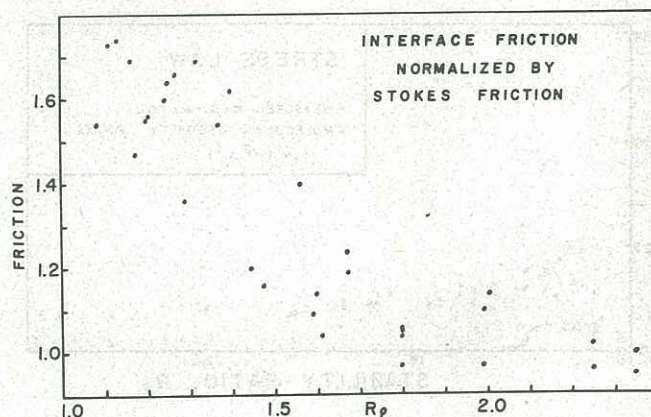
Figure 5 The interface friction parameter, F, defined by A3, plotted as a function of the density ratio, R_ρ.

In Figure 6 is shown the measured dependence of F on the initial velocity difference u_0 imposed across the interface, in the form of a contour plot of $F(u_0,R)$. A general tendency is apparent for the interface friction to be smaller following a large amplitude wave, although the primary variation of F is with R_ρ.
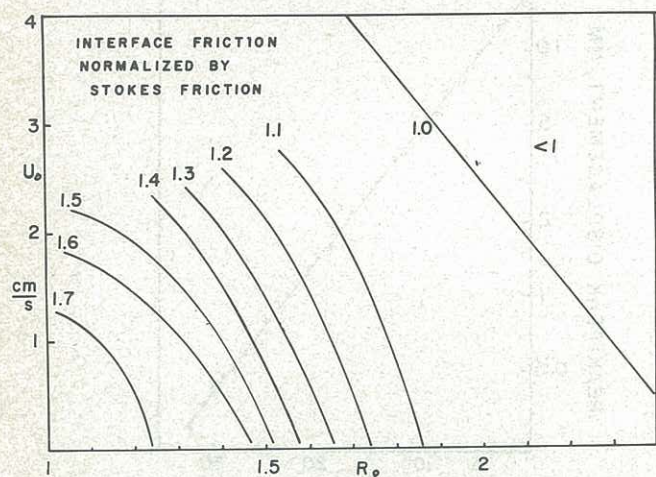
Figure 6 Contours of F, the normalised interface friction, as a function of R_ρ and initial wave velocity amplitude $\frac{1}{2}u_0$.

3.2   Linear Shear Law

If the variation of interface friction with wave amplitude is neglected, one can assume that the interfacial stress τ is proportional to the velocity difference Δu imposed across it; $\tau = K\Delta u$. The coefficient K can be calculated from the observed decay rate using A4. The observed values are plotted as circles on Figure 7. K is relatively constant, even though the fluxes vary by several orders of magnitude.

The observed K is compared with that expected from molecular viscous stress across the interface, assuming that the velocity shear is spread uniformly throughout the interface of thickness δh. This "viscous K", $K_V = \rho\nu/\delta h$, is shown as the crosses in Figure 7, and is found to be much larger than the observed K at small R_ρ when the interface is thin, and much smaller when the interface is thick.
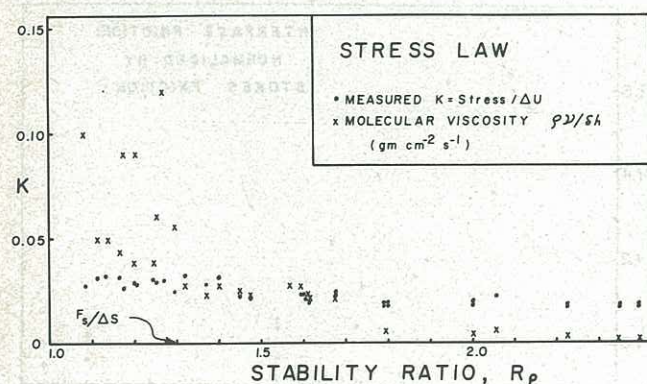
Figure 7 The measured interface stress coefficient K = stress/Δu (circles), compared to that expected from molecular viscosity (crosses), and that expected from the finger mass exchange, $F_S/\Delta S$.

The momentum transfer which is associated with the finger mass exchange can be estimated roughly $K_F \simeq \rho w(\text{finger}) \simeq F_S/\Delta S$, and is shown (in order of magnitude) on Figure 7. A "momentum transfer coefficient" $K/K_F = K\Delta S/F_S$ can be estimated using the measured flux law of Griffiths and Ruddick (1980), and is found (because of the near constancy of K) to vary roughly as R_ρ^6, from about 40 at low R_ρ to ~10^4 at high R_ρ.

4   CONCLUSIONS

From the decay rate of a standing internal gravity wave excited in a two-layer sugar-salt fingering system, it was found after correcting for the effects of wall friction, that:

(1) over the full parameter range measured, the observed stresses and the stresses expected from molecular friction are both much larger than the Reynolds stresses which could arise in the interface due to the finger mass exchange;

(2) the observed stresses are not consistent with those which could arise through the action of molecular viscosity. The Stokes boundary layer thickness is typically 2 mm. At low R_ρ, the interface is ~1 mm thick, and we expect a Stokes layer to occur, yet the measured friction is 1.7 times what we expect (Figure 6). As R_ρ becomes large, the interface thickens to 10 mm or more, and we observe a linear shear through the interface. However, the observed friction is again larger than the expected viscous friction (Figure 7);

(3) the stress is found to be proportional to the velocity difference across the interface, roughly independent of R_ρ and the interface thickness.

The decay rates measured in experiments where no fingering took place agreed with the viscous computations in Appendix A. The extra friction observed in the experiments reported must be associated with the finger activity but cannot be due to the finger flux Reynolds stresses. It must then be inferred that this extra friction is associated with the turbulent convection in the mixed layers, by a mechanism not understood.

5   REFERENCES

BARBER, E.J. (1966). Calculation of density and viscosity of sucrose solutions as a function of concentration and temperature. J. Nat. Cancer Inst. Monographs Vol. No. 21, pp 219-239.

CHEN, C.F. and R.D. SANDFORD (1977). Stability of time dependent double-diffusive convection in an inclined slot. J. Fluid Mech. Vol. No. 83, pp 83-95.

GRIFFITHS, R.W. and B.R. RUDDICK (1980). In press, J. Fluid Mech.

LAMB, H. (1932). Hydrodynamics, sixth ed., Cambridge University Press.

RUDDICK, B.R. and T.G.L. SHIRTCLIFFE (1979). Data for double diffusers: Physical properties of aqueous salt-sugar solutions. Deep-Sea Res. Vol. No. 26A, pp 775-787.

6   ACKNOWLEDGEMENTS

helpful discussions on the subject, and Ross Wylde-Browne and Derek Corrigan for very able technical and graphics assistance. This paper was written while the author was a guest investigator at the Australian Institute of Marine Science.

7   APPENDIX A

Viscous Decay of an Interfacial Wave

The equations of motion and boundary conditions (Lamb, 1932) for a two-layer irrotational, inviscid wave in a closed "rigid lid" container can be easily solved for a standing wave. The inviscid velocity is brought to zero at the walls, and matched across the (sharp) density interface by oscillatory Stokes boundary layers (Lamb, 1932), and the energy dissipation rate in these layers is easily computed. (alternatively, if a law for the interfacial stress, τ, is assumed to be $\tau = K\Delta u$, the dissipation rate per unit area is then $K\Delta u^2$.) When the total dissipation rate for the walls plus interface is compared to the average energy in the wave, we get an equation of the form $dE/dt = -2qE$, so that the amplitude (proportional to $\sqrt{E}$) decays at a rate q, where q is found to be:

$$q = \sqrt{\nu\sigma/2}\ [Q_0 + Q_S] + (2kK)/(\rho \tanh kH) \quad (A1)$$

where $Q_0 = k \tanh kH \begin{bmatrix} (2 \sinh^2 kH)^{-1} & \text{top, bottom} \\ +(kB \tanh kH)^{-1} & \text{sides} \\ +(\pi \tanh kH)^{-1} & \text{ends} \\ -H/(L \sinh^2 kH) & \text{ends} \end{bmatrix}$

$Q_S = k/(2 \tanh kH)$
H = layer depth, 14 cm

L = tank length, 44.4 cm
B = tank breadth, 20.1 cm
k = $2\pi/L$
σ = wave frequency, radian/s
$\nu = (\tfrac{1}{2}\sqrt{\nu_1} + \tfrac{1}{2}\sqrt{\nu_2})^2$, "average" layer viscosity
$\rho = \tfrac{1}{2}(\rho_1 + \rho_2)$, average layer density

The Q_0 term comes from wall friction, the Q_S from interface friction assuming a Stokes boundary layer, and the term proportional to K is due to interface friction assuming the linear stress law. For the tank used, the contributions to the computed decay rate were found to be:

$$q = \sqrt{\nu\sigma/2}\ (.119\ cm^{-1}) \begin{bmatrix} 10.6\% & \text{top, bottom} \\ 41.7\% & \text{sides} \\ 12.2\% & \text{ends} \\ 35.4\% & \text{Stokes at interface} \end{bmatrix} \quad (A2)$$

For experiments in which fingering did not occur, the decay rate normalised by $\sqrt{\nu\sigma/2}$ was found to be in the range .112 to .122 cm^{-1}, in excellent agreement with the calculations A2, considering that molecular diffusion should weaken the frictional effect of the interface.

The observed decay rate, corrected for wall friction and normalised by the decay rate expected from a Stokes layer at the interface is:

$$F = (q - Q_0\sqrt{\nu\sigma/2})\ /\ (Q_S\sqrt{\nu\sigma/2}) \quad (A3)$$

If the linear stress law $\tau = K\Delta u$ is assumed to hold at the interface, then K may be computed from the observed decay rate by:

$$K = (q - Q_0\sqrt{\nu\sigma/2})\ (\tanh kH)\ /2k \quad (A4)$$