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Turbulent Flow Between a Rotating Disk and a Stationary Wall

J.W. POLKOWSKI
Development Engineer, Brown, Boveri & Co. Ltd., Baden, Switzerland

SUMMARY Turbulent flow between a rotating disk and a stationary wall with radial outflow has been investigated in this paper. A gap between a disk and a wall was assumed big enough to distinquish three regions, disk boundary layer, wall boundary layer and a core. Moment coefficient as well as other relevant physical quantities have been found as functions of Reynolds number, coolant mass flow rate and geometry (rotating and/or stationary shroud). Cooling system of gas turbine rotors is one of many engineering applications of the problem discussed here.

1 INTRODUCTION

There are basically two methods of approaching the problem of a turbulent flow over rotating disk. One approach is based on semiempirical relationships, developped for a turbulent flow, hence velocity profiles within the boundary layers satisfy power (or logaritmic) law. Then e.g. von Karman integral method can be used to solve equation of motion in the boundary layers.

Another approach is to make no assumptions about the velocity profiles etc. and to solve the set of governing equations over the whole flow between the rotor and stationary wall. Promising attempt has been made by Owen et.al. in [1], [7], but even for a very simple geometrical model of the flow, the authors admittet that quantitative agreement with experimental data has not been satisfactory, due to difficulty in establishing the adequate turbulence model and resulting distribution of Reynolds stresses.

In this paper the first method is applied. It is assumed, that velocity profiles in the boundary layers satisfy 1/n power law and that the gap between a disk and stationary wall is greater than the sum of maximum thickness of boundary layers (this condition is satisfied in most of actual situations). Consequently the core flow exists between the boundary layers, which is assumed to rotate like a solid body.

2 NOTATION

r, Θ, z	radial, circumferential and axial directions in cylindrical coordinates system
R	disk radius [m]
s	shroud length in z direction
$s'=s/R$	dimensionless length of shroud
$Re=\omega R^2/\nu$	Reynolds number
$C_M=M/\frac{1}{2}\rho\omega^2R^5$	moment coefficient
$C_m=\dot{m}/\rho R^3\omega$	mass flow coefficient
ρ	density [kg/m^3]
ω	disk angular velocity [sec^{-1}]
β	core flow angular velocity [sec^{-1}]
$\beta/\omega=1-K$	ratio of core to disk angular velocity
δ	boundary layer thickness
$\eta=\delta/z$	dimensionless boundary layer thickness

Suffixes

cool-coolant; d-disk; w-stationary wall; sh-shroud; in-inlet; out-outlet; rot-rotor; cas-casing; f.d.-free disk

3 GENERAL CONSIDERATION AND BASIC ASSUMPTIONS

The following four regimes can be distinguished from the point of view of the amount of coolant throughflow

a) no coolant throughflow ($\dot{m}_{cool}=0$)
b) $0<\dot{m}_{cool}<\dot{m}_{f.d.}$ ($\dot{m}_{f.d.}$-mass flow rate induced by a free disk)
c) $\dot{m}_{cool}\approx\dot{m}_{f.d.}$
d) $\dot{m}_{cool}>\dot{m}_{f.d.}$

Model of flow with the core, as described above, constitutes a good approximation in case s) as well as in case c); in the latter the problem is reduced to the solution of one only, disk boundary layer. This corresponds precisely to zero core rotation. In case b) there is some radial outflow in the core. V_{rav} in the core is usually of the order from less than 1m/sec to ~ 3 m/sec. Case d) happens very rarely, as $\dot{m}_{cool}$ is usually not greater than about $\dot{m}_{f.d.}$. The method of solution in case d) would be similar to a free disk, except that the boundary condition for V_r in the disk boundary layer at its outer edge should be equal to some $V_r>0$. This would be necessary only if total mass flow rate exceeded significantly $\dot{m}_{f.d.}$.
It is assumed in the present analysis that ρ=const. and μ=const.

Radial and circumferential components of velocity in the disk boundary layer are assumed as follows:

$$V_r=\alpha r\omega\left(\frac{z}{\delta}\right)^{\frac{1}{n}}\left(1-\frac{z}{\delta}\right)=\alpha r\omega\left(\eta^{\frac{1}{n}}-\eta^{\frac{n+1}{n}}\right) \tag{1}$$

$$V_\theta=r\omega\left[1-k\left(\frac{z}{\delta}\right)^{\frac{1}{n}}\right]=r\omega\left(1-k\eta^{\frac{1}{n}}\right) \tag{2}$$

with the boundary conditions

$$z=0:\ V_r=0;\ V_\Theta=r\omega \tag{3}$$

$$z=\delta:\ V_r=0;\ V_\Theta=r\omega(1-k) \tag{4}$$

For a very small z, $V_r\approx\alpha r\omega\eta^{1/n}$. In the relative coordinate system, attached to the rotor, we have

$$V_{\theta rel} = V_\theta - r\omega = -k\, r\omega\, \eta^{\frac{1}{n}} \tag{5}$$

$$V_{r rel} = V_r \approx \alpha\, r\omega\, \eta^{\frac{1}{n}} \tag{6}$$

Resultant relative, reference velocity can be written as

$$V_{ref} = \left(V_r^2 + V_{\theta rel}^2\right)^{0.5} = r\omega\, \eta^{\frac{1}{n}} \left(k^2+\alpha^2\right)^{0.5} \tag{7}$$

$$V_{ref} = r\omega\left(k^2+\alpha^2\right)^{0.5} \quad as \quad \eta = 1 \tag{7'}$$

Empirical 1/7 power law for a pipe has a form

$$u_{max}/V_* = 8.74\left[V_* R/\nu\right]^{\frac{1}{7}} \tag{8}$$

where $v_*^2=\tau_w/\rho$

Exponent other than 1/7 may give better agreement with experimental data for very high Reynolds number. Therefore the expression $A(v.R/\nu)^{1/n}$, instead of $8.74(v.R/\ )^{1/7}$ has been applied in the present analysis.

Putting $V_{ref}=U_{max}$ and $\delta=R$ we obtain the expression for a wall shear stress along the disk

$$\tau_0 = A^{-2n/n+1}\ \rho\, V_{ref}^{2n/n+1} (\nu/\delta)^{2/n+1} \tag{9}$$

putting $A'=A^{-2n/n+1}$ and substituting (7') into (9) we obtain

$$\tau_0 = A'\rho\left(\frac{\nu}{\delta}\right)^{\frac{2}{n+1}} (r\omega)^{\frac{2n}{n+1}} \left(\alpha^2+k^2\right)^{\frac{n}{n+1}} \tag{9'}$$

From (5) and (6) we have

$$V_r/V_{\Theta rel} = \tau_r/\tau_\Theta = -\alpha/k \tag{10}$$

and from (10)

$$\tau_\theta = \tau_0\left[1+\left(\frac{\alpha}{k}\right)^2\right]^{-0.5} \tag{11}$$

Having given τ_0 from (9') we obtain

$$\tau_\theta = A'\rho\left(\frac{\nu}{\delta}\right)^{\frac{2}{n+1}} (r\omega)^{\frac{2n}{n+1}}\, k\left(k^2+\alpha^2\right)^{\frac{n-1}{2n+2}} \tag{12}$$

Velocity profiles in the wall boundary layer are as follows:

$$V_r = \alpha' r\omega(1-k)\,\eta^{\frac{1}{n}}(1-\eta) = \alpha' r\omega(1-k)\left(\eta^{\frac{1}{n}} - \eta^{\frac{n+1}{n}}\right) \tag{14}$$

$$V_\theta = r\omega(1-k)\,\eta^{\frac{1}{n}} \tag{15}$$

with the boundary conditions

$$z=0:\ V_r=0;\ V_\Theta=0 \tag{16}$$

$$z=\delta:\ V_r=0;\ V_\Theta=r\omega(1-k) \tag{17}$$

Similar consideration to that for a disk boundary layer leads to the following relations

$$V_{ref} = r\omega(1-k)\left(1+\alpha'^2\right)^{0.5} \tag{18}$$

$$\tau_0 = A'\rho\left(\frac{\nu}{\delta}\right)^{\frac{2}{n+1}} \left[r\omega(1-k)\right]^{\frac{2n}{n+1}} \left(1+\alpha'^2\right)^{\frac{n}{n+1}} \tag{19}$$

$$\tau_\theta = \tau_0\left(1+\alpha'^2\right)^{-0.5} = A'\rho\left(\frac{\nu}{\delta}\right)^{\frac{2}{n+1}} \left[r\omega(1-k)\right]^{\frac{2n}{n+1}} \left(1+\alpha'^2\right)^{\frac{n-1}{2n+2}} \tag{20}$$

$$\tau_r = \alpha'\tau_\theta \tag{21}$$

4 METHOD OF SOLUTION

The integral momentum equations for boundary layers will now be applied to solve the problem.

For a stationary, axially symmetrical flow the Navier Stokes equations in radial and circumferential direction can be written as

$$V_r\frac{\partial V_r}{\partial r} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial \tau_{zr}}{\partial z} \tag{22}$$

$$V_r\frac{\partial V_\theta}{\partial r} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_\theta V_r}{r} = \frac{1}{\rho}\frac{\partial \tau_{z\theta}}{\partial z} \tag{23}$$

continuity equation for ρ=const has a form

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0 \tag{24}$$

All other derivatives of the stress tensor components on the right hand side of eq's (22) and (23) can be omitted in this case, and for simplicity the following notation is adapted:

$$\tau_{z\theta} = \tau_\theta \quad ; \quad \tau_{zr} = \tau_r$$

Equations (22) and (23) can be integrated along z from 0 to the outer edge of the boundary layer (δ). It is assumed that pressure is constant along z and it is only a function of r. In some papers, e.q. Dorfman in [5], terms containing V_z in equations (22) and (23) are omitted in the solution of the problem of an anclosed disk considered to be negligible. This is unnecessary simplification, as the set of equations can easily be solved, retaining those terms.

As a results of integration of eq. (22) and (23), using (24), we obtain

$$\frac{d}{dr}\left(r\int_0^\delta V_r^2\,dz\right) - \int_0^\delta V_\theta^2\,dz = -\frac{r}{\rho}\tau_r - \frac{r}{\rho}\delta\frac{dp}{dr} \tag{25}$$

and

$$\frac{d}{dr}\left(r^2\int_0^\delta V_r V_\theta\,dz\right) = -\frac{r^2}{\rho}\tau_\theta \tag{26}$$

respectively.

After substituting eqs. (1) and (2) into eqs. (25) and (26), and then after integration and rearrangement of terms, we get

$$\alpha^2\omega^2 a\left(3r^2\delta + r^3\frac{d\delta}{dr}\right) - r^2\omega^2\delta\left(bk^2-2kc+1\right) + r^2\omega^2\delta(1-k)^2 = -\frac{r}{\rho}\tau_r \tag{27}$$

$$\alpha\omega^2 e\left(4r^3\delta + r^4\frac{d\delta}{dr}\right) = -\frac{r^2}{\rho}\tau_\theta \tag{28}$$

where

$$a = \frac{n}{n+2} + \frac{n}{3n+2} - \frac{n}{n+1}\ ;\ b = \frac{n}{n+2}\ ;\ c = \frac{n}{n+1}\ ;\ d = \frac{n}{2n+1}$$

$$e = c - d + k(0.5c - b) = \frac{n^2}{2n^2+3n+1} - k\frac{n^2}{2n^2+6n+4}$$

Eqs. (28) and (29) describe the problem in the disk boundary layer. There are two equations with two unknowns, α and δ. k can be considered as parameter, and after solving a similar set of eqs. for a wall boundary layer, its value can be determined from the third equation, which is:

$$M_{rot} = M_{cas} \qquad for\ \dot{m}_{cool} = 0 \tag{29}$$

$$M_{rot} - M_{cas} = \Delta\dot{m}(\vec{r}\times\vec{V}) \quad for\ \dot{m}_{cool} > 0 \tag{30}$$

Eqs. (27) and (28) can easily be solved analytically, anticipating that $(r/\delta)(d\delta/dr)$=const. This leads to the relation δ=const r^m where the exponent m can be determined from dimensional analysis, and m=n-1/n+3.

The solution of eqs. (27) and (28) is given by

$$\delta_d = A^{-\frac{2n}{n+3}}\left(\frac{5n+11}{n+3}\right)^{-\frac{n+1}{n+3}} (\alpha e)^{-\frac{n+1}{n+3}}\, k^{\frac{n+1}{n+3}} \times \left(k^2+\alpha^2\right)^{\frac{n-1}{2(n+3)}} \left(\frac{\nu}{\omega}\right)^{\frac{2}{n+3}} r^{\frac{n-1}{n+3}} \tag{31}$$

$$\alpha_d = k\left\{\left[\frac{2}{n+1} - k\frac{2}{n+2}\right] \Big/ \left[\left(\frac{n}{n+2} + \frac{n}{3n+2} - \frac{n}{n+1}\right)\frac{4n+8}{n+3}k + \left(\frac{n^2}{2n^2+3n+1} - k\frac{n^2}{2n^2+6n+4}\right)\frac{5n+11}{n+3}\right]\right\}^{0.5} \tag{32}$$

Substituting eq. (31) into (12) we get

$$\tau_{\theta d} = \left(\frac{5n+11}{n+3}\right)^{\frac{2}{n+3}} A^{-\frac{2n}{n+3}} \rho k^{\frac{n+1}{n+3}} (\alpha^2+k^2)^{\frac{n-1}{2(n+3)}} \times (\alpha e)^{\frac{2}{n+3}} \nu^{\frac{2}{n+3}} \omega^{\frac{2(n+2)}{n+3}} r^{\frac{2(n+1)}{n+3}} \tag{33}$$

Eq. (33) can be expressed as

$$\tau_{\theta d} = B\rho\nu^{\frac{2}{n+3}} \omega^{\frac{2(n+2)}{n+3}} r^{\frac{2(n+1)}{n+3}} \tag{33'}$$

Having given τ_0, friction moment can be found

$$M_d = \int_0^R 2\pi r^2 \tau_\theta \, dr = \frac{2\pi(n+3)}{5n+11} B\rho\nu^{\frac{2}{n+3}} \omega^{\frac{2(n+2)}{n+3}} R^{\frac{5n+11}{n+3}} \tag{34}$$

From eq. (34), the moment coefficient can be found as

$$C_{Md} = M/0.5\rho\omega^2R^5 = \frac{4\pi(n+3)}{5n+11} B Re^{-\frac{2}{n+3}} \tag{35}$$

On the basis of eqs. (14) to (21), a similar procedure in solving the problem for a stationary wall boundary layer leads to the following relations:

$$\delta_w = A^{-\frac{2n}{n+3}} T^{-\frac{n+1}{n+3}} \alpha'^{-\frac{n+1}{n+3}} (1+\alpha'^2)^{\frac{n-1}{2(n+3)}} \times \left[\frac{\nu}{\omega(1-k)}\right]^{\frac{2}{n+3}} r^{\frac{n-1}{n+3}} \tag{36}$$

$$\alpha' = \left[\frac{4(n+3)(n+1)(3n+2)}{n^2(5n+11)(3n+2) - 8n^3(n+2)}\right]^{0.5} \tag{37}$$

$$\tau_w = A^{-\frac{2n}{n+3}} T^{\frac{2}{n+3}} \rho\nu^{\frac{2}{n+3}} [\omega(1-k)]^{\frac{2(n+2)}{n+3}} \times \alpha'^{\frac{2}{n+3}} (1+\alpha'^2)^{\frac{n-1}{2(n+3)}} r^{\frac{2(n+1)}{n+3}} \tag{38}$$

where $T = \left(\frac{n}{n+2} - \frac{n}{2(n+1)}\right)\left(\frac{5n+11}{n+3}\right)$

From (38) moment can be written as

$$M_w = \int_0^R 2\pi r^2 \tau_{\theta w} \, dr = \frac{2\pi(n+3)}{5n+11} B'\rho\nu^{\frac{2}{n+3}} \times [\omega(1-k)]^{\frac{2(n+2)}{n+3}} R^{\frac{5n+11}{n+3}} \tag{39}$$

where $B' = A^{-\frac{2n}{n+3}} T^{\frac{2}{n+3}} \alpha'^{\frac{2}{n+3}} (1+\alpha'^2)^{\frac{n-1}{2(n+3)}}$

and the moment coefficient

$$C_M = \frac{4\pi(n+3)}{5n+11} B'(1-k)^{\frac{2(n+2)}{n+3}} Re^{-\frac{2}{n+3}} \tag{40}$$

Putting (1-k)=0, i.e. k=1, eqs. (31) to (35) constitute a solution for a free disk; eqs. (36) to (40) are irrelevant in this case, as $\tau_{\Theta w}=M_w=0$.

If k, found from the relation $\tau_{\Theta d}=\tau_{\Theta w}$, is inserted into eqs. (31) to (40), then this set of eqs. constitutes a solution for an enclosed disk with no throughflow ($\dot{m}_{cool}=0$) and with negligible effect of shroud.

For the flow with throughflow ($0<\dot{m}_{cool}<\sim\dot{m}_{f.d.}$) it is assumed, that V_Θ of coolant is equal to $r\omega(1-k)$ (core flow rotation), hence eq. (30) can be written as

$$M_{rot} - M_{cas} = \dot{m}_{cool}(r_{out}V_{\theta out} - r_{in}V_{\theta in}) \tag{30'}$$

Eq. (30') is an approximate solution of the problem with throughflow.

The effect of shroud is taken into account in a following way:

$$M_{rot} = M_d + M_{dsh} \tag{41}$$

$$M_{cas} = M_w + M_{wsh} \tag{42}$$

where M_d and M_w are given by eqs. (37) and (42) respectively. It is assumed, that τ_Θ=const along the shroud, and equal to $\tau_{\Theta d}(R)$ or $\tau_{\Theta w}(R)$ for a disk or wall shroud respectively. Bearing in mind that "streamlines" within the boundary layers are almost tangent to the shroud (V_r much smaller than V_Θ), this assumption seems to be a reasonable approximation. Consequently M_{dsh} and M_{wsh} can be written as

$$M_{dsh} = \tau_{\theta dR} \, 2\pi R^2 S_d \tag{43}$$

$$M_{wsh} = \tau_{\theta wR} \, 2\pi R^2 S_w \tag{44}$$

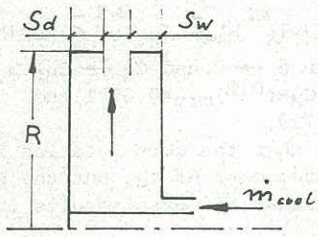

Figure 1 Schematic presentation of the geometry of the system

Using eqs. (33), (35), (38) and (40) to (44), eq. (30'), after dividing each term by $0.5\rho\omega^2R^5$, takes on the form

$$2\pi B\left(\frac{n+3}{5n+11} + S_d'\right) - 2\pi B'(1-k)^{\frac{2n+4}{n+3}}\left(\frac{n+3}{5n+11} + S_w'\right) = C_{\dot{m}} Re^{\frac{2}{n+3}}\left[(1-k) - \left(\frac{r_{in}}{R}\right)^2 \frac{\omega_{in}}{\omega}\right] \tag{30''}$$

from which k can be determined, and then the torque from eq. (41). For A=8.74 and n=7, eq. (30") has the form

$$(0.0891 + 0.4098 S_d')k^{0.8}(\alpha^2+k^2)^{0.3}(\alpha e)^{0.2} - (0.0678 + 0.3116 S_w')(1-k)^{1.8} = C_{\dot{m}} Re^{0.2}\left[(1-k) - \left(\frac{r_{in}}{R}\right)^2 \frac{\omega_{in}}{\omega}\right] \tag{30'''}$$

where

$$\alpha = k\left(\frac{0.25 - 0.222k}{1.8783 - 0.8196k}\right)^{0.5}; \quad e = 0.4083 - 0.3403k$$

5 PRESSURE AS A FUNCTION OF RADIUS

It is assumed, that $\delta p/\delta z=0$ in the gap and that pressure gradient depends only on the core flow conditions. Pressure gradient can be written as

$$dp/dr = \rho(V_\theta^2/r - V_r \, \partial V_r/\partial r) \tag{45}$$

Term $V_z \, \delta V_r/\delta_z$ is omitted in the relation (45), as $\delta V_z/\delta_z$ is assumed to be zero within the core flow. Term $V_r \, \delta V_r/\delta_r$ is zero (or almost zero) if there is no throughflow ($\dot{m}=0$), whereas in the presence of coolant flow ($\dot{m}>0$), it is negligible in comparison to V_Θ^2/r, as V_{rav} is usually of the order of less than 1 m/sec to max. 2 to 3 m/sec. This simplifying assumption was verified by the sample calculations for a few actual cooling systems of gas turbine rotors. Consequently eq. (45), after integration takes on the simple form

$$p = 0.5\rho\omega^2(1-k)^2 r^2 + C$$

where C can be found from the known pressure at r_{in} or r_{out}.

6 CONCLUSIONS

The relationship between a torgue coefficient and other physical quantities has been found in the form $C_M = f(Re, S_d', S_w', C_{\dot{m}})$.
The solution is easily adaptable for a particular range of Reynolds number, as the constants A and n can be determined e.q. from the logaritmic law, which is claimed to be in excellent agreement with experimental data for any Reynolds number.
If $S_d=0$, then a free disk and a closed system with $\dot{m}_{cool}=0$ constitute the limiting cases of the general solution. This can be presented by the following inequality.

if $S_d=0$, $S_w \gtrless 0$, then

$$(C_M Re^{0.2})_{min} = (C_M Re^{0.2})_{\dot{m}=0} \leq C_M Re^{0.2} < (C_M Re^{0.2})_{f.d.}$$

When $S_w \rightarrow \infty$, then $\beta \rightarrow 0$ and $C_M \rightarrow C_{Mf.d.}$, for any $\dot{m}_{cool}$. For n=7, $(C_M Re^{0.2})_{min}=0.0371$ and $(C_M Re^{0.2})_{f.d.}=0.0729$.
When S_d increases, then the core rotation increases, resulting in the decrease of τ_d, but the prevailing effect of the increase of C_{Mdsh} results in the increase of C_{Mrot}.
Fig. 2-4 present relationship between the torgue coefficient and shroud length, mass flow rate coefficient and core rotation, for n=7.
The torgue coefficient of a free disk, $C_{Mf.d.} = 0.0729\ Re^{-0.2}$ (for n=7) in the present analysis, whereas the value often quoted in the literature is $C_M' = 2\ C_M = 0.146\ Re^{-0.2}$, which is almost identical (in many papers the moment of both sides of a rotating disk is considered).
A comparison of the computation, based on the present analysis with the experimental results, obtained by Daily and Nece [3] (n=7, $\dot{m}_{cool}=0$, $S_d=0$) are presented below

S_w'	β/ω	$C_M Re^{0.2}$	β/ω presented	$C_M Re^{0.2}$ calculation
0.025	0.490	0.0755	0.470	0.0388/0.0776*
0.0636	0.464	0.0772	0.447	0.0394/0.0788*
0.10	0.445	0.080	0.428	0.0413/0.0826*
0.2145	0.411	0.0873	0.382	0.0447/0.0884*

*) 2 x $C_M Re^{0.2}$

An agreement with experiments is fairly good, though the effect of the shroud is accounted for in a very simplistic way in the present analysis.

When $0<\dot{m}_{cool}<\dot{m}_{f.d.}$, the more precise solution would be obtained, if the flow within the core were included into the set of basic equations, resulting in nonlinear relationship between V_Θ and r in the core. It seems to be a reasonable supposition, that when $\dot{m}_{cool} \approx \dot{m}_{f.d.}$, then $C_{Md} \approx C_{Mf.d.}$ and $\beta=0$ (provided that $S_d=0$), while according to the present analysis C_{Md} (corresponding to $\dot{m}_{cool}=\dot{m}_{f.d.}$) is 6.5% less than $C_{Mf.d.}$, and $\beta/\omega=0.14$. Consequently it can be estimated, that the difference between the more precise solution and the present one, in which two separate boundary layers are considered, bounded by integral form of moment of momentum equation (30), is from zero for $\dot{m}_{cool}=0$ to about 6.5% for $\dot{m}_{cool}=\dot{m}_{f.d.}$.
When $\dot{m}_{cool} \gtrsim \dot{m}_{f.d.}$, then the inward flow along the stationary wall should not occur, and probably this is a sufficient condition for preventing an ingress of hot gases from the main stream, which is a trouble making phenomenon in the cooling system of gas turbine rotors.

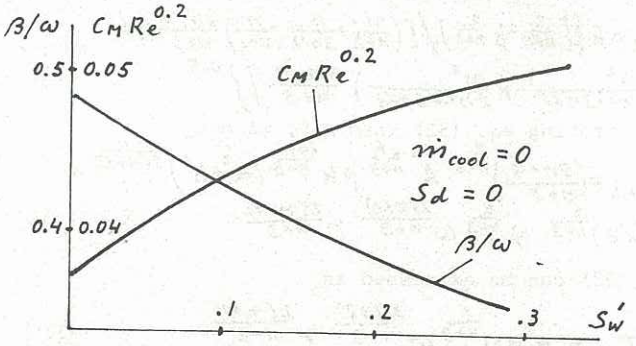

Figure 2 C_M and β/ω as functions of S_w'

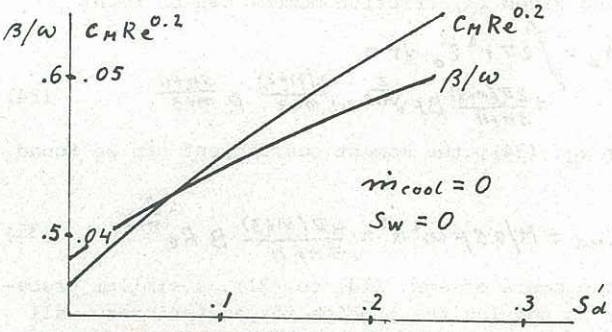

Figure 3 C_M and β/ω as functions of S_d'

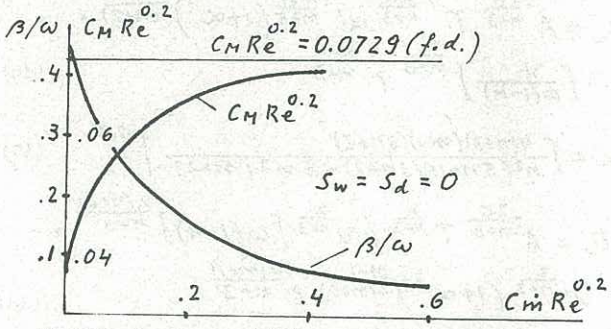

Figure 4 C_M and β/ω as function of $\dot{m}$

7 ACKNOWLEDGEMENT

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