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Stability of Elastico-Viscous Poiseuille Flow with Flexible Boundaries

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SUMMARY Stability of elastico-viscous plane Poiseuille flow with flexible boundaries is analysed. The study is restricted to elastico-viscous liquids with short memories. By requiring the fluid and flexible boundary motion to be compatible at the interface, the eigen-value problem is posed and then solved by a graphical method. Sets of neutral stability curves are obtained for various values of flexible boundary and fluid elasticity parameters. The nature of influence on flow stability depends on the values of these parameters.

1 INTRODUCTION

Kramer(1960) reported substantial drag reduction in water for cylindrical bodies covered with flexible boundaries. This triggered considerable theoretical research work by Benjamin(1960), Landahl(1962), Kaplan(1964), Gyorgyfalvy(1967), Babenko and Kozlov(1972), Pathak and Chaturvedi (1977) and others.

Out of various viscous drag reducing techniques, the use of flexible boundaries needs lot of further detailed research because of its possible practical applications in water borne and air borne vehicles (Bushnell et.al.1976). The term flexible boundaries implies a wide class of rubber like elastic surfaces which posses the property of interacting with the flow system. A proper coupling between the flow and the flexible boundary may have significant stabilising and drag reducing effects. Flexible boundary research may also play a significant role in bioengineering and analytical study of biological flow systems which usually have flexible boundaries.

Flows of fluids with memory are also becoming increasingly important in context of drag reducing properties of very dilute solutions of high molecular weight polymers. In view of the above it would be interesting to study the interaction of flexible boundary and elastico-viscous fluid. In the present paper, the authors investigate the stability of elastico-viscous plane Poiseuille flow with flexible boundaries. In section 2 the elastico-viscous stability equation, the flexible boundary model, the associated boundary conditions and solution of the resulting eigen-Value problem are discussed. Results which bring out the effect of flexible boundary parameters and fluid elasticity are presented in section 3. Finally, conclusions are given in section 4.

2 THE PROBLEM AND ITS SOLUTION

It has been shown by Chun and Schwarz (1968) and Walters(1962) that for the second-order fluids and Walters's fluid A' and B' with short memories, the stability equation takes the following form

$$(U-c)(\alpha^2\phi - \phi'') + \phi U'' + \frac{\phi U}{S}$$

$$= \left(\frac{U-c}{S} - \frac{1}{i\alpha R}\right)(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi), \qquad (1)$$

where U is the undisturbed velocity in x-direction, ϕ is the amplitude, α the wave number, c the wave velocity, R the Reynolds' number, 1/S a small fluid elasticity parameter and primes denote differentiation with respect to y. For a vanishing 1/S, (1) reduces to the Orr-Sommerfeld equation.

The flow under consideration is the plane Poiseuille flow for which $U = 1 - y^2$. As in the classical viscous problem, the consideration is restricted to even solution corresponding to antisymmetric disturbances which cause instability of fluid motion.

The stability equation(1) has one relevant even 'inviscid' solution ψ_1(Stuart,1954) and one 'viscous' solution ψ_2(Fong and Walters ,1965). These two basic solutions must be appropriately combined to satisfy the boundary conditions of the problem under study.

2.1 Flexible Boundary Model

The model of the flexible boundary to be considered is a taut membrane supported over a foam like elastic base. The mechanical behaviour of this model is determined by its mass per unit length m, damping per unit length d_0, tension T and spring constant K_0. The dynamic equation of such a model is

$$T\frac{\partial^2\eta}{\partial x^2} - m\frac{\partial^2\eta}{\partial t^2} - d_o\frac{\partial\eta}{\partial t} - K_o\eta = p_b, \qquad (2)$$

where η is the boundary displacement in y direction and p_b is the pressure at the boundary.

The flexible boundary undergoes displacements in response to travelling pressure wave disturbances of the form

$$p_b = \hat{p}_b \exp[\, i\alpha\,(x - ct)\,]\,, \qquad (3)$$

where p_b is the amplitude of pressure wave disturbance. The travelling wave type displacements of the flexible boundary can be expressed in the form

$$\eta = \hat{\eta}\;\exp\,[\,i\alpha\,(x - ct)\,]\,, \qquad (4)$$

$$\xi = \hat{\xi}\;\exp\,[\,i\alpha\,(x - ct)\,]\,,$$

where $\hat{\eta}$ and $\hat{\xi}$ are the amplitudes of displacements from the mean position of the flexible boundary in y and x directions respectively.

Using the accoustical concept of admittance the normal and travelling wave admittance of the flexible boundary can be respectively expressed as

$$Y_{11} = -\frac{\partial\eta/\partial t}{p_b} = \frac{i\alpha\hat{\eta}c}{p_b}\,, \qquad (5)$$

$$Y_{12} = -\frac{\partial\xi/\partial t}{p_b} = i\alpha\hat{\xi}c\,.$$

Defining the propagation velocity of disturbances in membrane as $c_{om} = (T/m)^{1/2}$, the cut-off frequency as $\omega_o = (K_o/m)^{1/2}$ and the wave propagation velocity for the composite model as $c_o = \sqrt{(c_{om}^2 + \omega_o^2/\alpha^2)}$ and using(3), the expressions(5) take the form

$$Y_{11} = -\frac{ic}{m\alpha(c_o^2 - c^2 - icd_o/\alpha)}\,, \qquad (6)$$

$$Y_{12} = 0.$$

2.2 Boundary Conditions

Boundary conditions at the flexible boundary are obtained by satisfying the requirement of compatibility of fluid and flexible boundary motion at the interface. These after linearisation takes the form

$$\psi_{1b} + A\,\psi_{2b} = c\hat{\eta}\,, \qquad (7)$$

$$\psi'_{1b} + A\,\psi'_{2b} = -U'_b\,\hat{\eta}\,,$$

where A is a constant and subscript b indicates the value at the mean position of the flexible boundary.

Using (5), the equations (7) become

$$\psi_{1b} + A\,\psi_{2b} = \frac{Y_{11}\,\hat{p}_b}{i\alpha}\,, \qquad (8)$$

$$\psi'_{1b} + A\,\psi'_{2b} = -U'_b\,\frac{Y_{11}\,\hat{p}_b}{i\alpha c}\,,$$

where $\hat{p}_b = c\psi_{1b} + U'_b\,\psi_{1b}$, (Benjamin, 1960).

Elimination of A from (8) leads to

$$\frac{\psi_{2b}(\xi_b)}{\xi_b\,\frac{d}{d\xi}[\psi_{2b}(\xi_b)]} = \frac{1}{1-y_c}\left[\frac{Y_{11}\,\hat{p}_b/i\alpha - \psi_{1v}}{-U'_bY_{11}\,\hat{p}_b/i\alpha c - \psi'_{1b}}\right],$$

which can be written as

$$D(\lambda\,,\,\xi_b) = E(\alpha^2,\,y_c,\,Y_{11})\,. \qquad (9)$$

For any given value of λ and Y_{11}, the complex equation (9) can be solved graphically for ξ_b, α and y_c. Fong and Walters(1965) have expressed the function D as

$$D = (F_r + iF_i)\left[1 + \lambda(G_r + iG_i)\right], \qquad (10)$$

where $\lambda = (\alpha\,RU'_c)^{2/3}/S$.

Values of F_r, F_i, G_r and G_i are tabulated for ξ ranging from -0.9 to 4.9. For a given λ, the real and imaginary parts of $D(\lambda,\xi_b)$ are plotted on a polar diagram. The function $E(\alpha^2, y_c, Y_{11})$ is then computed with the help of a computer program and plotted on the same polar diagram. For prescribing Y_{11}, the mass coefficient m, following Landahl (1960), is assumed proportional to α^{-2} for $\alpha < 0.8$ and equal to a constant value 2 for $\alpha > 0.8$. The intersection of the two curves gave the desired eigen values ξ_b, α and y_c. Reynolds number R is then calculated from the relation

$$\xi_b = (1-y_c)\,(\alpha\,RU'_c)^{1/3}\,. \qquad (11)$$

Sets of neutral stability curves are obtained for variations in flexible boundary and fluid elasticity parameters.

3 DISCUSSION OF RESULTS

Typical neutral stability curves for λ = 0.08 and their response to variations in flexible boundary parameters c_o and d_o is illustrated in Figures 1 and 2. The dotted curve corresponds to the rigid boundary case and is given for the sake of comparison. From such neutral stability curves, the critical Reynolds number R_c for any combination of λ, c_o and d_o could be found out.

Figure 3 depicts the effect of changes in d_o (keeping c_o = 0.75) on the stability of flow for various values of λ. It can be seen that R_c increases with increase in d_o for all λ.

Figure 4 shows the effect of variations in c_o (keeping d_o = 0.025) on the flow stability. It is found that for Newtonian fluid increase in c_o causes a continuous increase in R_c. For elastico-viscous fluids($\lambda \neq 0$), an increase in c_o upto a certain value ($\approx$ 0.55 here) leads to increase in R_c and thereafter any further increase in c_o causes a decrease in R_c.

Figure 5 illustrates explicity the combined effect of λ and d_o (for c_o = 0.75) on flow

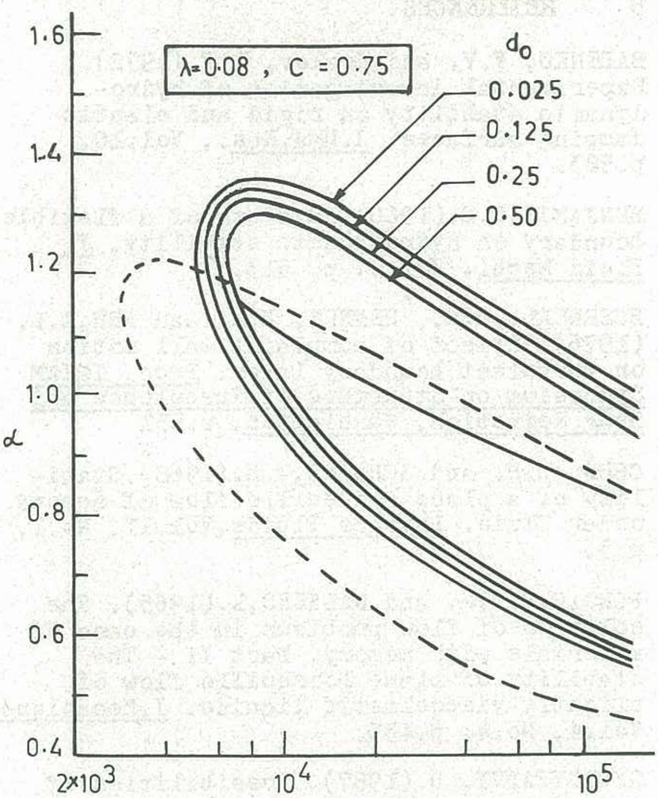

Figure 1 Neutral Stability Curves for various d_o

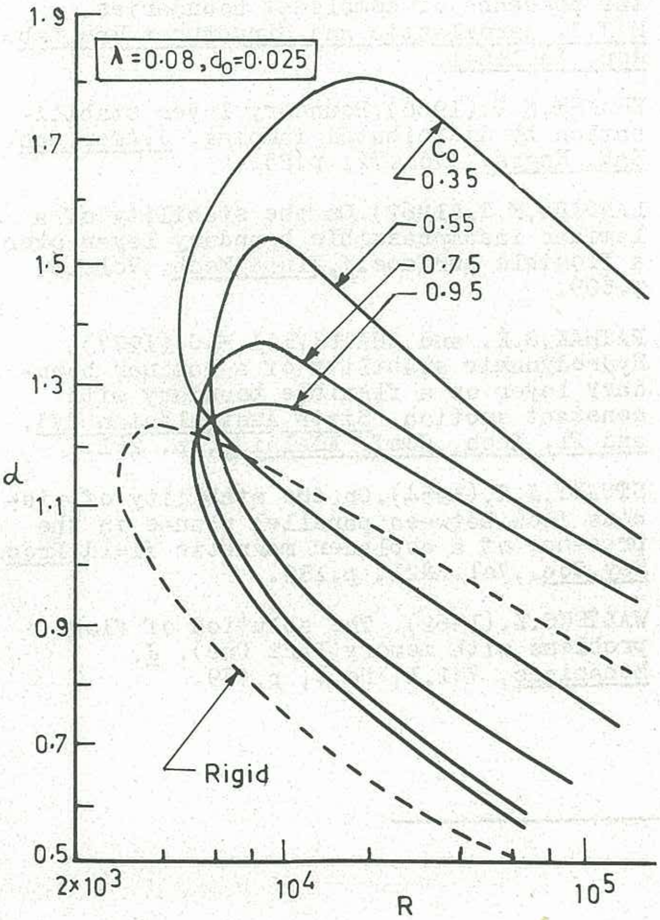

Figure 2 Neutral Stability Curves for various c_o

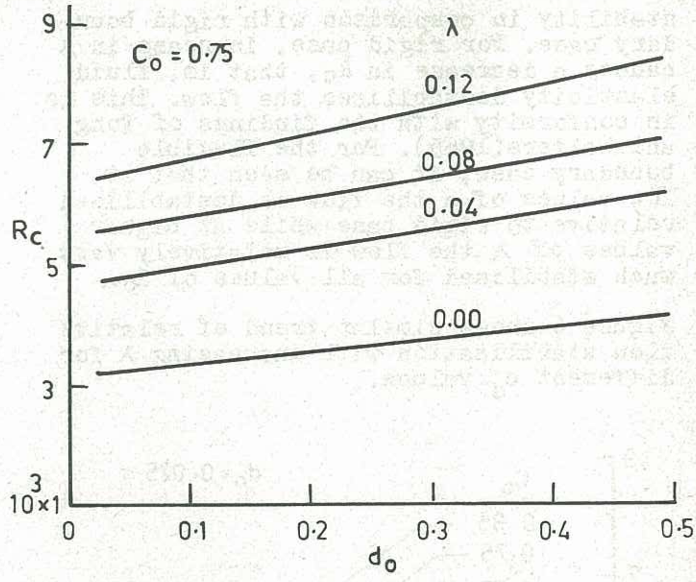

Figure 3 Effect of d_o on R_c

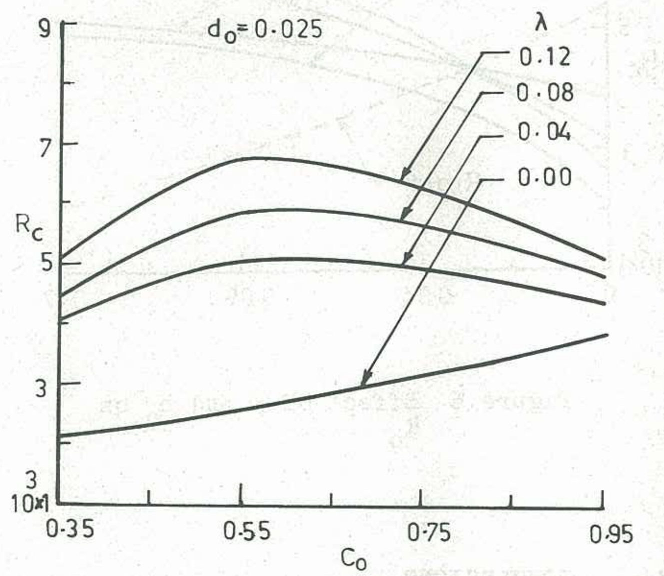

Figure 4 Effect of λ and d_o on R_c

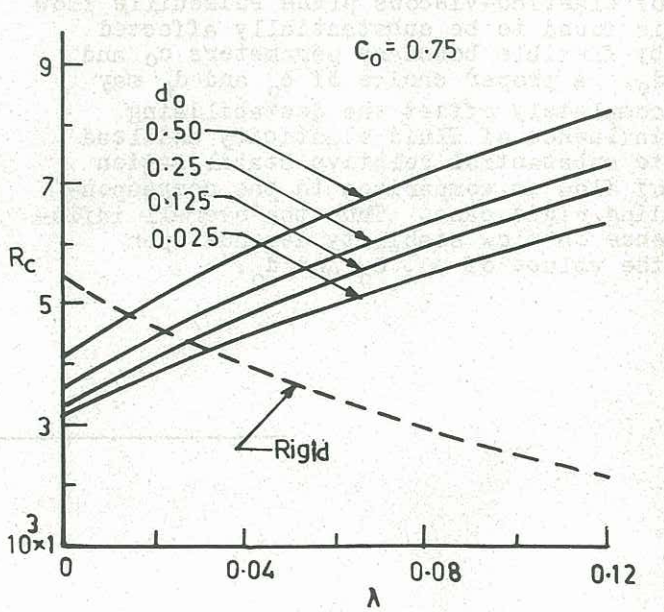

Figure 5 Effect of λ and c_o on R_c

stability in comparison with rigid boundary case. For rigid case, increase in λ causes a decrease in R_c, that is, fluid elasticity destabilises the flow. This is in conformity with the findings of Fong and Walters(1965). For the flexible boundary case, it can be seen that at low values of λ the flow is destabilised relative to rigid case while at higher values of λ the flow is relatively very much stabilised for all values of d_o.

Figure 6 shows similar trend of relative flow stabilisation with increasing λ for different c_o values.

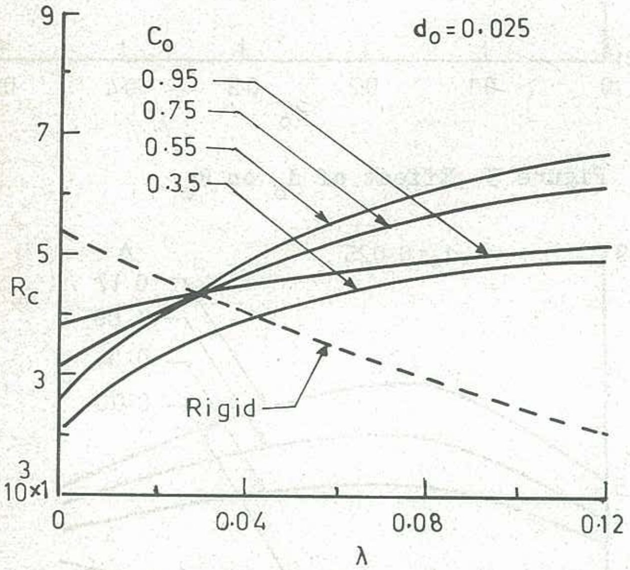

Figure 6 Effect of λ and c_o on R_c

4 CONCLUSIONS

The critical Reynolds number R_c in case of elastico-viscous plane Poiseuille flow is found to be substantially affected by flexible boundary parameters c_o and d_o. A proper choice of c_o and d_o may completely offset the destabilising influence of fluid elasticity and lead to substantial relative stabilisation of flow in comparison to the corresponding rigid case. Thus the overall influence on flow stability depends upon the values of λ, c_o and d_o.

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