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The Dynamic Behaviour of Propeller Anemometers

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SUMMARY A model equation is developed to predict the response of propeller anemometers in turbulent flow. The equation is nonlinear, and is shown to describe anemometer motion correctly for the situations which are easily simulated in a wind tunnel. Implications for the measurement of turbulent flows are discussed.

1 INTRODUCTION

The propeller anemometer is used to measure wind structure over a large range of windspeeds in a wide range of applications. These instruments are cheaper than laser anemometers and more robust than hot-wire sensors. They have low starting speed and, unlike cup anemometers, can be used to measure wind direction. However a number of anomalous features of propeller anemometer behaviour have been reported in the literature. These include a non-cosine response to wind direction, changes in directional response and calibration constant with wind speed and changes in time constant with wind speed and direction. The purpose of this paper is to develop a model equation of motion for the anemometer which accounts for these anomalies and thereby allows a more accurate recovery of the wind velocity vector from anemometer signals.

Particular attention is paid to the Gill anemometer, a lightweight helicoid driving a d.c. tachometer generator. The experimental results presented below refer to the model 27101, made by R.M. Young Co.

Notation

I	-	moment of inertia of rotating parts
P	-	effective pitch
R	-	propeller radius
U	-	wind speed
U_s	-	stall speed of propeller
ω	-	rate of rotation of propeller
θ	-	angle between propeller axis and wind vector
ρ	-	air density
ν	-	air kinematic viscosity

2 THE EQUATION OF MOTION

An equation of motion for the anemometer is given by equating the rate of change of angular momentum of the moving parts to the sum of the various torques arising in an unsteady wind flow. If it is assumed that the aerodynamic forces on the propeller depend only upon the instantaneous state of flow (i.e. no added mass or similar effects) then dimensional considerations lead to an expression for the aerodynamic torque of the form

$$Q_A = \rho U^2 R^3 q_c \tag{1}$$

Here q_c is a nondimensional torque coefficient which can be written, for each particular propeller,

$$q_c = q_c(\lambda, \theta, Re) \tag{2}$$

where
$$\lambda = \omega R/U, \tag{3}$$
$$Re = UR/\nu.$$

It is implied that U and θ are values averaged over the swept area and blade-passing interval of the propeller.

Now the ideal propeller anemometer (with no mass or shaft friction) would always run in a state of zero aerodynamic torque so that in steady or unsteady flow

$$\lambda(t) = \lambda_0(\theta, Re)$$

where λ_0 is the solution of the steady flow expression

$$q_c(\lambda_0, \theta, Re) = 0.$$

If a real instrument always stays close to this ideal behaviour we may expect that $|\lambda - \lambda_0| << 1$ so that q_c may be expanded

$$q_c = \left|\frac{dq}{d\lambda}\right|_{\lambda_0} (\lambda - \lambda_0),$$

leading to

$$Q_A = \rho\frac{R^4}{P} Ug(\theta)\ (Uf(\theta) - \omega P) \tag{4}$$

where $f(\theta) = \lambda_0 P/R$, $g(\theta) = -[dq/d\lambda]_{\lambda_0}$. Note that in the last step the dependence of f and g on Reynolds number has been dropped. This is because the lift and drag coefficients of the propeller blades are not expected to be strongly affected by Re over the operating range of the propeller.

The real instrument also has a frictional torque which opposes shaft rotation. This must vary according to the load carried by the bearings and their rate of rotation, but it is found that friction is adequately accounted for if this retarding torque is assumed to be a constant, Q_S (which may vary between anemometers). Equating the torques Q_S and Q_A with the rate of change of angular momentum now gives the desired model equation of motion;

$$LP\frac{d\omega}{dt} = U\frac{g(\theta)}{g(o)}\ [Uf(\theta) - P\omega] \mp U_s{}^2 \tag{5}$$

where $g(o)L = I/\rho R^4$ and $g(o)U_S{}^2 = PQ_S/\rho^4$, the sign being chosen to oppose that of ω. Equation (5) is the basis of this paper. It has three unknown functions, which can be evaluated by wind-tunnel calibration as explained below.

An interesting point is that for a frictionless anemometer the model equation can be written

$$\frac{Lg(o)}{g(\theta)}\frac{d\omega}{dx} + \omega = Uf(\theta)/P\ ,$$

where $U = {}^{dx}/dt$. Thus the anemometer is a linear first-order velocity transducer, but only if the wind run x is the independent variable (as we might expect from its design).

3 ANEMOMETER RESPONSE

3.1 Steady Axial Flow

The constant P can be chosen so that f(o) = 1, when in steady axial flow (5) reduces to

$$P\omega \;=\; U \mp U_s^{\,2}/U \qquad (6)$$

In high-speed tests ($U \gg U_S$) P may be evaluated as the ratio U/ω - it is therefore the effective pitch of the propeller. Blade element theory could be used to predict the relationship between P and the geometric pitch (in terms of lift and drag coefficients). Since it is much easier to find P by experiment, we note here only that the theory indicates that blade drag should cause P to be a few percent higher than its geometric value. The propeller radius also has an effect - it is found that P tends to decrease with radius R (R.M. Young, 1972; Lindley et al, 1974b; Jackson, 1976).

At slower speeds it is found experimentally that ${}^{U}/\omega$ depends strongly on U - for example Brook (1974) found that at 1 m/s this calibration ratio was 14% higher than P. Precisely this behaviour is predicted by (6). If U and ω are measured over the speed range 0-1 m/sec, then plotted as Uω versus U^2, a fitted straight line intercepts the axis Uω = 0 at U_s^2. Since this procedure optimises the fit of equation (6) to the actual anemometer response at low speed it does not necessarily correspond to the speed at which the propeller first starts to rotate, usually called the threshold speed. Although in practice the two speeds are found to have similar magnitudes (Jackson, 1976), to distinguish U_s it will be referred to as the stall speed.

At such low speeds it is difficult to measure U accurately. Jackson (1976) and Milson (1978) solved this problem by towing propellers through still air. Some of Milsons results are plotted in Figure (1) in the manner suggested by (6). The agreement is good down to speeds close to the stall speed.

3.2 Steady Oblique Flow

Equation (5) now gives

$$\frac{\omega(\theta)}{\omega(o)} = \frac{f(\theta) \mp U_s^{\,2} g(o)/U^2 g(\theta)}{1 \mp U_s^{\,2}/U^2} \qquad (7)$$

so that at high speeds ω(θ)/ω(o) = f(θ), and the function f is recognised as just the directional response of the frictionless propeller. In principle f(θ) and g(θ) can be found directly from the torque coefficient q_C - this requires Q_A to be measured with the anemometer forced to rotate at various ω for each U and θ. Wyngaard et al. (1971) used this method to evaluate similar unknowns for a cup anemometer (although they did not notice that their data show a good agreement with the formulation used here in (4)).

It is easier to determine f(θ) by high-speed calibration (except that this will not work near θ = 90°). The resulting shape of this function is well documented. It shows large departures (up to 15%) from the ideal form f(θ) = cosθ, the error increasing as the propeller radius decreases. The author has obtained results which agree well with the manufacturers calibration for angles θ < 90°, but for θ > 90° finds a dependence of f(θ) on windspeed. Hicks (1972) reports this same problem for all θ with an anemometer fitted with a shaft extension, so it is likely to be caused by interference between the wake of the shaft and the propeller downstream.

Equation (7) predicts that there is always a small range of angles near 90° where the propeller does not rotate, and that this range of stall should decrease as U increases. This is also observed in practice - at 87°, a 230 mm Gill propeller will rotate at 6 m/sec but not at 2 m/sec.

3.3 Unsteady Motion

Solutions to (5) when U is constant but ω varies are easily obtained. If a small braking torque which slows the anemometer to speed ω_0 (but without stalling) is suddenly released, the anemometer response is

$$\omega \;=\; \omega_1 + (\omega_0 - \omega_1)e^{-\frac{Utg(\theta)}{Lg(o)}}$$

so by recording ω we can find the time constant T = Lg(o)/Ug(θ). In axial flow L = UT, which is the expression normally used to define the response length of the propeller. Measurements on the Gill propeller give L = 1.0 ± .05 m, depending on propeller radius. For oblique flow it is convenient to speak in terms of the axial response length, defined as UTcosθ. The ratio of the axial response length for some arbitrary θ to its value for θ = 0 is then found to be g(o)cosθ/g(θ), which shows the physical significance of the function g(θ). The ratio ${}^{g(\theta)}/g(o)$ has been measured earlier - Figure (2) summarises the results of Hicks (1972), Duchon et al, and Jackson (1976). The behaviour of this function near θ = π/2 is important for anemometers used to measure velocities transverse to the mean flow direction. Hicks postulated that it has the form $(\cos\theta)^{\frac{1}{2}}$ near θ = π/2, but there is too much scatter in the data to confirm this. The author prefers to use $(\cos(0.98\theta))^{\frac{1}{2}}$, which does not vanish at θ = π/2.

3.4 Motion in Turbulent Flow

When U makes small fluctuations about a mean $\overline{U}$ it is clear from (4) that the change in aerodynamic torque for a small increase in U is greater than the corresponding change for the same decrease in U. Thus the mean value of ω is greater than that obtained by substituting $\overline{U}$ for U in (5). This overspeeding is a well-known characteristic of rotating anemometers. The magnitude of the error can be obtained by perturbation methods in the cases where the fluctuations in velocity about the mean are relatively small (Kaganov and Yaglom, 1976). If we consider speed changes only (θ = 0) it is not difficult to show that

$$P\overline{\omega} \;=\; \overline{U} - \frac{U_s^{\,2}}{\overline{U}} + \frac{1}{\overline{U}}\int_0^\infty E(s)\,\frac{L^2s^2 - U_s}{L^2s^2 + \overline{U}^2}\,ds + \ldots$$

$$\overline{P^2(\omega-\overline{\omega})^2} \;=\; \frac{1}{2\overline{U}^2}\int_0^\infty E(s)\frac{(\overline{U}^2 + U_s^{\,2})^2}{L^2s^2 + \overline{U}^2}\,ds + \ldots$$

where E(s) is the power spectral density of U(t) at angular frequency s, and the overbars denote a time average. The ideal values are given by $L = U_S = 0$, so in practice the mean speed may be over- or underestimated according to the relative magnitudes of L and U_S. A first-order linear instrument with time constant $L/\overline{U}$ would give a similar expression for the mean square, but would predict $P\overline{\omega} = \overline{U}$.

Anemometers used to measure transverse wind components operate near θ = π/2. It is interesting to determine whether simple solutions to (5) could also be obtained for $\theta \sim \pi/2$. The response functions can then be approximated by

$$f(\theta) = a \sin\phi,$$

$$g(\theta)/g(o) = b + c/\sin\phi/$$

where $\phi = \theta - \pi/2$, and a,b,c are constants. Writing the axial component of velocity as $U_A = U \sin\phi$, (5) becomes

$$LP \frac{d\omega}{dt} = (Ub + c/U_A/)(aU_A - P_\omega) \mp U_s$$

We can now postulate, for example, that U be replaced by $\overline{U}$, and $/U_A/$ by some function of its r.m.s value σ_ω, say $\sigma_\omega h(\sigma_\omega/\overline{U})$ - this is equivalent to saying that for $\theta \sim \pi/2$ the response length distribution $g(\theta)/g(o)$ becomes a function only of the intensity of transverse turbulence $I = \sigma_\omega/\overline{U}$. Equation (8) is then (omitting the U_s term)

$$\frac{Le}{\overline{U}} P \frac{d\omega}{dt} + P_\omega = aU_A \qquad (9a)$$

with an effective response length given by

$$\frac{L_e}{L} = [b + cIh(I)]^{-1} \qquad (9b)$$

This is first-order and linear, and would allow transverse spectra to be computed directly from the anemometer signals.

Now Fichtl and Kumar (1974) computed effective response lengths from the ratio of the power spectral density of the output of a vertical anemometers $E_a(s)$ to the p.s.d. of the transverse wind velocity, E(s). They found that this ratio was well represented by

$$\frac{E_a(s)}{E(s)} = [1 + (s\, L_e/\overline{U})^2]^{-1},$$

which is exactly what (9a) would predict. They also showed that L_e is a function of I (and indeed this idea is theirs) - their data has been replotted here as L_e/L versus I, in Figure (3). Also shown is their best fit line ($b = 0$, $h \propto I^{-1/3}$), but there seems to be no reason to reject a straight-line fit. A tentative choice is $L_e/L = 0.05 + 1.5\, I$ (where the value $b = 0.05$ also fits Figure (2)). It is concluded that there are reasonable grounds for expecting (9a) to be a good model of transverse propeller behaviour, but that it should be used with caution until the function L_e/L is more clearly defined.

Finally, when fluctuations in U or θ are large we can resort to numerical solutions of the equation of motion. The anemometer output P_ω has been calculated in this way for the flow $\theta = 0$, $U = \overline{U} + \Delta u \sin(2\pi \overline{U} t/\gamma)$ for various values of Δu and wavelength γ (and with $\overline{U} = 10$ m/s, $U_s = .15$ m/s, and $L = 1.0$ m). The phase shift and attenuation were found to be quite noticeable when $\Delta u/\overline{U} = 0.2$, even for a wavelength $\gamma = 16$ m, but both were predicted well by first-order linear theory with a time constant $L/\overline{U}$. For $\Delta u/\overline{U} = 0.4$, P_ω became appreciably non-sinusoidal so the linear theory would be less valid. The output for $\Delta u/\overline{U} = 0.8$ is shown in Figure (4), where the distortion has become severe.

Also found was the response of an anemometer set at 90° to the flow $U = 10$ m/s, $\theta = 0.3 \sin(2\pi Ut/\gamma)$ for which the ideal response would be $U\sin\theta$. Here the numerical solution used the function $f(\theta)$ given by the manufacturer, and $g(\theta)/g(o) = [\cos(0.98\theta)]^{1/2}$.

There is not much distortion of the shape of the response, but now there is large phase shift and attenuation even for $\gamma = 16$ L.

4 INTERPRETATION OF SIGNALS

It is now assumed that (5) does provide a reliable model of propeller anemometer behaviour, so that it can be used to infer U and θ from measurements of anemometer signals. We have found that axial velocities can be calculated easily only in the special cases of low turbulent intensity and $\theta \sim 0$ or $\theta \sim \pi/2$. However in general it will be necessary to measure simultaneously the rotation rates of three anemometers of differing orientation in order to find the three components of wind velocity.

Since the model equation (5) is not linear in U, it cannot be used to find unique values of velocity - that is, a given signal $\omega(t)$ can be generated by more than one velocity vector. This can be seen by rewriting (5) to solve for $Uf(\theta)$;

$$(2Uf(\theta) - P_\omega)\, \mathrm{sign}[2Uf(\theta) - P_\omega]$$
$$- \left(\left|P^2\omega^2 + \frac{4g(o)}{g(\theta)} f(\theta)[LP\dot{\omega} \pm U_s^2]\right|\right)^{1/2} \qquad (10)$$

A unique value of $Uf(\theta)$ can be found only if there is some additional means of sensing the sign of $(2Uf - P_\omega)$.

The obvious way of resolving this problem is to assume that $(2Uf - P_\omega)$ and P_ω have the same sign. This is incorrect only if $(2Uf - P_\omega)$ changes sign, when the wrong solution may be chosen for some of the time.

It remains to calculate three velocity vectors from three values of ω (from say three orthogonal anemometers). Equation (10) shows the way to proceed. First, an assumed value of θ is used with $\omega(t)$ and $\dot{\omega}(t)$ to give a solution for $Uf(\theta)$ for each anemometer. An iterative solution for U and θ is then required (as pointed out by Horst (1973)). The new value of θ is then used to recalculate the $Uf(\theta)$, until the process converges. Details of a successful procedure are given by Jackson (1976). This solution must be carried out to find the velocity at each time step, so use of a computer is essential.

One point which is often overlooked is that anemometers used in an array may interfere with one another when any angle of attack θ exceeds $3\pi/4$ (approximately). This means that ideally the directional response should be determined with the other array anemometers present, and also that this response will not be a function only of the angle θ.

5 CONCLUSIONS

A model equation of motion has been developed for the propeller anemometer which correctly predicts the anomalous features of the behaviour of this instrument. This equation is nonlinear so that in general the statistical properties of the wind velocity cannot be obtained from corresponding properties of the anemometer signals. It is then necessary to solve the model equation to give the effective windspeed $Uf(\theta)$ at each time step. Since computer analysis is necessary in any case to derive the actual velocity components from the $Uf(\theta)$ values from three anemometers, there is little extra effort involved in accounting for the effects of friction and propeller inertia. The model equation shows how to compute these effects, and thus allows the use of propeller arrays in turbulence of larger magnitude and frequency than has previously been possible.

The anemometer behaviour is least well described when the anemometer axis is nearly normal to the wind flow, and moreover the model equation suggests that decoding of signals can then be ambiguous. To avoid these difficulties it is suggested that propeller arrays should be mounted so that no anemometer is normal to the mean wind direction.

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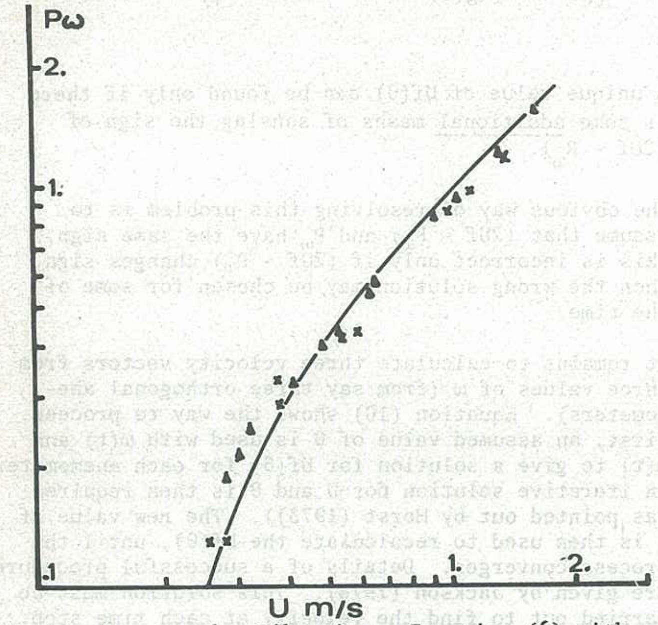

Fig. 1 Low-speed calibration. Equation (6) with U_S = 0.2 m/s (solid line) compared with Milson data for two propellers

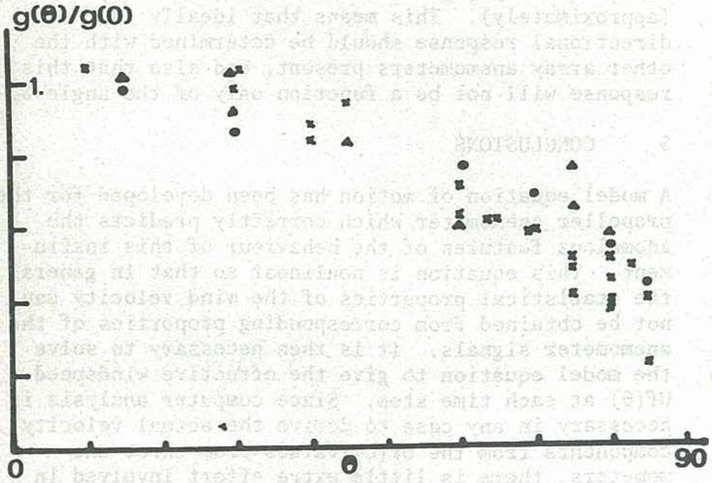

Fig. 2 Change in response function with angle: Jackson(1976), Duchon et al, Hicks(1972)

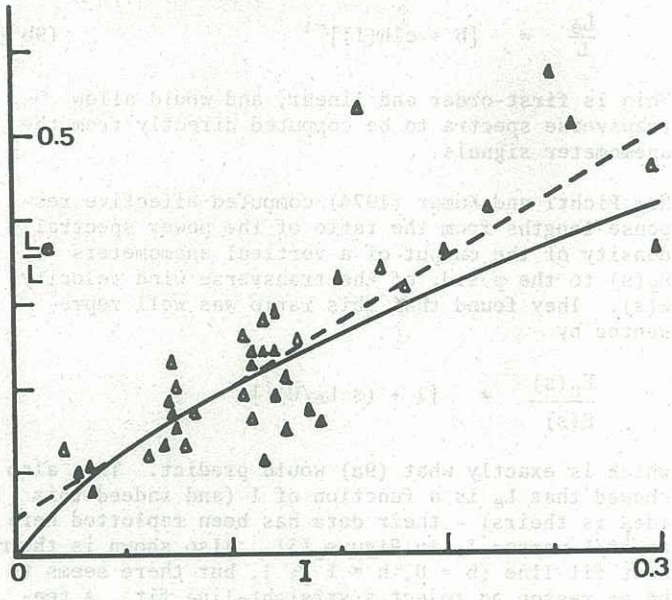

Fig. 3 Change in effective response length with intensity of transverse turbulence (- - - fitted line of Fichtl and Kumar)

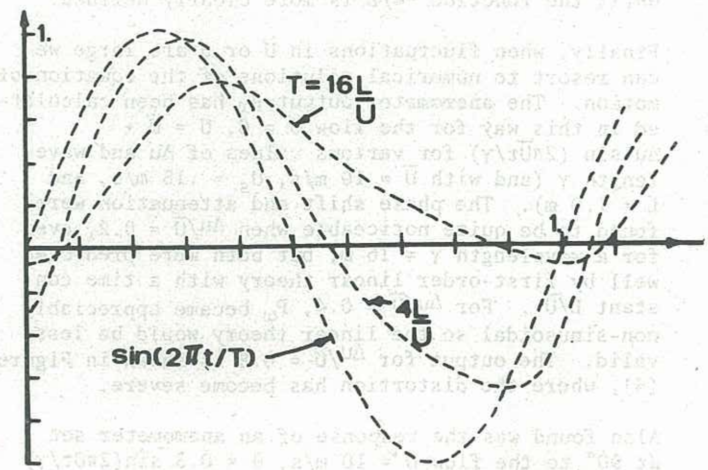

Fig. 4 Axial anemometer response: $(P\omega-\bar{U})/\Delta u$ versus $2\pi t/T$, when $\theta=0$, $U=10+8\sin(2\pi t/T)$