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Progress with Modelling Diurnal Temperature Profiles in the Upper Ocean

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1 INTRODUCTION

Many oceanographers have reported that, under conditions of low wind and bright sunlight, the nominally iso-thermal mixed upper layer of the ocean develops a well-marked temperature rise in the top 5-20 m. This is often termed "afternoon effect" from the time of day when it usually first becomes noticeable. Defence interest in the phenomenom stems from its influence on sonar performance. Because the near-surface temperature rise is accompanied by an increase in sound velocity,its practical effect is to diffract a sonar beam downward. In consequence the beam can miss its target altogether except at unsatisfactory low ranges.

Methods of calculating the instantaneous overall heating or cooling rate to an accuracy of say 10-20% are well established. Subject to some qualifications the net heat added cumulatively during the day can be identified with the total water heat rise. However where basic knowledge is lacking is in the vertical transport of heat through the water, which determines the vertical temperature profiles built up. This transport is commonly several orders greater than can be accounted for by molecular thermal conductivity, and can only be ascribed to water turbulence. The last is a complex phenomenon which is only partially understood at present, so that empirical "models" for turbulent heat transport must serve in place of soundly-based physical laws.

The present model starts at sunrise with a completely-mixed turbulent upper layer, and continues with the wind and sun conditions initially mentioned. It predicts the partial or complete suppression, by early afternoon, of turbulence over a localised depth range within the top 20 m. The thermally-insulating layer so formed is accompanied by the accumulation of heat above, giving rise to types of near-surface temperature profile which are actually observed.

While the model deals satisfactorily with the day-time case where there is overall net heating, work has yet to be completed on the night-time case where there is net surface cooling and convective over-turning in consequence. Hence the present paper is to be regarded as a "progress report" only.

2 UNITS AND NOTATION

Three primary S.I. units are used, of distance (m), time (hr) and temperature (C); all others are derived therefrom. Kinematic quantities are used wherever appropriate, e.g. stress (m^2hr^{-2}) and diffusivity (m^2hr^{-1}). The unit of heat per unit area of ocean surface (C m) is defined as that required to raise the temperature of a 1 m layer of sea water by 1 C; 1 C m = 4.0 MJ m^{-2}. In this way the text equations are left uncluttered by conversion factors, and the concepts put forward are more readily understood. An exception is made for wind speed which in the present context is most conveniently expressed in knots.

The symbols used, with meanings and units, are as follows

Symbol	Meaning and Unit
A	Dimensionless universal constant originally proposed by Nee and Kovasznay (1969)
E	Kinetic energy of turbulence per unit mass of fluid, m^2hr^{-2}
g	Acceleration due to gravity, 127 x 10^6m hr^{-2}
$\bar{I}$	Unit vector in the vertically downward direction
K	Eddy diffusivity, mechanical or thermal, m^2hr^{-1}
Q	Downward heat flux intensity transported by fluid conduction/convection, C m hr^{-1}
QR	Radiant (solar) ocean heating rate at particular depth, C hr^{-1}
QS	Heat flux intensity (non-solar) entering ocean surface, C m hr^{-1}
T	Ocean temperature rise, C
t	Elapsed time from 0600, hr
U	Wind speed (knot)
$\underline{W}$	Ocean horizontal velocity vector, m hr^{-1}
z	Ocean depth, m
z_{max}	Depth to bottom of nominally mixed ocean layer, m
α	Volume coefficient of expansion, C^{-1}
ε	Rate of dissipation of turbulent energy, m^2hr^{-3}
θ	Latitude (angle)
$\overline{\tau}$	Ocean horizontal stress vector, m^2hr^{-2}
$\overline{\tau_0}$	Surface stress vector due to wind, m^2hr^{-2}
Ω	Angular velocity of earth's rotation, 0.262 hr^{-1}

3 THE MODEL

Our aim is to study the vertical transport of heat through the ocean in response to penetrative (solar) heating and surface cooling. For reasons that emerge shortly we must also study the vertical transport of momentum in response to the surface wind stress and vertically-distributed Coriolis force. Each of these transports is involved with ocean turbulence in two distinct ways:

(1) The rate of gain of turbulent kinetic energy E is identified with the net rate of energy loss from the bulk motion system. We can identify a gain due to work done against fluid friction,

and a loss due to the formation of potential energy of buoyancy (stratification) associated with the downward transport of heat in a thermally expansive medium. Turbulent energy is also dissipated directly to heat. These elements are assembled in the equation

$$\frac{\partial E}{\partial t} = \overline{\tau} \cdot \frac{\partial \overline{W}}{\partial z} - g\alpha Q - \varepsilon \qquad (1)$$

Since however there is no theoretically established formulation for turbulence dissipation we attempt to develop our model without it; ε is omitted henceforth.

(2) The vertical transports of heat and momentum are mainly controlled by turbulent diffusion. The transports due to molecular diffusion, i.e., to conventional coefficients of viscosity and thermal conductivity, are too small to make an effective contribution. Unfortunately there is no theoretically established formulation for turbulent diffusion. Hence this is modelled empirically most commonly as an "eddy diffusivity" coefficient K (dimensions L^2T^{-1}) and we do the same, making no distinction between K (heat) and K (momentum) at the present stage of modelling.

By definition

$$\overline{\tau} = K \frac{\partial \overline{W}}{\partial z} \qquad Q = K \frac{\partial T}{\partial z}$$

which when combined with (1) give

$$\frac{\partial E}{\partial t} = K \left\{ \left| \frac{\partial \overline{W}}{\partial z} \right|^2 - g\alpha \frac{\partial T}{\partial z} \right\} \qquad (2)$$

We now come to the central feature of the model. We recognize K as an alternative scalar measure to E of the same complex entity, viz. turbulence, and therefore look for some interrelationship between them. Direct proportionality is out of the question because of their differing dimensions, L^2T^{-1} and L^2T^{-2} respectively. However (2) suggests a line of enquiry. It provides external (bulk) information on the rate of change of E; should not the same information determine the rate of change of K? Here there is room for manoeuvre; the determination could involve some function of K rather than K itself, and also some other function of $\frac{\partial E}{\partial t}$.

Simple dimensional considerations lead to

$$2 \frac{\partial}{\partial t} (K^{\frac{1}{2}}) \equiv K^{-\frac{1}{2}} \frac{\partial K}{\partial t} = A \text{ (sign)} \left| \frac{\partial E}{\partial t} \right|^{\frac{1}{2}} \qquad (3)$$

where A is expected to be a universal dimensionless constant. The right-hand-side expression allows $\frac{\partial E}{\partial t}$ to assume negative values; (2) implies that such may well occur under conditions of strong heating and light stress. Inserting the value of $\frac{\partial E}{\partial t}$ given by (2), (3) becomes

$$\frac{\partial K}{\partial t} = AK \text{ (sign)} \left| \left| \frac{\partial \overline{W}}{\partial z} \right|^2 - g\alpha \frac{\partial T}{\partial z} \right|^{\frac{1}{2}} \qquad (4)$$

or

$$\frac{\partial K}{\partial t} = A \text{ (sign)} \left| |\overline{\tau}|^2 - g\alpha KQ \right|^{\frac{1}{2}} \qquad (5)$$

with K not permitted to fall below molecular value.

Under neutrally buoyant conditions, that is to say with the temperature gradient term omitted, (4) reduces to

$$\frac{\partial K}{\partial t} = AK \left| \frac{\partial \overline{W}}{\partial z} \right| \qquad (6)$$

This corresponds to the "generation term" for K in a model developed by Nee and Kovasznay (1969) for neutrally-buoyant aerodynamic situations. Hence the present model may be regarded as that of Nee and Kovasznay generalised for non-zero buoyancy. From experimental laboratory results they assign to the same constant A a value in the order of 0.1; this value has been found satisfactory here, up to the present.

4 SOLUTION OF THE DIFFUSION EQUATIONS

The standard one-dimensional diffusion equation for temperature is

$$\frac{\partial T}{\partial t} = QR(t) + \frac{\partial}{\partial z} \left(K \frac{\partial T}{\partial z}\right) \qquad (7)$$

with initial conditions $t = 0$, $T = 0$, $\frac{\partial T}{\partial z} = 0$ and boundary conditions $z = 0$, $\frac{\partial T}{\partial z} = QS(t)$; $z = z_{max}$, $\frac{\partial T}{\partial z} = 0$.

The corresponding equation for horizontal velocity, a two-dimensional vector, is

$$\frac{\partial \overline{W}}{\partial t} + 2\,\Omega \sin\theta\; \overline{i} \times \overline{W} = \frac{\partial}{\partial z} \left(K \frac{\partial \overline{W}}{\partial z}\right) \qquad (8)$$

with initial conditions $t = 0$, $\overline{W} = 0$, $\frac{\partial \overline{W}}{\partial z} = 0$ and boundary conditions $z = 0$, $\frac{\partial \overline{W}}{\partial z} = \overline{\tau}_0$; $z = z_{max}$, $\frac{\partial \overline{W}}{\partial z} = 0$.

The second term on the left-hand side is of course Coriolis acceleration due to the earth's rotation. The surface stress τ_0 (m^2hr^{-2}) is calculated from the wind speed U (knot) using a standard bulk formula

$$\tau_0 = 5.25\, U^2$$

based upon a friction factor of .0013 and air and sea relative densities of .0012 and 1.02 respectively.

Following Nee and Kovasznay we treat K like T as a transportable scalar. Hence (7) is adapted by substituting K for T and the "generation term" (4) for QR(t). The resulting diffusion equation for K is

$$\frac{\partial K}{\partial t} = AK \text{ (sign)} \left| \left| \frac{\partial \overline{W}}{\partial z} \right|^2 - g\alpha \frac{\partial T}{\partial z} \right|^{\frac{1}{2}} + \frac{\partial}{\partial z} \left(K \frac{\partial K}{\partial z}\right) \qquad (9)$$

and we use this to close (7) and (8)

There are well-established methods for the numerical solution of (7) and (8), see for example Razelos (1973). Of these the Crank-Nicholson semi-implicit solution is used here, in conjunction with the tridiagonal matrix form of Gaussian elimination. Equation (9) likewise calls for a semi-implicit solution, but its non-linearity in K excludes the standard elimination technique. Pending development of a suitable alternative, a simple explicit solution is used. Some instability results, as is common with such solutions, but its level is found tolerable here.

As already mentioned in the Introduction, the upper ocean layer is assumed to be well-mixed at sunrise, so that the initial values of temperature and velocity can without loss of generality be taken as zero; see again (7) and (8). However it is necessary to assume an initial uniform value of diffusivity, and in the absence of night-time analysis this is estimated from wind speed using a conventional empirical formula adapted from

Sverdrup (1942, table 64):

$$K = \begin{cases} 0.41\,U^2 \\ 0.05\,U^3 \end{cases} \text{ whichever is less} \qquad (10)$$

5 RESULTS

5.1 Test Conditions

The R.N. Directorate of Naval Oceanography (DNOM, 1968) has collected an extensive data set comprising some 25,000 bathythermograph soundings taken at intervals of 15 minutes or less by naval teams at Ocean Weather Stations INDIA and JULIET during the years 1964 through to 1967. Towards ultimately testing the present model against this large data base, a sample ocean location is taken at latitude 56N, intermediate between O.W.S. INDIA (59N) and O.W.S. JULIET (52½N), for a fine midsummer day with cloud cover not exceeding two-eighths. Two tests are made, for wind speeds of 5 and 10 knots in some constant direction. The net solar heating regime (after surface reflection) is taken as half-sine-wave with noon peak 0.7 C m hr^{-1}; the penetration-depth relationship is assumed to be as for ocean type 1B in the optical classification scheme of Jerlov (1968, table XXI). Surface cooling is taken to be constant at 0.1 C m hr^{-1}, and ocean temperature at 15C corresponding to a volume coefficient of expansion of 0.0002 C^{-1}. The mixed layer depth is assumed constant at 100 m, but this is unlikely to be critical since the significant developments are found to occur much closer to the surface.

The grid spacings found suitable for numerical analysis are: depth element 1 m to depth 20 m, thence 2 m to 30 m, 5 m to 50 m, and 10 m to 100 m; time interval 15 min from 0600 to 1000 hr, thence 30 min to 1800 hr. Both sets of spacings have been tested by subdivision; negligible variation in the results has been found.

5.2 Temperature Results

Figures 1a and 1b show these for wind speeds of 5 and 10 knot respectively, from 0600 hr local apparent time every two hours to 1800 hr. To imply that the same winds had blown overnight, the 0600 diffusivity values are calculated from them using (10). Further modelled results, not reported in detail, have been obtained for wind speeds of 15 and 20 knot; the peak surface temperatures are found to be 0.14C and 0.09C respectively, occurring at 1600 hr. Such small temperature changes would probably escape detection in practice.

Figures 1a and 1b show clearly the modelled development pattern of transient thermoclines, as these temperature profiles are commonly called. The area within the temperature-depth curve must be consistent with the heating regime assumed, so that no new knowledge is gained here. However in regard to the depth of formation of the transient, the results may be checked against the following DNOM observational findings:

(1) The depth of formation* (determined mainly by wind mixing) is 3(6){9} m for a mean wind speed of 5(10){15} knot over the 3 hours prior to formation.

(2) Significant transients, i.e. those clearly visible on bathythermograph traces, are commonly about 6 m thick, rarely less than 3 m, and not often more than 9 m.

(3) For wind speeds over 15 knot transient formation is unusual.

*Assumed from other comments in DNOM (1968) to be to the top of the transient.

There is seen to be some broad resemblance between the model results and these observations, although the sensitivity of the former to wind speed is greater than the latter would lead us to expect. Much depends upon the way in which top and bottom transient depths are defined. The top depth is almost impossible to identify in the modelled temperature curves. If however the bottom depth is taken to correspond to some least observable temperature rise, a reasonably positive result is obtained. The DNOM temperature profiles are reported to 0.1C; adopting this in the model, the bottom depths for a 5(10) knot wind are seen from the figures to be 6(12) m at 1400 hour and 8(16) m at 1800 hour; for comparison the DNOM values can range from 6(9) m to 12(15) m at an unspecified time. Clearly extensive testing against day-to-day oceanographic/meteorological data is called for, and attention is currently being given to this.

5.3 Diffusivity Results

These results, also shown in Figures 1a and 1b, give the best insight into the behavioural pattern of the model. Starting at 0600 hr turbulence is generated immediately by wind stressing. However, it is not much reduced at first by potential energy formation, since the half-sine-wave heating is still small and so therefore is the downward flux of heat and buoyancy. Hence the 0800 hr levels of diffusivity are higher than the 0600 ones. Later however as the heating increases, the second trend overcomes the first and the diffusivity falls. Indeed it is seen to collapse spectacularly through several orders of magnitude from 1000 - 1200 hr onward, but selectively over a limited range of depth. At the lower wind speed of 5 knot the collapse is right down to the molecular level; it is as though a sheet of highly-insulating material were inserted at the appropriate ocean depth in the middle of the day, and the remainder of the day's heating allowed to accumulate above. The same effect occurs with a 10 knot wind but at a greater depth and to a less marked degree.

Once formed, the stilled layer tends to deepen, slowly at first and more rapidly as late afternoon cooling sets in. Meanwhile the water above the layer becomes more turbulent and better mixed. By the end of the day the surface turbulence recovers to the order of its assumed starting value.

5.4 Influence of Initial Diffusivity

The influence of this currently unknown and, in the absence of night-time analysis, unpredictable parameter is explored in Figure 1c. These results are for a 5 knot wind at 1600 hr only, but with a range of initial diffusivities. The last are conveniently expressed as equivalent wind speeds, using (15) for conversion. The range adopted is 5, 7, 10, 15, 20 knots, of which the first speed is the same as for Figure 1a.

Since the resulting 1600 hr temperature curves differ only slightly, it is concluded that for practical purposes the influence is unimportant. This is fortunate, since it means that ignorance of the 0600 diffusivity is no bar to setting up a temperature prediction scheme. As before, a better appreciation of the situation is gained from the diffusivity curves; comparing those of Figures 1a and 1c it is seen that each step-up in wind speed merely delays the development of the modelled behavioural pattern by about three-quarters of an hour.

5.5 Velocity Results

The velocity hodographs of Figure 2 are included for completeness. The synoptic velocity/depth diagrams

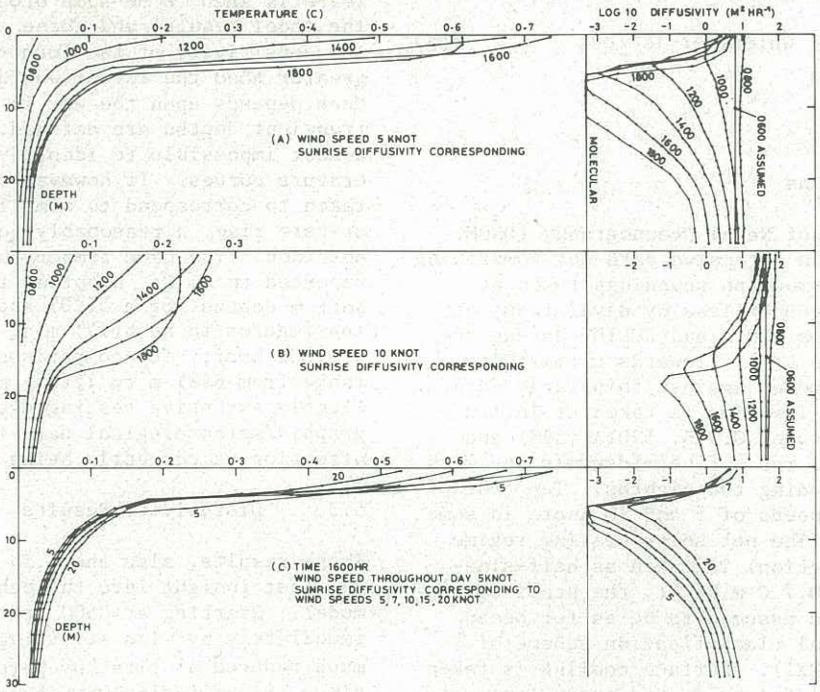

Figure 1 Temperature and velocity results. Conditions: Latitude 56N, cloud cover not exceeding two-eighths, half-sine-wave solar heating with peak 0.7 C m hr^{-1} penetrating as for Jerlov optical classification 1b, surface cooling 0.1 C m hr^{-1} constant, ocean temperature 15C. Wind speeds as indicated.

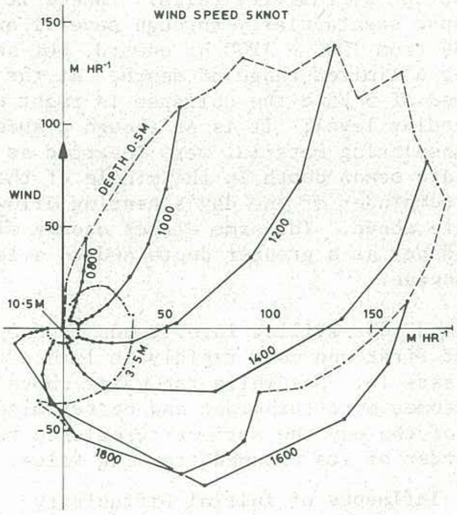

Figure 2. Velocity results.
Conditions as for Figure 1a.

(Ekman spirals) are shown in full line at two-hourly intervals; the marked points are the mean velocity vectors for successive 1 m layers down to 11 m. These curves are well-behaved except for the crossover of the 1600 and 1800 hr ones. Additionally, the diurnal behaviour of three of the layers, centering on 0.5, 3.5 and 10.5 m, is shown in broken line at half-hourly intervals. These curves correspond to the early part of the well-known graphical solution presented by Ekman (1905 , Figures 3-6 incl), for velocity response to a suddenly-applied wind stress. The 0.5 m and 10.5 m curves are unexceptional, but the 3.5 m one, in a zone where the turbulence is recovering rapidly, exhibits a striking reversal of direction between 1600 and 1800 hr.

6 CONCLUSION

A simple eddy-diffusivity model for the upper ocean is proposed. It rests on the physical hypothesis that the turbulent diffusivity responds to the same combination of external (bulk) parameters as does the turbulent kinetic energy. At the present stage of modelling some simplifications are made; no distinction is drawn between mechanical and thermal diffusivities, and dissipation of turbulent energy directly to heat is ignored. Nevertheless the model is able to reproduce the type of near-surface heat accumulation which can be observed on calm sunny days, ascribing it to the partial or complete suppression of turbulence immediately beneath. The modelled temperature profiles have the type of dependence on wind speed reported for a large number of R.N. bathythermographic observations.

As well as in offering some physical insight into turbulence, the model is interesting in its near-freedom from adjustment constants. There is only one, to which other investigators have already ascribed a value found suitable here.

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