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An Alternative to Diffraction Theory for Evaluating Inertial Wave Forces on Cylinders Standing on the Sea Floor

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1 INTRODUCTION

The problem of wave force evaluation on any marine structure was first tackled by Morison et al.(1950) with respect to marine piles. Their proposed formula for wave force evaluation is basically empirical in nature as it relies on empirical data to provide an estimation on the drag and inertial coefficients incorporated in the formula. Since then this formula has been widely accepted by engineers. The extensive use of Morison's equation has generated many efforts to measure these semi-empirical parameters. As a result, there exists a glut of published data for determining the two force coefficients. Unfortunately, inspite of this wide experience there are still serious conflicts and uncertainties regarding the reliability of the method as well as of the published data.

Theoretically, the drag and inertial coefficients should be predictable respectively by means of boundary layer and potential flow analyses. In the present paper, we shall devote our attention solely to the prediction of the inertial wave force component or its associated inertial coefficient.

The calculation of the inertial wave force component for a vertical surface piercing cylinder standing on the sea bottom was first reported by MacCamy and Fuchs(1954). Since then, many other solutions have been reported for other object shapes as well as for different states of submergence. The use of diffraction theory exclusively in these solutions has given rise to a school of thought which thinks that Morison's equation should be limited to bodies small with respect to the wave length such that the presence of body does not modify the flow field in which the structure is immersed. In other words, Morison's equation precludes fluid-structure interaction, and that fluid-structure interaction can only be accounted for in diffraction models. As a result, limits has been set on the range of validity for Morison's equation despite the fact that diffraction solutions can always be rearranged into a form which yield expressions for C_I to preserve the use of Morison's equation in such situations.

Viewed from another perspective, Morison's equation represents an ad hoc extension of non-oscillatory concepts into oscillatory flow situations. Results to date indicate that it is adequate in all situations if the inertial force coefficient can be predicted analytically. Obviously, in such situations one may not see the obvious need for an empirical expression. However, its use does convey a certain degree of convenience in certain situations. This will be illustrated towards the end of the present paper. In situations in which the coefficients are derived solely from experimental measurements, Morison's equation should only be used with the same wave theory with which the coefficients have been evaluated from experimental data. This has been demonstrated in CERC(1966) which shows that the use of wave velocities and accelerations given by different wave theories yield different magnitudes for the empirical coefficients. The same is obviously true also for theoretically deduced force coefficients. The underlying difference is that theoretically deduced coefficients normally is accompanied by a known range of validity. Such is not the case for experimentally derived coefficients. When this latter is used, caution must always be exercised if extrapolating sufficiently far from the range in which the experiments were conducted.

In what follows, we shall examine the theoretical evaluation of inertial wave force or its associated coefficient using the wave pressure approach and to demonstrate that Morison's equation does confer certain advantages when use in conjunction with submarine pipeline design.

2 HORIZONTAL WAVE FORCE AND OVERTURNING MOMENT ON VERTICAL SURFACE PIERCING CYLINDER

The initial success of diffraction analysis seems to have precluded the exploration of another approach for evaluating the inertial wave force component. Generally speaking, diffraction analysis is relatively involved and time consuming. Its use normally demands a level of sophistication in mathematics beyond the comprehension of most design engineers. With this in mind, we shall examine the alternative approach of inertial wave force prediction founded on the concept of wave pressure. This latter approach has been used successfully in the past for computing wave forces acting on breakwaters.

Taking the wave pressure to be acting on the projected area of the surface piercing cylinder standing on the sea bottom, the maximum (inertial) wave force acting on the cylinder of diameter D can be written as

$$F_{h,max} = D\int_{-d}^{\eta} 2p_{max}dz$$

$$= D\,\frac{\rho g H}{k}\,\frac{\sinh k(d+\eta)}{\cosh kd} \qquad (1)$$

wherein the factor of 2 is introduced on the basis that total reflection occurs at the front face of a surface piercing cylinder.

Though the value of η is not known a priori, its knowledge is not necessary if we are only interested in deriving the inertial coefficient. On comparing this to the expression for the inertial force term from Morison's equation, which is of the form

$$F_{h,max} = \rho C_I (\tfrac{1}{4}\pi D^2) \int_{-d}^{\eta} \frac{\partial u}{\partial t}\, dz$$

$$= \tfrac{1}{4}\pi D^2 C_I \frac{\rho g H}{2} \frac{\sinh k(d + \eta)}{\cosh kd} \qquad (2)$$

we arrive at an expression for C_I as follows:

$$C_I = \frac{4L}{\pi^2 D} \qquad (3)$$

It is obvious that this expression differs significantly in form from the corresponding result deduced on the basis of diffraction analysis, given below to facilitate comparison:

$$C_I = \frac{4}{\pi^3}\left(\frac{L}{D}\right)^2 \frac{1}{\left\{J_1'^2\left(\frac{\pi D}{L}\right) + Y_1'^2\left(\frac{\pi D}{L}\right)\right\}}$$

$$= \frac{4}{\pi^3}\left(\frac{L}{D}\right)^2 \frac{1}{\left|H_1'\left(\frac{\pi D}{L}\right)\right|} \qquad (4)$$

A plot of the results given by both (3) and (4) are shown in Figure 1. From this, it can be seen that relatively good agreement exists in the range for which $D/L > 0.2$ between the inertial coefficients deduced on the basis of two very different approaches. For $D/L < 0.2$, values predicted by (3) become larger than the non-oscillatory limit of 2. This non-oscillatory limit actually corresponds to the case of $L \to \infty$ or $D/L = 0$. On assuming that values for C_I cannot be larger than the flow limit everywhere, the following distribution for C_I can be recommended for practical applications:

$$C_I = \begin{cases} 2.0 & \text{for } D/L < 0.2 \\ \dfrac{4}{\pi^2}\dfrac{L}{D} & \text{for } D/L > 0.2 \end{cases} \qquad (5)$$

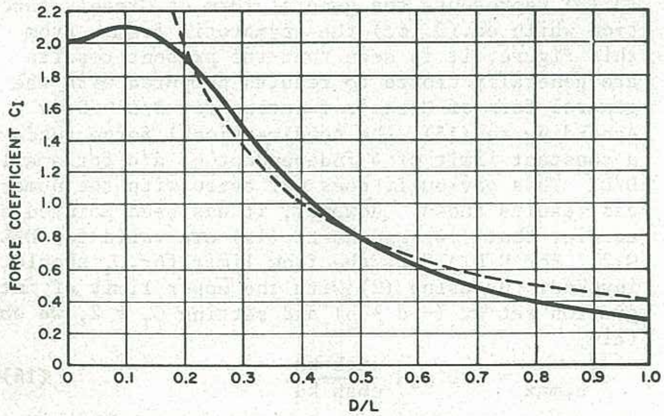

Figure 1 Comparison between exact and approximate inertial force coefficients

It can be seen from Figure 1 that (5) approximates (4) rather remarkably over the entire range of D/L shown.

From the above considerations, it can be seen that Morison's equation cannot be considered to be valid within a certain D/L range only if we allow for a variable C_I to be introduced. However, we may say that Morison's equation with a C_I equal to that of non-oscillatory flow is valid only for the range $D/L < 0.2$. Now, since we have established a very simple closed form solution for $F_{h,max}$, the continual use of the inertial force coefficient may seem redundant. We may use the following formulas directly for the dimensionless horizontal force:

$$\frac{F_{h,max}}{\rho g (D/2)^2 (H/2)} = \frac{2\pi \sinh k(d + \eta)}{\cosh kd} \qquad \text{for } D/L < 0.2 \qquad (6a)$$

$$\frac{F_{h,max}}{\rho g (D/2)^2 (H/2)} = \frac{8}{kD}\frac{\sinh k(d + \eta)}{\cosh kd} \qquad \text{for } D/L > 0.2 \qquad (6b)$$

For actual design calculations in situations for which the wave height is known, η can be set equal to $\frac{1}{2}H$ for the purpose of evaluating $F_{h,max}$. In Figure 2, we have compared (6) with the exact solution of MacCamy and Fuchs (1954) as well as numerical results obtained by the finite element method given by Zienkiewicz et al.(1978). For the purpose of this comparison, η has been set to zero since no values have been quoted. It can be seen from Figure 2 that the agreement between (6) and other solutions are remarkable.

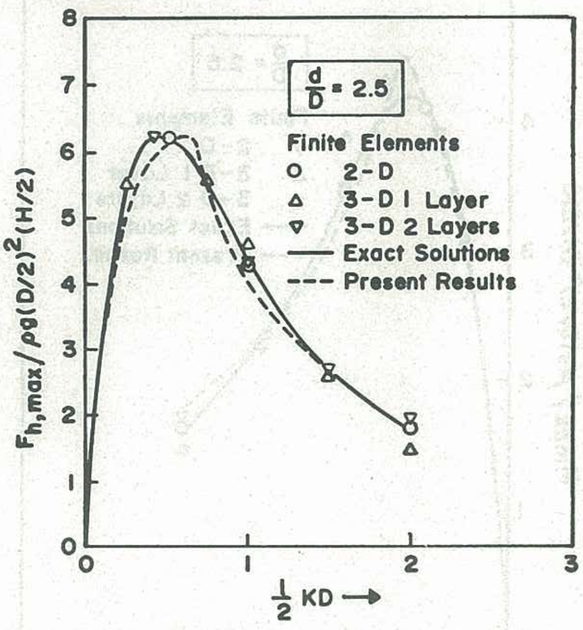

Figure 2 Horizontal forces on a surface piercing cylinder

The overturning moment for a surface piercing cylinder for $D/L > 0.2$ can be written as

$$M_{max} = \int_{-d}^{\eta} 2Dp_{max}(d + z)\, dz$$

$$= \frac{\rho g H D}{\cosh kd}\left\{(d + \eta)\frac{\sinh k(d + \eta)}{k} + \frac{1}{k^2}\left[1 - \cosh k(d + \eta)\right]\right\} \qquad (7)$$

On non-dimensionalizing, (7) becomes

$$\frac{M_{max}}{\rho g (D/2)^2 (H/2) d} = \frac{8}{\cosh kd}\left\{\left(1 + \frac{\eta}{d}\right)\frac{\sinh k(d + \eta)}{kD} + \frac{D/d}{(kD)^2}\left[1 - \cosh k(d + \eta)\right]\right\} \qquad (8)$$

For $D/L < 0.2$, the appropriate expression for M_{max} is given by

$$M_{max} = \tfrac{1}{2}\pi D^2 \int_{-d}^{\eta} \frac{\partial u}{\partial t}(d + z)\, dz$$
$$= \frac{\pi}{4}\,\frac{\rho g H D^2}{\cosh kd}\left\{(d + \eta)\sinh k(d + \eta) + \frac{1}{k}\left[1 - \cosh k(d + \eta)\right]\right\} \qquad (9)$$

which can be non-dimensionalized as

$$\frac{M_{max}}{\rho g (D/2)^2 (H/2) d} = \frac{2\pi}{\cosh kd}\left\{\left(1 + \frac{\eta}{d}\right)\sinh k(d + \eta) + \frac{D/d}{kD}\left[1 - \cosh k(d + \eta)\right]\right\} \qquad (10)$$

In Figure 3, (8) and (10) have been plotted for the case of $\eta = 0$ in comparison with the exact solution of MacCamy and Fuchs (1954) and numerical results of Zienkiewicz et al.(1978). Again, remarkable agreement has been achieved.

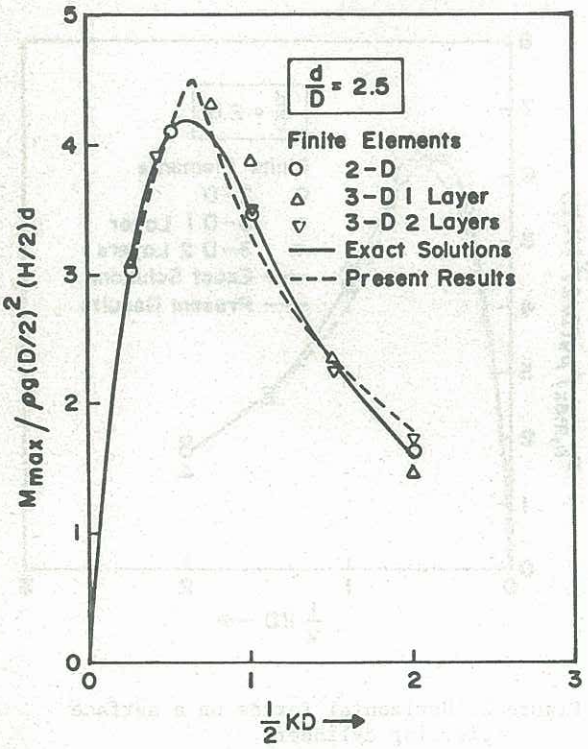

Figure 3 Overturning moments on a surface piercing cylinder

Though Figure 1 indicates that C_I given by both the present approach as well as diffraction analysis tends to zero as $D/L \to \infty$, diffraction analysis actually yields a zero force per unit width of projected area in this limiting condition. This is consistent with the model which takes the entire plan form of the cylinder into consideration. As under such conditions no surrounding fluid medium can be present and hence no wave induced forces. The treatment based on wave pressure, on the other hand, gives a constant force per unit length, given by

$$f_{h,max} = \frac{F_{h,max}}{D} = \frac{\rho g H}{k}\,\frac{\sinh k(d + \eta)}{\cosh kd}$$
$$= \frac{\rho g H}{k}(\tanh kd \cosh k\eta + \sinh k\eta) \qquad (11)$$

On assuming that $k\eta \ll 1$, (11) becomes

$$f_{h,max} = \frac{\rho g H}{k}\tanh kd + \rho g H \eta \qquad (12)$$

which, on setting $\eta = \frac{1}{2}H$, yields

$$f_{h,max} = \frac{\rho g H}{k}\tanh kd + \tfrac{1}{2}\rho g H^2 \qquad (13)$$

This is identical to the formula recommended by Nagai(1969) for breakwater design, after extensive comparison with model test data.

3 HORIZONTAL WAVE FORCE AND OVERTURNING MOMENT ON SUBMERGED VERTICAL CYLINDER

To pursue the validity of the present approach a step further, the horizontal wave force acting on a submerged vertical cylinder standing on the seabed and its associated overturning mement are calculated using the present approach and compared to numerical results reported by Garrison(1978) via numerical evaluation of Green's functions, Hogben and Standing(1974) via boundary integrals and Zienkiewicz et al.(1978) via finite element methods.

For the present purpose, (1) can be modified to read

$$F_{h,max} = \int_{-d}^{-d+h} 2Dp_{max}\, dz = D\,\frac{\rho g H}{k}\,\frac{\sinh kh}{\cosh kd} \qquad (14)$$

where h denotes the height of the cylinder. Non-dimensionalizing by $\rho g (D/2)^2 (H/2)$, we obtain

$$\frac{F_{h,max}}{\rho g (D/2)^2 (H/2)} = \frac{8}{kD}\,\frac{\sinh kh}{\cosh kd} \qquad (15)$$

For comparison with the numerical results reported by Garrison(1978), we set $h = \frac{1}{2}D$ and evaluate the RHS of (15) for d/h ratios used by Garrison, viz. 1.5, 1.75, 2.0, 2.5, 3.0 and 4.0. Figure 4 shows a plot of the present results superposed on to the results reported by Garrison(1978) using two forms of the Green's function. The equation number in the figure refers to equations in Garrison's paper. Eq. (3.13) represents the general form of Green's function while eq.(3.16c) the asymptotic form. From this figure, it is seen that the present results are generally closed to results computed with the general form of Green's function for $D/L > 0.3$. According to (15), the nondimensional force tends to a constant limit of 4 independent of d/h for small D/L. This obviously does not agree with the numerical results shown. However, it has been pointed out earlier that (14) and hence (15) are valid for $D/L > 0.2$. For $D/L < 0.2$, the flow limit for C_I should be invoked. By using (2) with the upper limit of integration set at $(-d + h)$ and setting $C_I = 2$, we obtain

$$F_{h,max} = \tfrac{1}{4}\pi D^2 \rho g H\,\frac{\sinh kh}{\cosh kd} \qquad (16)$$

which can be non-dimensionalized to yield

$$\frac{F_{h,max}}{\rho g (D/2)^2 (H/2)} = \frac{2\pi \sinh kh}{\cosh kd} \qquad (17)$$

On using (17) for the region of $D/L < 0.2$ and fairing the two curves at the discontinuity, the complete nondimensional force acting on a submerged cylinder sitting on the seabed derived from the wave pressure approach is shown in Figure 4. The present results are generally more conservative with maximum over-estimation of the inertial wave force of about 30%. This level of accuracy is normally deemed acceptable in marine works in view of the gross inadequacies existing in the collection of environmental data for inputting into the force computation.

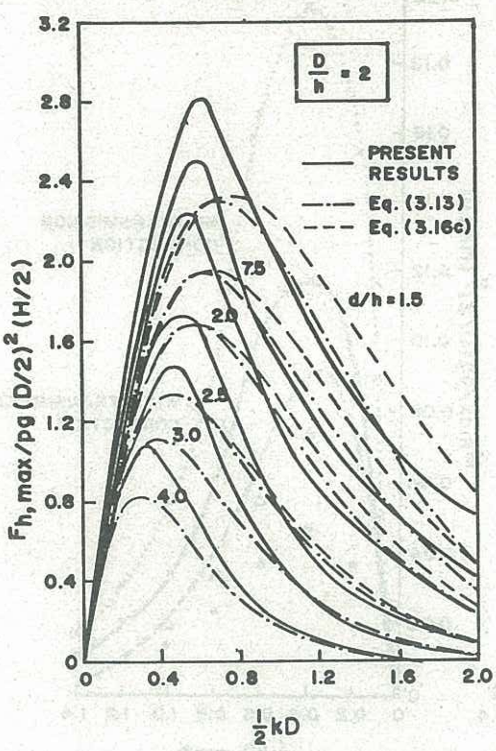

Figure 4 Horizontal forces on submerged cylinders

Figure 5 shows comparisons between horizontal and vertical forces as well as overturning moments induced by a progressive wave train on a submerged cylinder predicted by the present approach and numerical results reported by Zienkiewicz et al.(1978) using finite elements as well as Hogben and Standing(1974) using the method of boundary integrals. For the horizontal force, the same trend as seen in Figure 4 between the present results and numerical results is again observed.

The overturning moment for a submerged cylinder can be derived using (7) and (9) by setting the upper limit of integration at $(-d+h)$ rather than at η. On making this change, (9) and (7) yield

$$M_{max} = \frac{\pi}{4}\frac{\rho g H D^2}{\cosh kd}\left\{h \sinh kh + \frac{1}{k}(1 - \cosh kh)\right\} \quad \text{for } D/L < 0.2 \qquad (18)$$

and

$$M_{max} = \frac{\rho g H D}{\cosh kd}\left\{\frac{h}{k}\sinh kh + \frac{1}{k^2}(1 - \cosh kh)\right\} \quad \text{for } D/L > 0.2 \qquad (19)$$

Non-dimensionalizing by $\rho g(D/2)^2(H/2)d$, (18) and (19) become respectively

$$\frac{M_{max}}{\rho g(D/2)^2(H/2)d} = \frac{2\pi}{\cosh kd}\left\{\frac{h}{d}\sinh kh + \frac{D/d}{kD}(1 - \cosh kh)\right\} \quad \text{for } D/L < 0.2 \qquad (20)$$

and

$$\frac{M_{max}}{\rho g(D/2)^2(H/2)d} = \frac{8}{\cosh kd}\left\{\frac{h/d}{kD}\sinh kh + \frac{D/d}{(kD)^2}(1 - \cosh kh)\right\} \quad \text{for } D/L > 0.2 \qquad (21)$$

(20) and (21) have been used for plotting the comparison for overturning moment shown in Figure 5.

The present results are seen to be some twice that given by Hogben and Standing(1974) and Zienkiewicz et al.(1978). This large difference between the present and numerical results can be corrected for if we take into account of the wave transmission above the submerged cylinder. It may be argued, see for instance Chue(1980), that transmission for an infinitely long (two-dimensional) submerged breakwater of width D is given by

$$\chi_t = \frac{(1 - \frac{h}{d})\{\frac{h}{d} + (1 - \frac{h}{d})(\frac{\pi D}{L})^2\}}{\frac{h}{d} + (1 - \frac{h}{d})^2(\frac{\pi D}{L})^2} \qquad (22)$$

The correctness of (22) can be demonstrated by the fact that for $h/d = 1$, $\chi_t = 0$, implying that when the submerged cylinder extends to and above the sea surface no wave energy will be transmitted past the cylinder. At the other limit, when $h/d = 0$, $\chi_t = 1$, implying that if the cylinder is infinitesimally thin, all wave energy will be transmitted past the cylinder. Notice that both these limiting solutions are independent of D/L. It may be pointed out that the deductions from (22) are opposed to that given by John(1949) for an infinitesimally thin plate floating on the sea surface. The difference between an infinitesimally plate lying on the surface and another on the bottom is that the former does possess wave dampening properties as it interrupts the continuous movement of the surface undulations of the waves whereas the latter is not expected to behave distinctly different from the influence already exerted by the sea bottom.

Using (22), the transmitted wave height at the leeward edge of the cylinder is given by

$$H_t = \chi_t^{\frac{1}{2}} H \qquad (23)$$

Assuming the transmitted wave height induces a force opposite to that of the incident waves, it may be shown that the correction factor to be used is given by $\frac{1}{2}(2 - \chi_t^{\frac{1}{2}})$ which is equivalent to $\frac{1}{2}(1 + \chi_r^{\frac{1}{2}})$ where χ_r denotes the reflection coefficient. When this is applied to the overturning moments computed via (20) and (21), it may be seen from Figure 5 that the corrected moments are now in much better agreement with the numerical results shown. The maximum error now in no case exceeds 50%, with the maximum values occurring where the slope of the curve is steepest. The above correction factor is not recommended for horizontal forces computed via (15) and (17) as this results in an over-correction especially for the region where relatively good agreement now exists.

4 VERTICAL WAVE FORCE ON SUBMERGED VERTICAL CYLINDER

The submerged vertical cylinder is also subjected to a periodic wave force on its top face. Since this face is horizontal, the wave pressure acting on each element of the surface is constant and the total vertical force can be evaluated as follows:

$$F_{v,max} = p_{max}A = \frac{\pi}{4}D^2\frac{\rho g H}{2}\frac{\cosh kh}{\cosh kd} \qquad (24)$$

which can be non-dimensionalized as

$$\frac{F_{v,max}}{\rho g(D/2)^2(H/2)} = \frac{\pi \cosh kh}{\cosh kd} \qquad (25)$$

A plot of (25) for the case with $h/d = 0.3$ and $D/d = 0.6$ as a function of $\frac{1}{2}kD$ is shown in Figure 5, together with numerical results reported by Hogben and Standing(1974) via boundary integrals and Zienkie-

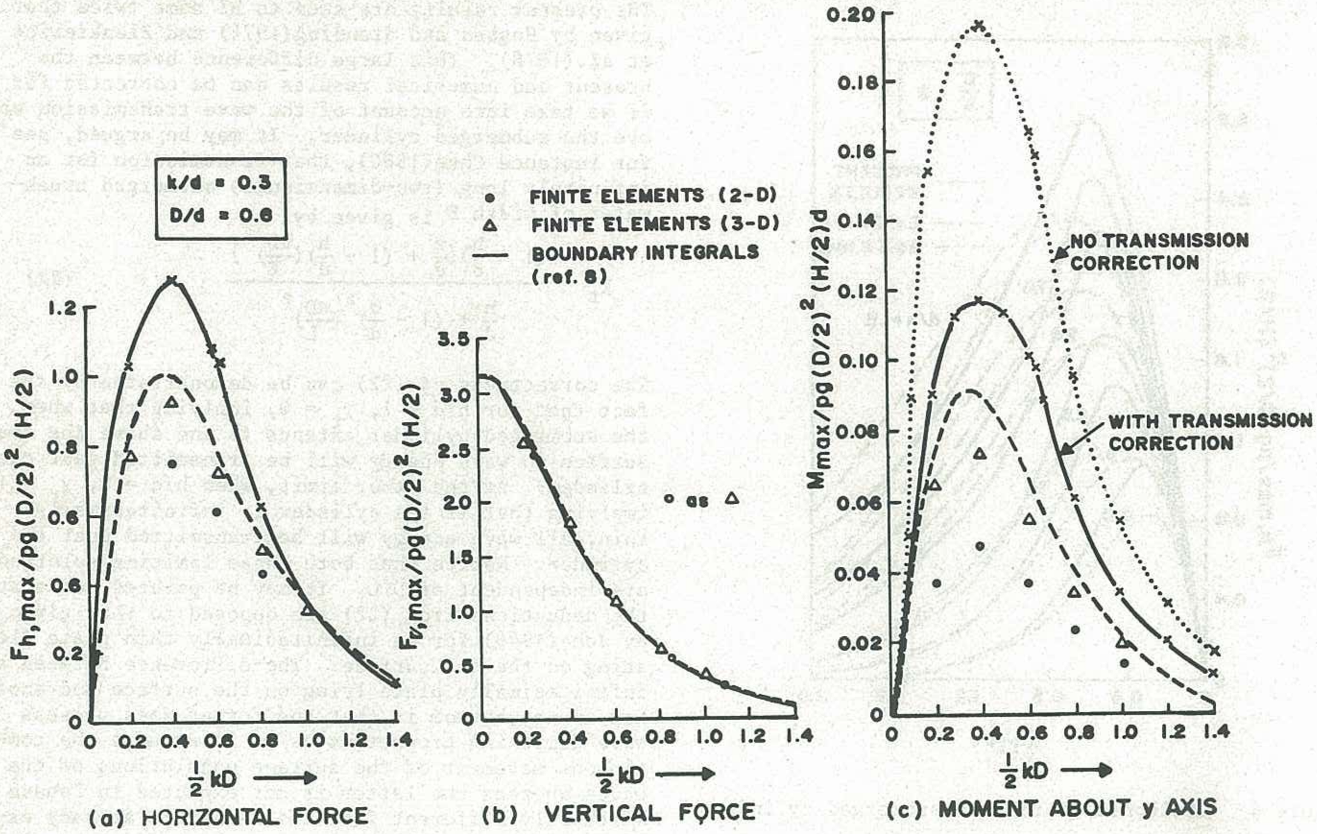

Figure 5 Horizontal and vertical forces and overturning moments on a submerged cylinder

wicz et al. (1978) via finite element methods. The agreement between (25) and the numerical results is excellent.

5 INERTIAL COEFFICIENTS FOR SUBMARINE PIPELINES

It has been shown in §2 that the surface piercing cylinder solution taken to the limit of $D/L \to \infty$ yields the correct force per unit length for surface piercing breakwaters. The same procedure can obviously be applied to (14) to yield the force per unit length on submerged structures resting on the seabed. This is given by

$$f_{h,max} = \frac{F_{h,max}}{D} = \frac{\rho g H}{k}\frac{\cosh kh}{\cosh kd} \qquad (26)$$

A class of submerged structures of particular importance in recent years is the submarine pipelines. (26) should be applicable to both partially embedded and unspanned submarine pipelines lying on the seabed. Taking first the latter situation, the height of the pipeline above the seabed is equal to D, the pipe diameter. Thus, the force per unit length acting on the unspanned submarine pipeline is

$$f_{h,max} = \frac{\rho g H}{k}\frac{\cosh kD}{\cosh kd} \qquad (27)$$

In the practice for submarine pipeline design, the Morison equation has often been used, together with some empirically derived inertial coefficients. The use of the Morison equation has caused much controversy in that the force coefficients, usually assumed to be constant, have indicated quite a large scatter. For the use of the Morison equation, a reference acceleration must be defined. If the mean acceleration over the projected area offered by the pipeline resting unspanned on the seabed is used, it may be shown that the inertial coefficient in conjunction with (27) is given by

$$C_I = \frac{4}{\pi^2}\frac{L}{D} \qquad (28)$$

which is the same as (3) deduced earlier for a surface piercing cylinder standing on the seabed. This expression should be restricted to $D/L > 0.2$ since it tends to infinity as $D/L \to 0$. It is therefore necessary to also limit the application of (27) to $D/L > 0.2$. For $D/L < 0.2$, $C_I = 2$ should be applicable. The foregoing discussions indicate that (26) should also be restricted to situations for which $h/L > 0.2$.

In the case of a partially embedded submarine pipeline, it can be shown that for a depth of embedment e

$$C_I = \frac{4}{\pi^2}\frac{L}{D}\left(1 - \frac{e}{D}\right) \qquad (29)$$

if the entire cross-sectional area of the pipe is used in evaluating the volume in Morison's equation instead of just using the portion protruding above the seabed. The reference acceleration is again taken as the mean acceleration over the projected area. The use of the entire cross-sectional area in computing the volume represents no approximation as in the final force relation after C_I is back-substituted into Morison's equation the volume in the inertial force term and the volume incorporated into the C_I expression cancel each other out. This assumption is made solely for the purpose of keeping (29) in a simpler form as well as to avoid referencing to handbooks for expressions for the volume of a partially embeded cylinder. (29) should be applicable to $D/L > 0.2(1 - e/D)$. For $D/L < 0.2(1 - e/D)$, C_I should be kept constant at the flow limit of 2.

6 CONCLUSION

The wave pressure approach for calculating wave forces and overturning moments has been shown to yield adequate engineering results for cylinders standing on the sea floor, irrespective of whether the top of the cylinder stays submerged or protrudes above the sea surface. It was further shown that the results for the surface piercing cylinder gives the correct limit for a vertical wall extending above the sea surface as $D \to \infty$. This was taken as the basis for extending the submerged cylinder results into situations simulating a submarine pipeline lying unspanned or partially embedded on the seabed.

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