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The Vortex Shedding Process Behind a Circular Cylinder

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SUMMARY A new look at the flow behind a circular cylinder has provided an insight into the vortex shedding process. Using aluminium particles and time exposure photography a sequence of instantaneous streamline patterns of the flow behind a cylinder can be obtained. These show that the conventional closed cavity which is formed during the starting flow is open and instantaneous "alley-ways" of fluid are formed which penetrate the cavity during the vortex shedding process. Also using dye to obtain streaklines, the experiment also shows how layers of dye and hence vorticity are convected into the cavity behind the cylinder and how they are eventually squeezed out.

1 INTRODUCTION

A deep insight into the mechanism of vortex shedding can be obtained by studying the instantaneous streamline patterns at various phases of the vortex shedding cycle. Instantaneous streamline patterns have often been ignored in unsteady flow studies and streaklines are more often used. The relationship between instantaneous streamlines and streaklines is extremely complex. Streaklines can be used to give an idea of where the vorticity in a flow resides but tells us very little about the surrounding flowfield and the entrainment processes. Instantaneous streamlines can be obtained photographically by short time exposures which give a field of short streaklines. It can be shown that over very short time intervals, streaklines, pathlines and instantaneous streamlines are identical.

2 AN INTERPRETATION OF PRANDTL'S MOVIE USING PROPERTIES OF CRITICAL POINTS

Prandtl (see Shapiro and Bergman (1962)) produced a movie of flow around bodies using aluminium particles on a free surface. Each frame of the movie was produced by an extremely short time exposure and no streaklines can be seen (i.e. no instantaneous streamlines). The authors therefore decided to multiplically expose 40 consecutive frames of the movie on one photographic plate and repeat this for various parts of the shedding cycle behind the bodies. A series of instantaneous streamline pictures was obtained and is shown in figure 1 for a cylinder starting from rest.

Various salient features of the flow patterns become obvious. These are critical points (i.e. points where the slope of the instantaneous streamlines become indeterminate). These points have been classified by Perry and Fairlie (1974) but for the work described here, certain properties of these critical points and other flow pattern features are important. These properties are as follows:

(1) Viscous critical points are points of zero vorticity. These occur only on the boundary where the no slip condition applies. To the first order approximation in time, these critical points translate with uniform velocity relative to the boundary. To see these critical points, the observer must be stationary relative to the boundary. Relative to any other observer, the critical point will disappear.

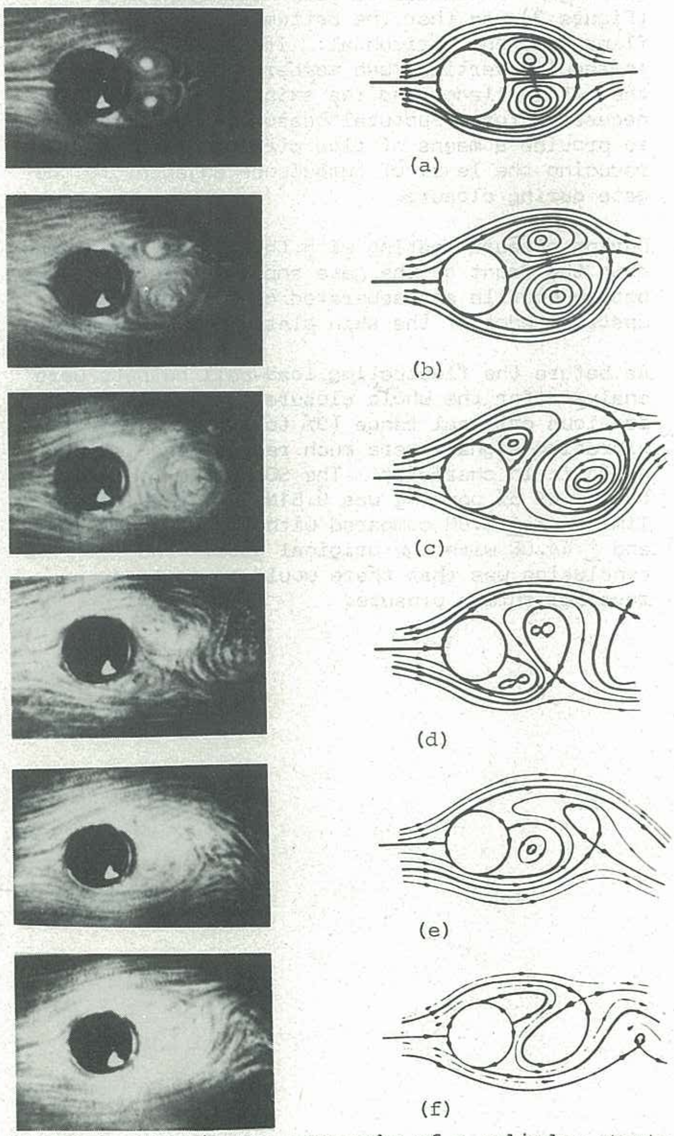

Figure 1 Flow in the near wake of a cylinder starting from rest. Observer moving with cylinder. Each picture obtained by multiplically exposing 40 consecutuive frames from Prandtl's movie. Author's interpretations also shown. Sequency of events begins at (a) and continues to (f).

(2) Inviscid critical points are singularities at the boundary where the no-slip condition has been relaxed. These can also be thought of as

critical points within the fluid i.e. critical points in mid air. To the first order approximation in time, these translate with uniform velocity equivalent to the velocity of the pressure maxima or minima. If an observer is moving with this velocity, then relative to him, the critical points are located at the pressure maxima (giving saddles) or minima (giving centres). Relative to any other observer, however, these critical points will be displaced from the pressure maxima or minima.

(3) Instantaneous streamlines, irrespective of whether they occur in steady or unsteady flow must obey continuity. Therefore, in two dimensional incompressible flow, the only allowable critical points are centres and saddles otherwise continuity would be violated. Foci, nodes and limit cycles are not allowed. There appears to be misconception in the literature that two-dimensional Kelvin-helmoholtz-like roll ups involve foci. This is incorrect. Trajectories surrounding the roll ups are closed.

(4) The geometry (or shape) of instantaneous streamlines and pathlines depend on the velocity of the observer whereas the geometry (or shape) of streaklines does not. The term "observer" is used in the present context to mean a camera capturing a short time exposure of a field of particles.

(5) In unsteady flows, if there exists a Galilean coordinate system in which the flow becomes steady, then relative to an observer moving with that coordinate system, the instantaneous streamlines and streaklines will coincide, provided the smoke or dye has deformed for a sufficient period of time. When this occurs, smoke will approximately align itself with the eigenvectors and separatrices of the critical points, but this will not be so if the observer is moving in any other coordinate system. (A separatrix is a line which joins saddle points).

(6) A vortex sheet is a streakline. In an incompressible flow, vorticity can only be generated at the boundaries (see Lighthill (1963)). Thus, if dye is injected at a boundary where the vorticity is generated, the dye will follow the vorticity. Vorticity, like dye, resides with the fluid particles (Batchelor (1970)).

(7) In principle, a streakline can never be broken. It represents a flexible barrier which fluid can never cross, even though in the actual flow situation, it may be stretched and become so highly convoluted that the original vortex sheet may become unrecognizable as a continuous sheet.

From these pictures and using property 1 and 3, a simple model for the flow in the cavity is proposed for the steady state oscillation as shown in figure 2. Similar models can be constructed for other bluff bodies. Only the separatrices are shown. As shown in figure 1, the cavity which forms during the starting-up process is closed and all the saddles are joined together. This is the classical picture of cavity flow but during the vortex shedding process, the cavity is open and the saddles are not necessarily joined by the separatrices and instant "alley-ways" of fluid penetrate the cavity. The formation of "alley-ways" can be seen in the computer plots of Fromm and Harlow (1963) and also in the flow visualization of Taneda (1978) who also used the short time exposure of aluminium particles to obtain streamline patterns behind bodies such as circular cylinders, elliptical cylinders and flat plates. Unfortunately, he presented his results without further interpretation.

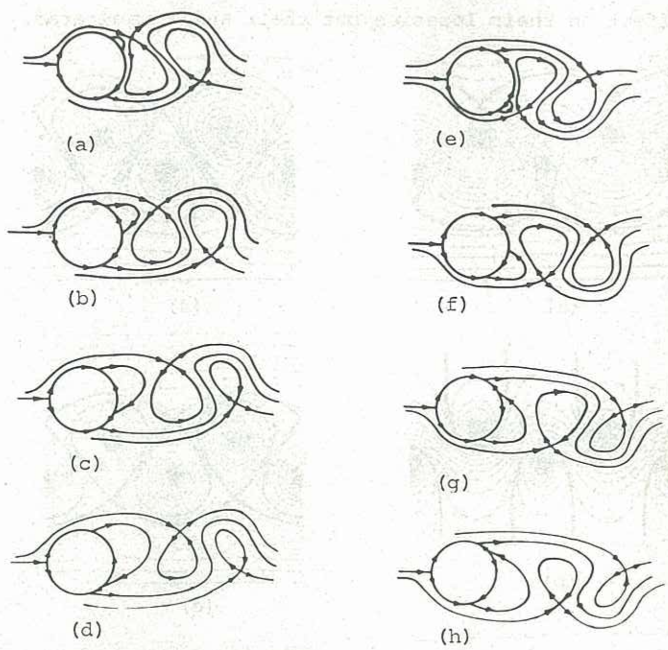

Figure 2 Proposed simple model for steady vortex shedding. Observer moving with the cylinder. Only separatrices shown. Sequence is from (a) to (h).

Flow behind other bodies such as the ellipse and bluff plate of Prandtl follow the same model, except in the case of the plate, the separation points on the body are fixed at the sharp edges.

In all these patterns, the observer is stationary relative to the body. Hence, critical points on the surface appear as critical points (property 1). Also, since the flow immediately behind the body is inactive (i.e. the velocity is low), the critical points within the fluid are approximately in their correct positions (by property 2). However, further downstream (2 or 3 wavelengths), the eddies accelerates rapidly until they reach their final phase velocity and the centres and saddles are displaced from their correct positions (property 2). To a stationary observer, the centres and saddles will merge to approach "centre-saddles" (where the separatrices form a cusp). To see these critical points as steady flows, the observer has to move with the phase velocity of the pressure maxima and minima.

3 PROPERTIES OF THE FAR WAKE

The different streamline patterns as seen by different moving observers can also be illustrated if we use von Kármán's (1912) stable vortex spacing (see also Milne-Thomson (1968) and Lamb (1945)) and solve for the complex potential, with the effect of the velocity of the observer included. The generated stream functions relative to different observers are shown in figure 3. In figure 3(b), the observer is moving with the free stream and faster than the eddies. The saddles for this case are at infinity. This pattern has been erroneously reported by Goldstein (1965) to correspond with the observer moving with the eddies. This error has been repeated in many textbooks. In figure 3(e), the observer is moving with the eddies and slower than the free stream.

It should be noted that in these patterns the centres (or vortices) do not move laterally - only the saddles. This is because the centres here are irregular, i.e. point vortices have been assumed which possess infinite tangential velocity at their origins. Hence, the motion of the observer has no effect on their location but their shape is altered.

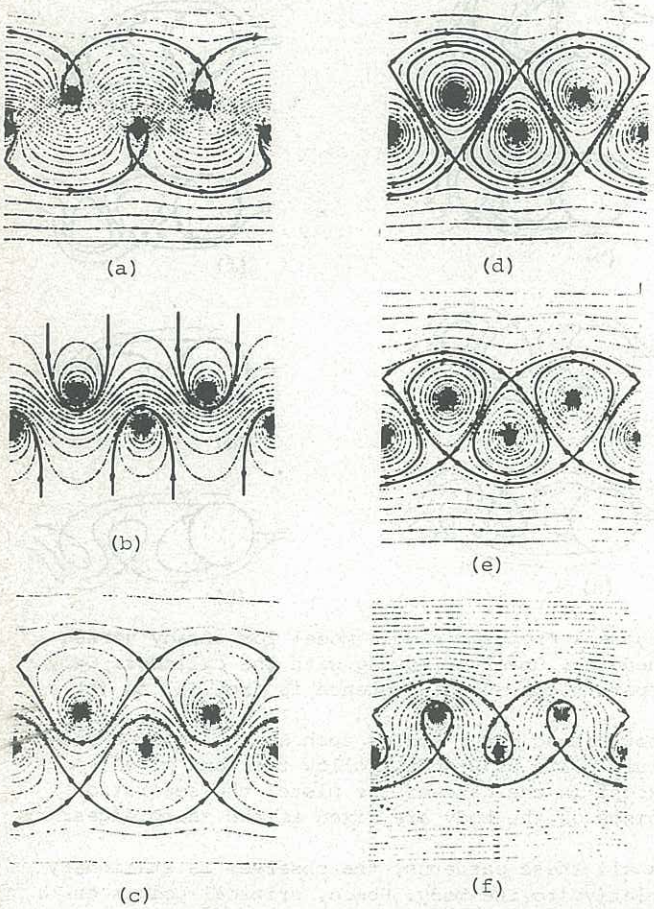

Figure 3 Karman vortex street as seen by different observers. Let $-V_E$ be the velocity of eddies relative to free stream. V_E is a positive number and positive velocity is from left to right.
(a) Observer moving at 1.0 V_E, i.e. observer is moving faster than free stream and faster than eddies.
(b) Observer moving at 0.0 V_E, i.e. observer is moving with the free stream but faster than the eddies.
(c) Observer moving at -0.4 V_E, i.e. observer is moving slower than free stream but faster than the eddies.
(d) Observer moving at -0.8 V_E, i.e. observer is moving slower than free stream but faster than the eddies. Happens to give almost connected separatrices.
(e) Observer moving at -1.0 V_E, i.e. observer is moving slower than free stream but moving with the eddies.
(f) Observer moving at -2.0 V_E, i.e. observer is moving slower than free stream and slower than the eddies.

Figure 4 shows the smoke pattern of Zdravkovich (1969) who introduced smoke close to the stagnation point of a cylinder. Using properties 4, 5, 6 and 7 one can see that the streakline which is a vortex sheet has aligned itself with the separatrices of the instantaneous streamline pattern. This pattern is steady.

It can also be seen that the smoke "eigenvectors" are approximately orthogonal at the saddles, hence showing that the flow is irrotational near the saddle (Perry & Fairlie 1974).

The Karman vortex street can be observed to occur in a variety of situations besides two dimensional wake flow. The authors produced a Karman vortex jet

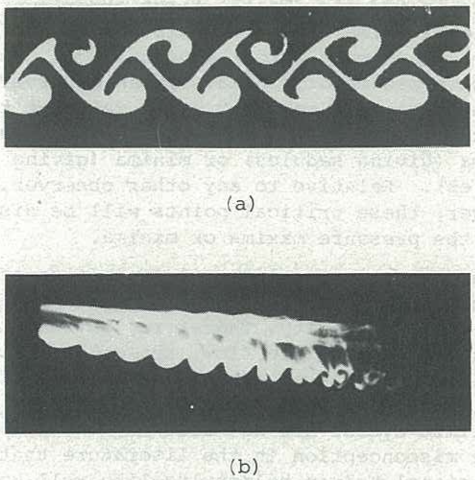

Figure 4 (a) Karman vortex street behind a circular cylinder (from Zdravkovich (1969)).
(b) Three dimensional Karman vortex street jet structure.

from an oscillating circular tube from which was issuing heavy smoke (see fig.5(b)). Note that the eddies are pointing in the opposite direction to that of Zdravkovich (1969).

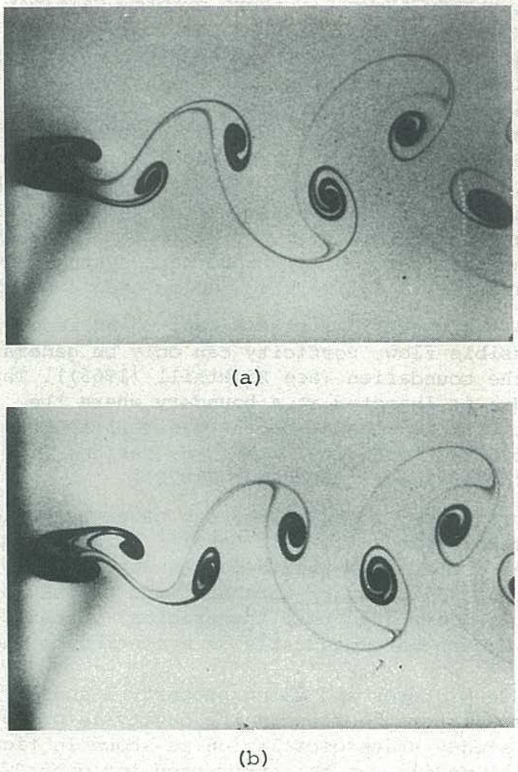

Figure 5 Dye traces in a Karman vortex street behind a circular cylinder. Blue dye corresponds to positive and red dye corresponds to negative vorticity. (Blue upper, red lower).

4 DYE EXPERIMENTS

The authors decided to carry out some experiments where positive and negative vorticity was coloured blue and red respectively. This was done by introducing blue dye on the upper surface of the cylinder and red dye on the lower surface. The cylinder was completely submerged in a water tunnel. Instantaneous streamlines were determined by a short time exposure of aluminium particles introduced into the flow and illuminated by a sheet of laser light.

Generally, the identations which formed on the vortex sheet corresponded with instantaneous alley ways. However, centres did not necessarily correspond with the roll up of the vortex sheet partly because the flow is unsteady and because it was discovered that many layers of dye and hence vorticity are convected into the cavity allowing centres to form below the main sheet. Figure 6 shows these layers. From property 7, a streakline can never be broken and fluid entering the cavity via the alley ways is bounded on one side by the dye interface. On the next cycle, the

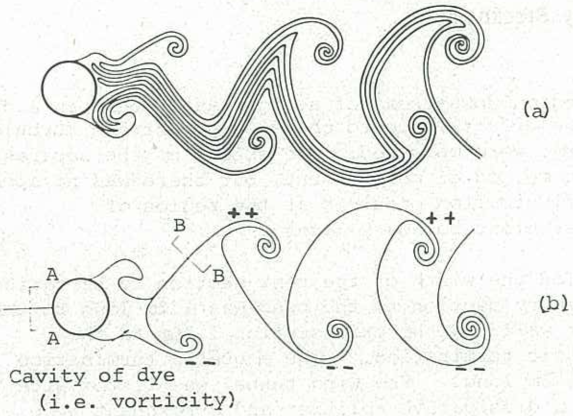

Figure 6 (a) Schematic "threading diagram" of vortex sheets in a Karman vortex street. (b) Simplified model. Flow of dye across A-A is equal to flow of dye across B-B.

same applies to the next body of fluid so the various bodies of fluid within the cavity are recognizable and separated from each other by a dye interface. These bodies of fluid form a queue and are successively stacked up one behind the other and move in jumps towards the body awaiting their turn to be "squeezed" out of the cavity and carried away by a Kelvin-Helmholtz-like roll up. Figure 7(a) shows a schematic "threading" diagram of the vortex sheets and demonstrates that every eddy produced is ultimately interconnected with every other eddy. The two sets of

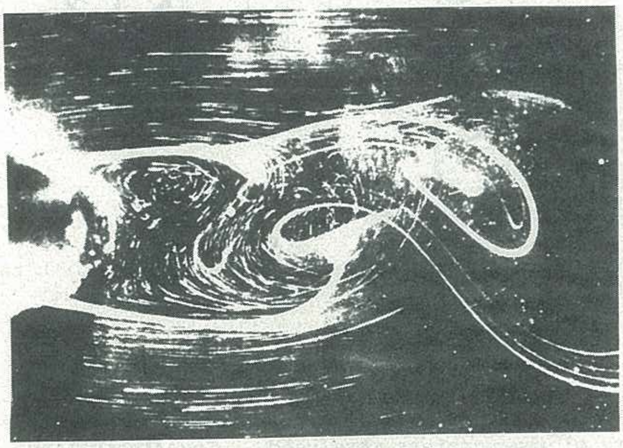

Figure 7 Instant streamlines and dye traces (streaklines or vortex sheets) behind circular cylinder.

vortex sheets intertwine with each other in the far wake. This ultimately forms what appears to be a single sheet of alternating sign folded on itself like the structures of Zdrakovich. This is illustrated in figure 7(b) and serves as a simplified model of the real case. Dye introduced at the stagnation point forms a cavity and by continuity this dye leaves the cavity as a thin sheet. However, in this model, there is no interchange of fluid in the cavity except by viscous diffusion. In reality, fresh fluid is continually being pushed into and squeezed out of the cavity.

Figure 7 shows instantaneous streamlines, together with the dye.

5 CONCLUSIONS

It can be seen that the unsteady cavity behind a vortex shedding cylinder is most complex. Rather than a stagnant "pool" of fluid, the cavity should be thought of as a region where vortex sheets are undergoing a multiple folding process and that each vortex which is shed is always ultimately connected back to the cavity by its own "umbilical cord" or "thread". This explains how vorticity resides underneath the separating vortex sheet. This vorticity is convected into the cavity, i.e. viscous diffusion although present, is not the basic mechanism. Properties of the far wake have been identified both experimentally and theoretically.

6 ACKNOWLEDGEMENTS

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