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Non-Newtonian Effects in Creeping Flow About a Sphere

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1 INTRODUCTION

The drag of spheres in creeping motion through non-Newtonian fluids may reflect both shear-thinning and elastic properties of these fluids. Most aqueous polymer solutions exhibit both types of properties but to very different extents. The literature on sphere motion through such fluids has been reviewed by Acharya et al (1976). While constant viscosity, weakly elastic fluids, such as silicone oils, have been known for some time, fluids of nearly constant viscosity and showing large elasticity have only recently been described by Boger and Binnington (1977). These two groups of fluids have more recently been used for a study of elastic effects in creeping sphere motion in the virtual absence of shear-thinning (Chhabra et al 1979/80).

The effects of shear-thinning alone are not so easily isolated from those of elasticity because it is not possible to demonstrate conclusively that elasticity is absent from aqueous polymer solutions. This leaves some uncertainty in sphere fall results obtained in "inelastic" shear-thinning polymer solutions. The use of the power law to describe shear-thinning behaviour adds additional uncertainties. New results and a discussion of theories using the power law are presented in this paper.

Highly shear-thinning and elastic fluids are readily prepared and have been used to investigate the combined effects of these properties on sphere motion by Acharya et al (1976) and by Sigli and Coutanceau (1977). Recently some success was achieved in predicting these effects by using the Carreau viscosity equation to describe fluid behaviour (Chhabra and Uhlherr, 1980).

Thus, the effects on sphere drag in creeping motion, of elasticity alone, shear-thinning alone and the two in combination are summarised in this paper.

2 CONSTANT VISCOSITY ELASTIC FLUIDS

The influence of fluid elasticity on the drag coefficient of a sphere in creeping motion was allowed for empirically by Chhabra et al (1979/80), by writing

$$C_D = (24/Re_o)X_e \qquad (1)$$

where X_e is a correction factor to account for the deviation of the drag from the Stokes value. Re_o is the usual Reynolds number based on the Newtonian or zero shear viscosity. In the absence of shear-thinning X_e is a function only of a Weissenberg number We arbitrarily defined as $2\theta V/d$ with θ the Maxwellian fluid relaxation time. The variation of X_e with We for all values of Re_o in the range 1.7×10^{-5} to 8.1×10^{-2} is shown in Figure 1. The results cover more than four decades of We from 1.66×10^{-4} to 6.37. For We<0.1 there is no significant reduction in drag due to elasticity while for We>0.1 the drag decreases significantly below the Newtonian value as We increases. For We>0.7 the drag becomes constant again but at a value of only ~74% of the Newtonian drag. The constant drag region We<0.1 can be identified with low levels of elasticity such that the fluid behaviour is second order and the relaxation time is shear independent. This behaviour was observed with three silicone fluids as well as with two separan/corn syrup solutions at low shear rates. It is broadly in agreement with theoretical perturbation solutions using viscoelastic constitutive equations. These solutions, all limited to low levels of elasticity, predict only very slight reductions in the drag below the Stokes value. The progressive reduction in drag for We>0.1 corresponds to increasing deviation of the first normal stress difference from a quadratic dependence on shear rate; ie increasing dependence of relaxation time on shear rate. This behaviour was observed with seven separan/corn syrup solutions at shear rates outside the second order region, including the two solutions mentioned above for which results extended into the second order region. Thus overlap of results was obtained using two chemically very different groups of fluids.

3 SHEAR-THINNING ELASTIC FLUIDS

Bird (1965) has suggested and Abdel-Khalik et al (1974) have shown that constitutive equations containing a characteristic time parameter should, in principle, be capable of representing both shear-thinning and elastic effects without recourse to independent measurements of normal stress. The viscosity equation proposed by Carreau (1972) is a four parameter equation containing an explicit time parameter.

$$(\eta-\eta_\infty)/(\eta_o-\eta_\infty) = [1 + (\lambda\dot{\gamma})^2]^{(n-1)/2} \qquad (2)$$

where λ is a characteristic fluid time, the power n has the same value and significance as the power law flow behaviour index in the shear-thinning region, and η_o, η_∞ are the zero shear rate and infinite shear rate viscosities respectively.

The viscosity η_∞ is difficult to measure, and can usually be omitted since $\eta_\infty \ll \eta_o$, or the viscosity of the solvent can be used as suggested by Abdel-Khalik et al (1974). The authors have used the Carreau viscosity equation with the stream function of Wasserman and Slattery (1964) and a variational principle to obtain X, defined in the same way as X_e in Equn (1). The drag correction factor X was calculated as a function of dimensionless fluid time $\Lambda(=2\lambda V/d)$, which is analogous to the Weissenberg number, the power law index n and a ratio of viscosities $(\eta_o-\eta_\infty)/\eta_o$. The result was compared with experiments using a wide range of chemically different polymer solutions, and agreement was found to be excellent, as shown for a typical case in Figure 2. Ten test fluids were used, having $0.40\leq n\leq0.81$ and $0.33\leq\lambda\leq19$(s), and correction factors X down to 0.1 were observed and predicted.

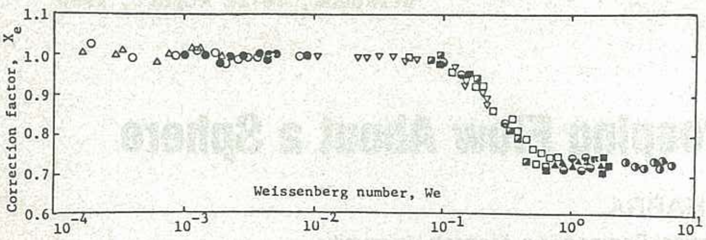

Fig. 1: Drag correction factor for creeping sphere motion through elastic, constant viscosity fluids.

A comparison of predicted and measured values of X for all fluids is shown in Figure 3. Agreement is generally better than ± 7.5%. In addition, values of first normal stress difference could be predicted which were in reasonable agreement with direct measurements in steady shear using a Weissenberg rheogoniometer. Bird's assertion appears to be well justified by these results.

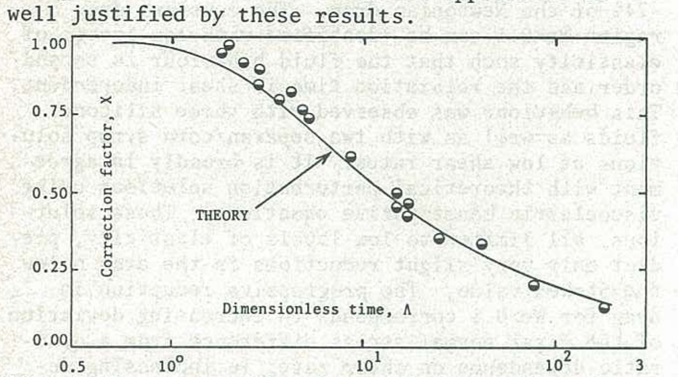

Fig. 2: Drag correction factor for creeping sphere motion through a Carreau model fluid.

It should be noticed that elasticity alone produces a maximum reduction in drag to 74% of the Newtonian value, while the combined effects of elasticity with shear-thinning produces a reduction in drag to 10% of the Newtonian value for very similar levels of elasticity (in terms of We based on Maxwellian relaxation time) and with the same definition of Re_o.

4. INELASTIC POWER LAW FLUIDS

Shear-thinning inelastic fluid behaviour is often described by the power law beacuse of the simplicity of this model. Frequently also zero shear viscosity is difficult to measure and so a more realistic fluid model cannot be used. Despite the severe limitations of the power law in describing regions of a flow field where the shear rate approaches zero, it continues to be applied to the problem of creeping sphere motion. It has not yet been determined over what proportion of the sphere surface this fluid model breaks down. Presumably if the areas of the sphere surface about the front and rear stagnation points and the volume of fluid far from the sphere do not contribute significantly to the total drag of a sphere, then a power law description of the flow field may yield an acceptable result for the drag.

4.1 Theoretical Solutions

Many authors have presented approximate solutions for the creeping motion of a sphere through a power law fluid and a good review of the area has been given by Acharya et al (1976). All solutions rely on different stream functions arbitrarily chosen only to satisfy the equation of continuity and the required boundary conditions and to contain a dependence on the power law index n. Most authors also applied a variational principle which allowed them to replace an a priori dependence of the stream function on n by an arbitrary parameter for which the dependence on n was to be determined. In

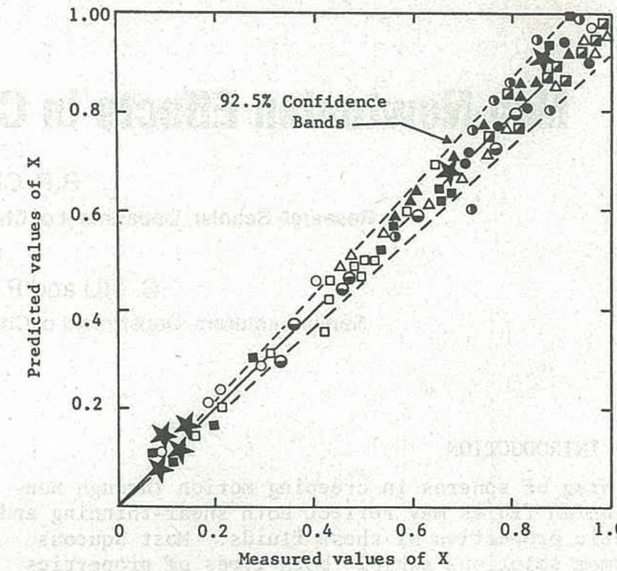

Fig. 3: Comparison of experiment with predictions for creeping sphere motion through ten elastic shear-thinning fluids modelled by the Carreau viscosity equation (Chhabra, Uhlherr, 1980).

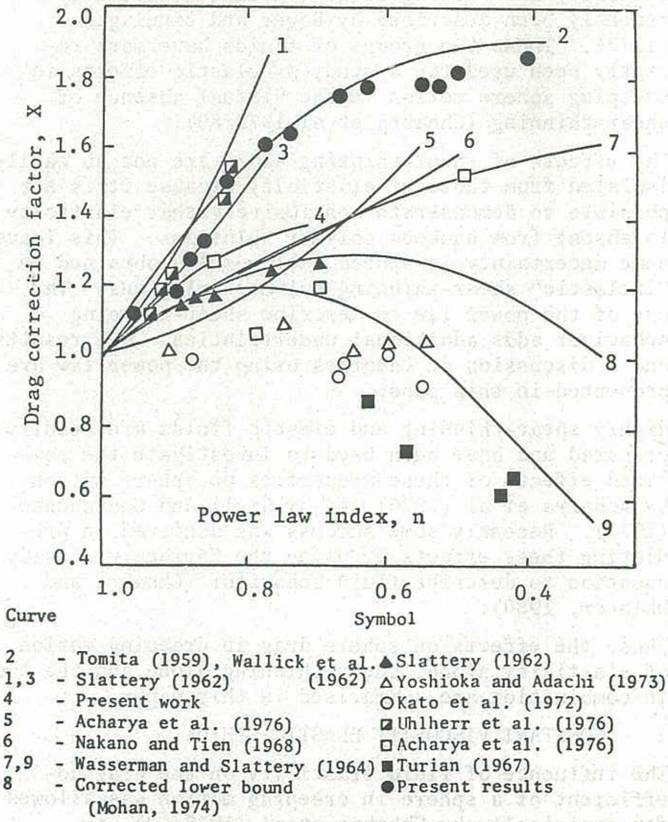

Fig. 4: Comparison of theoretical solutions and experimental data from the literature for X as a function of n.

this case, because of the minimization of energy dissipation rate, the resulting solutions are upper bounds on the drag. Seven solutions are compared in Figure 4. This figure shows a plot of X(n) given by $C_D Re/24$ where Re is the usual power law Reynolds number, $d^n V^{2-n} \rho/K$. Only Tomita (1959) and Acharya et al did not employ the variational principle and their solutions are not upper bounds on the drag. In addition, the latter authors introduced further approximations to allow them to give a closed form solution. One of the solutions included in Figure 4 was obtained by the authors using a stream function first proposed by Ziegenhagen (1964). This choice was based only on the fact that this stream function had not previously been used with the power law; it offers no particular advantages over other stream functions. The solution of Nakano and Tien (1968) was obtained for flow about Newtonian fluid spheres and the result shown in the figure is that for an infinite viscosity ratio of the internal to the external phase. Wasserman and Slattery (1964) in addition to supplying an upper bound for the drag, also calculated a lower bound based on another variational principle employing a trial stress profile. This lower bound was corrected by Mohan (1974).

Figure 4 shows the strong dependence of the drag correction factor on the choice of stream function. Since none of the solutions are rigorous, nor indeed can possibly be so because of the limitations inherent in the power law, the choice of the best result must be based on experiment.

4.2 Experimental Results

Sphere fall tests were carried out in twelve shear-thinning polymer solutions having $0.40 \leq n \leq 0.95$ as well as in three Newtonian fluids. The solutions were well described by the power law over a wide range of shear rate, encompassing the ranges of average shear rate (calculated as 2V/d) generated by all the spheres used. All the solutions were inelastic, in that normal force was not measurable under steady shear with an R16/19 Weissenberg rheogoniometer. In two of the solutions, normal force just became measurable at shear rates above 180 and 280 s^{-1}, which are very much higher than the shear rates generated in these fluids by any of the spheres used. Certainly in all cases the Weissenberg number was much less than 10^{-3}, and in view of the results for purely elastic fluids, elasticity can be assumed to be absent from the results under discussion here. Such uncertainty concerning the presence of elastic effects will always be encountered with aqueous polymer solutions.

Terminal velocity under gravity was measured for 20 different spheres ($1.59 \leq d \leq 12.69$ mm; $1190 \leq \rho_p \leq 16600$ kgm^{-3}). The data were corrected for wall effects by carrying out the measurements in five or more cylinders having different internal diameters, and extrapolating the velocity to a zero value of sphere to tube diameter ratio (Turian, 1967; Chhabra et al, 1977). All the usual precautions required by such experiments were observed (Chhabra et al, 1978/80; Chhabra and Uhlherr, 1979, 1980) and the results showed excellent reproducibility and accuracy; in the case of Newtonian fluids, deviation from the Stokes drag coefficient was less than ±2%. The results of the measurements are included in Figure 4. It must be observed that a single point in this figure represents an entire curve $C_D(Re,n)$.

Also shown in Figure 4 are experimental results reported in the literature. A large amount of scatter is evident and no entirely satisfactory explanation for this can be given. Most of the earlier authors did not characterise the elastic properties of their solutions. That these solutions were, in fact, elastic can be inferred from reported information of polymer species and concentrations. Only Acharya et al (1976) and Uhlherr et al (1976) fully characterised their solutions, showing that normal force was not measurable even at the highest shear rates generated by the falling spheres. For the results of other authors, elastic effects cannot be excluded as the cause of the scatter. The different treatment of wall effects by different authors may also be reflected in the scatter of the results, although the errors introduced are probably small.

The picture that emerges of creeping sphere motion through inelastic power law fluids is entirely unsatisfactory - from both the theoretical and the experimental aspect. We consider that the new experimental results reported here are among the most reliable to date from the point of view of completeness of fluid characterisation, terminal velocity measurement and wall correction. They indicate that the first approximation solution of Slattery (1962) shows the best agreement for $0.8 \leq n \leq 1.0$ and that Tomita's (1959) solution gives the best description of the drag for $0.4 \leq n \leq 0.8$. Uhlherr et al (1976) had previously reached the same conclusion for $n > 0.8$.

4.3 Average Shear Rate

The surface average shear rate for a sphere can be readily calculated from any stream function. Different stream functions give different results, and Stokes stream function leads to $\bar{\dot{\gamma}}$ of 2V/d. Not all the stream functions under discussion give this result when n = 1; those of Tomita and of Wasserman and Slattery do not, and give 1.33 V/d and 1.736 V/d respectively. These solutions therefore cannot be expected to accurately describe the flow field in the Newtonian case. That they lead to X=1 at n=1 is perhaps due to a compensation through the pressure drag term for deviation from the Stokes friction drag. Measurements of total drag tend to be insensitive to effects in the flow field and measurements of velocity distributions would be prefereable. These are, however, difficult to obtain close to the sphere surface, where the most important changes are expected. Sigli and Coutanceau (1977/78) have reported visual observations of the flow field about a sphere. They were concerned mainly with the effects of elasticity on the flow field relatively far from the sphere surface and no results are reported with which the validity of a stream function could be directly tested.

The power law Reynolds number $d^n V^{2-n} \rho / K$ arises naturally through non-dimensionalising the variables. This Reynolds number, like that based on zero shear viscosity, is independent of shear rate. It may be preferable to use a Reynolds number based on an average apparent viscosity which could conceivably better reflect the influence of shear-thinning viscosity on the drag. This Reynolds number $\overline{Re}$ is

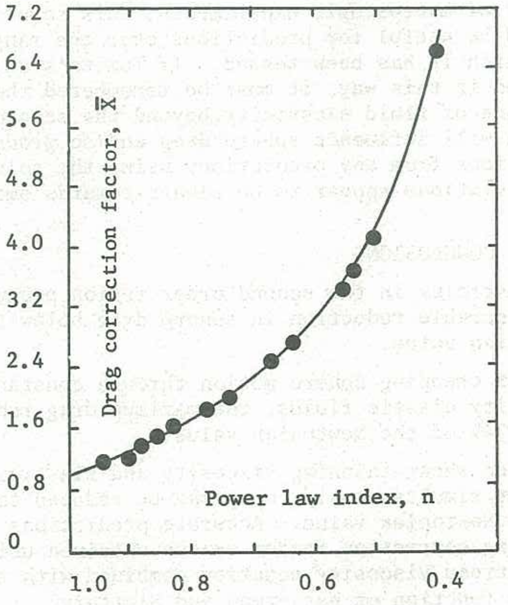

Fig. 5: Comparison of the present experimental results in terms of $\bar{X}$ with the theory of Tomita (1959), corrected by Wallick et al. (1962).

given by $dV\rho/\bar{\eta}$, where $\bar{\eta}$ is obtained at the surface average shear rate. A drag correction factor can be defined empirically as $\bar{X}$, given by $C_D\overline{Re}/24$.

This is tantamount to introducing a correction factor directly into the equation for drag force in the Stokes law regime for a Newtonian fluid

$$F_D = 3\pi dV\bar{\eta}\,\bar{X} \quad (3)$$

The correction factors X and $\bar{X}$ are related by a shear dependent multiplying factor, the value of which is a function only of n, and which can be calcualted for any stream function. For example, for the stream function of Tomita $\bar{\dot{\gamma}}$ is 1.33 V/n^2d and $\bar{X}/X$ is $(n^2/1.33)^{1-n}$. The main disadvantage of this entirely empirical approach is that theoretical solutions can nolonger be compared on a single plot of $\bar{X}(n)$ since each solution is now based on a different Reynolds number; solutions can only be compared in pairs. Similarly, experimental results can be compared with only one theoretical solution at a time in terms of its particular $\bar{X}(n)$ definition. This comparison was carried out for the present experimental results and each stream function from the literature, and it was found that slight improvements in agreement could be obtained. Maximum deviations were reduced from as much as 30-40% to 20-30% in most cases. However, for the case of Tomita's solution the procedure produced excellent agreement with the experiments over the whole range of n from 1.0 to 0.4. This is shown in Figure 5. This result may well be fortuitous in view of the shortcomings of Tomita's solution mentioned above, and it is not suggested that this solution gives the best description of the flow field. However, it is suggested that it gives the best macroscopic description of sphere drag in inelastic power law fluids for $0.4 \leq n \leq 1.0$.

The fact that all but one of the solutions employing variational principles and resulting in upper bounds for the drag give values of X smaller than Tomita's has yet to be explained. It may simply be due to the inadequacy of the power law in describing the flow field so that no theoretical solution based on this model can ever rigorously predict sphere drag; agreement with experiment must always be fortuitous. Nevertheless, once one solution has been identified that adequately describes the results of a large number of macroscopic experiments, this solution should be useful for predictions over the range for which it has been tested. If Tomita's solution is used in this way, it must be remembered that the presence of fluid elasticity beyond the second order region will influence sphere drag and so produce deviations from any predictions using the solution. The deviations appear to be always towards smaller drag.

5. CONCLUSIONS

1a Elasticity in the second order region produces no measurable reduction in sphere drag below the Newtonian value.

1b For creeping sphere motion through constant viscosity elastic fluids, the maximum drag reduction is to 74% of the Newtonian value.

2 When shear-thinning viscosity and elasticity are present simultaneously, drag may be reduced to 10% of the Newtonian value. Accurate predictions of the drag correction factor can be obtained using the Carreau viscosity equation combined with the stream function of Wasserman and Slattery.

3 If the power law must be used for the description of creeping sphere mtoion and the power law Reynolds number is used to calculate the drag coefficients, Tomita's theoretical solution gives the best value of total drag for $0.4 \leq n \leq 0.8$ and Slattery's first approximation solution is best for $0.8 \leq n \leq 1.0$, provided that the effects of fluid elasticity are absent.

Agreement with Tomita's solution can be further improved by the arbitrary use of a surface average apparent viscosity.

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