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Depth Control During Demand Changes in Open Channels

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I INTRODUCTION

It is sometimes required that flow changes in open channels be brought about in one of two ways:

(i) in which both the flow-rate and the depth are controlled in a prescribed manner at a particular channel section (e.g. the forebay of a hydro-power station) or,

(ii) in which, for one channel pool, the depth and/or the rate of change of depth are controlled within prescribed limits (e.g. in irrigation works, for channel protection and safety)

The question that then arises is how to operate the channel controls so as to satisfy these requirements. The answer to this question has commonly been sought empirically, as, for example, in Dewey and Madsen's (3) work on the California Aqueduct.

Wylie (7) showed that the concept of "valve-stroking" developed for closed-conduit flow, notably by Streeter (5, 9) and Propson (4) could be extended to open-channel flow and operating procedures for channel controls could be "designed" to achieve a desired flow change in a prescribed manner. He termed it "gate-stroking" in open-channel flow. This method was demonstrated for the case of a single channel pool with a control gate at each end. The manner of the changes in both flow-rate and depth at a particular channel section was specified, i.e. category (i) above. Bodley and Wylie (1) extended the method to any number of pools in series, separated by control gates.

This present paper examines the second way in which flow changes may be required to be made, namely when limits on water depth and/or rate of change of water depth during the flow change are specified, but the manner in which the flow-rate changes from its initial to its final value is not.

A single channel pool with control gates at each end is considered and a computational scheme developed for determining gate motions which will bring about a required flow change within specified constraints on maximum and minimum depths. The rate of change of depth is only indirectly constrained in this scheme. Another computational scheme has been developed which provides direct constraint on rate of change of depth as well as depth. It is not reported on here because of space limitations.

Two typical flow changes were studied: (i) flow initiation (zero to 169.92 m^3/s (6000 ft^3/sec)); (ii) flow arrest (169.92 m^3/s to zero). For both cases the maximum permissible change of depth was taken as (a) 0.075 m, (b) 0.15 m and (c) 0.225 m.

The results of all "gate stroking" calculations were confirmed by "analysis".

2 NOTATION

The following symbols are used in this paper:

A = area of cross section below water surface;

$C = \sqrt{gA/T}$ = wave celerity;

C^+, C^- = positive and negative characteristic lines, defined by Eq. 4;

g = acceleration due to gravity;

Q = flow-rate;

S = slope of energy grade line;

S_o = slope of channel bottom;

T = width of water surface;

t = time;

V = average velocity at cross section;

x = distance along channel; and

y = depth of water at channel section.

3 GOVERNING EQUATIONS

3.1 Unsteady Channel Equations

For one-dimensional flow in a prismatic channel of small slope, such that hydrostatic conditions may be assumed on any vertical line in the water, the unsteady momentum and continuity equations applied to a short control volume that extends across the channel yield (9):

$$gy_x + g(S - S_o) + VV_x + V_t = 0 \tag{1}$$

$$\frac{A}{T} V_x + Vy_x + y_t = 0 \tag{2}$$

in which y = depth of flow in the channel; V = average velocity over the cross-section; A = cross-sectional area, below water surface; T = width of the water surface; S_o = slope of the channel bed; S = slope of the energy grade line; g = acceleration due to gravity; x = distance along the channel; and t = time. Subcripts x and t indicate partial differentiation with respect to these independent variables. It is assumed that: (a) the energy losses are adequately represented by Manning's equation for steady uniform flow; and (b) a finite depth of water remains at each section in the channel at all times during the transient and that changes are never so rapid as to create a hydraulic bore.

The governing equations, Eqs. 1 and 2, cannot be solved analytically; however, several numerical methods have been developed for their solution (9). The solutions presented here were obtained by the method of characteristics. A particular advantage of this method is that the characteristic lines generated in the solution are essentially the paths, in the *xt*-plane, of surface disturbances. Knowledge of this fact can be a valuable aid in interpreting solutions.

In the method of characteristics, the partial differential equations, Eqs. 1 and 2, are recast as the following ordinary differential equations:

$$\pm \sqrt{\frac{gT}{A}}\frac{dV}{dt} + \frac{dV}{dt} + g(S - S_o) = 0 \qquad (3)$$

$$\frac{dx}{dt} = V \pm \sqrt{\frac{gA}{T}} \qquad (4)$$

respectively.

Schemes for solving these equations using a "characteristic grid" or a "rectangular grid" in the desired region of the *xt*-plane have been detailed elsewhere (e.g., Ref. 8). A rectangular grid was used in the work presented here. The selection of the grid and the choice of interpolation schemes are outlined in Appendix I.

3.2 Gate Equation

For flow through underflow gates with submerged outflow, Chow (2) gives the relation:

$$Q = C_D WH \sqrt{2g(y_1 - y_2)} \qquad (5)$$

in which Q = flow-rate; C_D = gate coefficients; W = gate width; H = gate opening; and y_1, y_2 = water depths upstream and downstream of the gate, respectively.

4 GATE STROKING WITH DEPTH AND/OR RATE OF CHANGE OF DEPTH CONSTRAINTS

Gate-stroking with these constraints can only be applied to a single channel pool. If the channel is composed of a number of pools in series, one pool may be selected for this kind of transient control. The effect of this transient can then be traced through the remaining pools by the method given by Bodley and Wylie (1).

In the nominated channel pool, the desired flow change is brought about in three phases. Looking at conditions in the pool in the *xt*-plane (Fig. 1) the initial and final steady flow conditions exist within the lower and upper hatched areas, respectively. The region between these hatched areas is occupied by the transient and is most easily described in terms of a particular example. Thus, if a reduction in flow-rate is desired, during which the water depth must not exceed a given maximum nor fall below a given minimum and the rates of change of water depth, rising and falling must not exceed given values, the three phases of the transient are as follows:

Phase I: the downstream depth is increased and the upstream depth decreased at the permissible rates, or less, to the limit depths or to predetermined depths within the allowable limits.

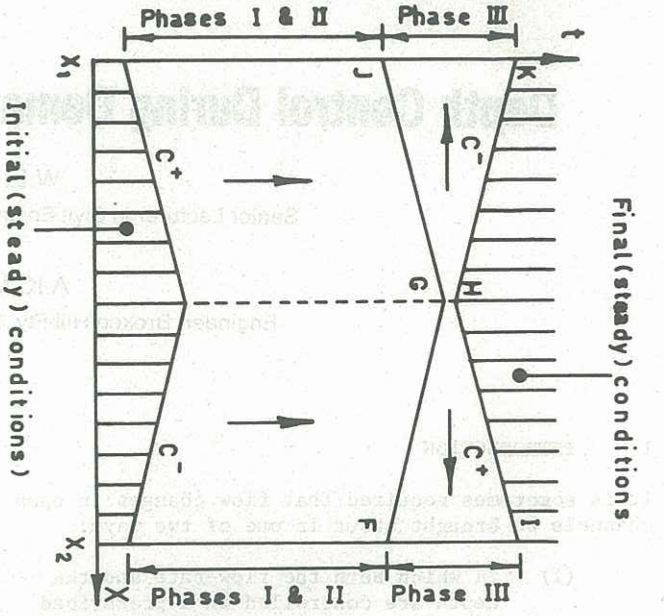

Figure 1 Channel Pool Viewed in the *xt*-Plane. Arrows indicate direction in which computation is advancing. The interval *GH* is one time step, Δt.

Phase II: the downstream and upstream depths and the consequent water surface profile, set in Phase I, are held until the flow-rate decreases to the desired new value, or marginally beyond it, at the mid-length of the pool - point *H* on the *xt*-diagram.

Phase III: downstream depth is decreased and upstream depth increased to the required final values in a manner that is computed from the known conditions along the bounding characteristics of the trapezia *FGHI* and *JGHK*. The duration of Phase III in this scheme is determined by the pool length, the wave celerity and the chosen time increment, Δt. The mean rate of change of depth in Phase III is therefore determined by the depth change to be effected during this Phase. Control over rate of change of depth is thus only indirect, in the computational scheme described here.

At the conclusion of Phase III, both flow-rate and depth variations at the two ends of the pool are known for the time intervals embraced by Phases I, II and III of the transient. If the depths over these time intervals are also known on the outside of the two gates (i.e. upstream of the upstream gate and downstream of the downstream gate; for example, they might both be reservoirs with depth constant) then the actual gate positions versus time can be computed if the gate calibrations are known. These computed gate positions versus time are the "stroking-designed" gate motions which will bring about the desired flow change within the specified constraints on depth and, indirectly, rate of change of depth.

At the conclusion of the gate motions, the flow is completely steady at the desired new condition with no residual disturbances, along the entire length of the pool. The volume of water within the pool is maintained approximately constant during the flow change.

5 VERIFICATION OF GATE STROKING SOLUTIONS

The standard characteristics method of analysis was used to provide verification of the gate stroking solutions. Data used in the analysis consisted of the specified initial conditions in the pool and the upstream and downstream depth variations within the pool for the duration of the transient. From these data, the analysis program computed the variations in upstream and downstream flow-rate throughout the transient and also the final steady-state conditions along the reach.

6 APPLICATIONS

6.1 Description of Test Channel and Test Conditions

Channel dimensions and other properties were chosen to be representative of the California Aqueduct - reach length = 7254.24 m (23,800 ft) bed slope = 0.000045; cross-section, trapezoidal with bottom width = 12.192 m (40 ft) and side slope 1 vertical: 1.5 horizontal; Manning's $n = 0.012$; radial underflow gates were assumed, 24.384 m (80 ft) wide and having a gate coefficient, C_D (in Eq. 5) = 0.75, independent of flow-rate. Reservoir conditions were assumed both upstream and downstream of the test pool.

The test conditions studied were:

(i) flow initiation, zero to 169.92 m^3/s (6000 ft^3/sec); duration of Phase I, 1015 s, with depth changes imposed linearly in that time; maximum change of depth (a) 0.075 m (≈ 0.25 ft), (b) 0.15 m and (c) 0.225 m,

(ii) flow arrest, 169.92 m^3/s (6000 ft^3/sec) to zero; duration of Phase I, 1015 s, with depth changes imposed linearly in that time; maximum change of depth (a) 0.075 m (≈ 0.25 ft), (b) 0.15 m and (c) 0.225 m.

In each case the initial and final depths at the downstream end of the pool was prescribed such that the backwater profiles intersected close to the mid-length of the pool. In this way the volume of water in the pool was maintained approximately constant throughout each flow change.

6.2 Typical Results: Two typical sets of results are given below

6.2.1 Flow initiation zero to 169.92 m^3/s, maximum depth change 0.15 m - case (i)(b) above.

The imposed depth changes at the ends of the pool are plotted in Fig. 2A. The (very nearly constant) depth at the mid-length of the pool is also shown. The resulting flow-rate changes are plotted in Fig. 2B and the required gate motions for the two gates in Fig. 2C.

The three phases of the transient can be clearly seen in each plot. As mentioned, the duration of Phase I was specified as 16.91 minutes (1015 s). The duration of Phase II is seen to be 16.66 minutes (999.6 s) and Phase III, 15.01 minutes (900.6 s).

6.2.2 Flow arrest, 169.92 m^3/s to zero, maximum depth change, 0.15 m - case (ii)(b) above

The imposed depth changes and the resulting flow rate changes are plotted in Fig. 3A and 3B, respectively. Mid-length depth (which is again nearly constant) is also shown in Fig. 3A. Required gate motions are plotted in Fig. 3C.

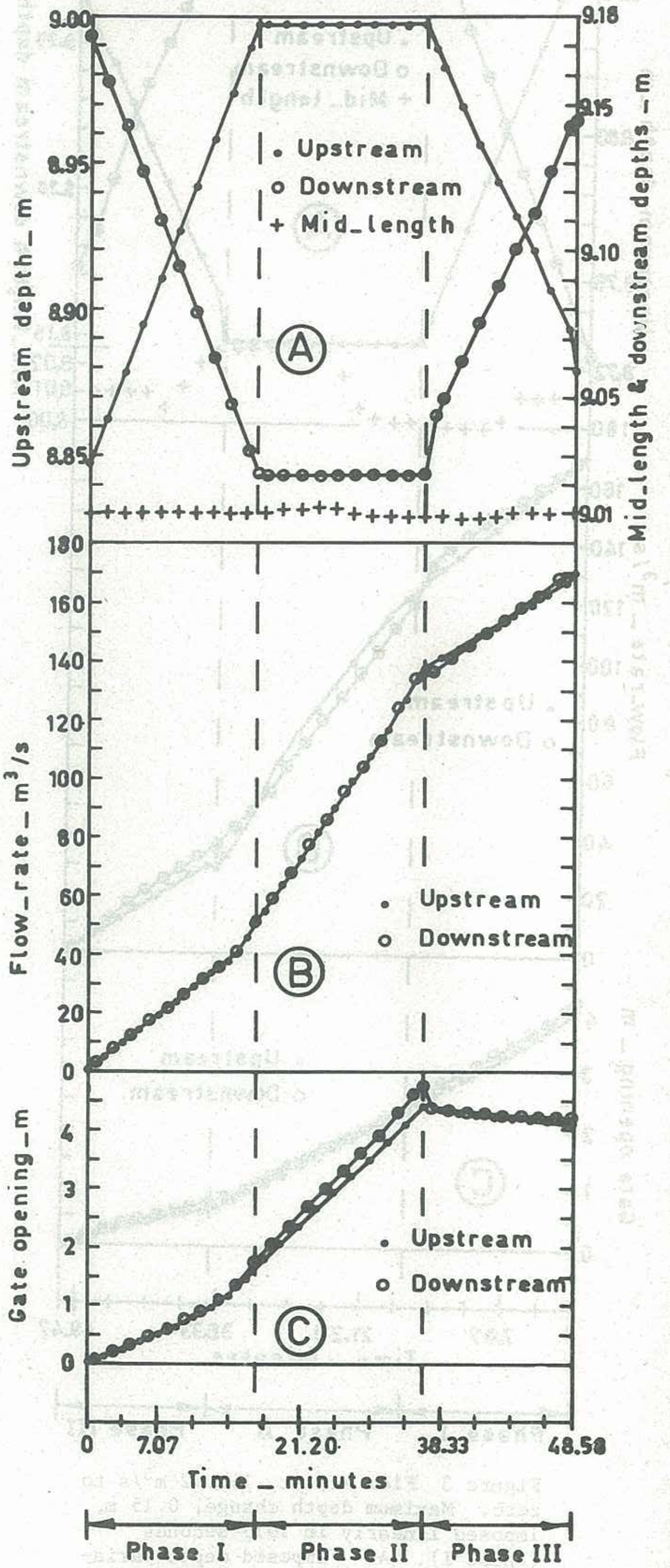

Figure 2 Flow Initiation, zero to 169.92 m^3/s. Maximum depth change, 0.15 m, imposed linearly in 1015 seconds (Phase I). A:- Imposed depth variations. B:- Resulting variation in flow-rate. C:- Stroking-designed gate motions.

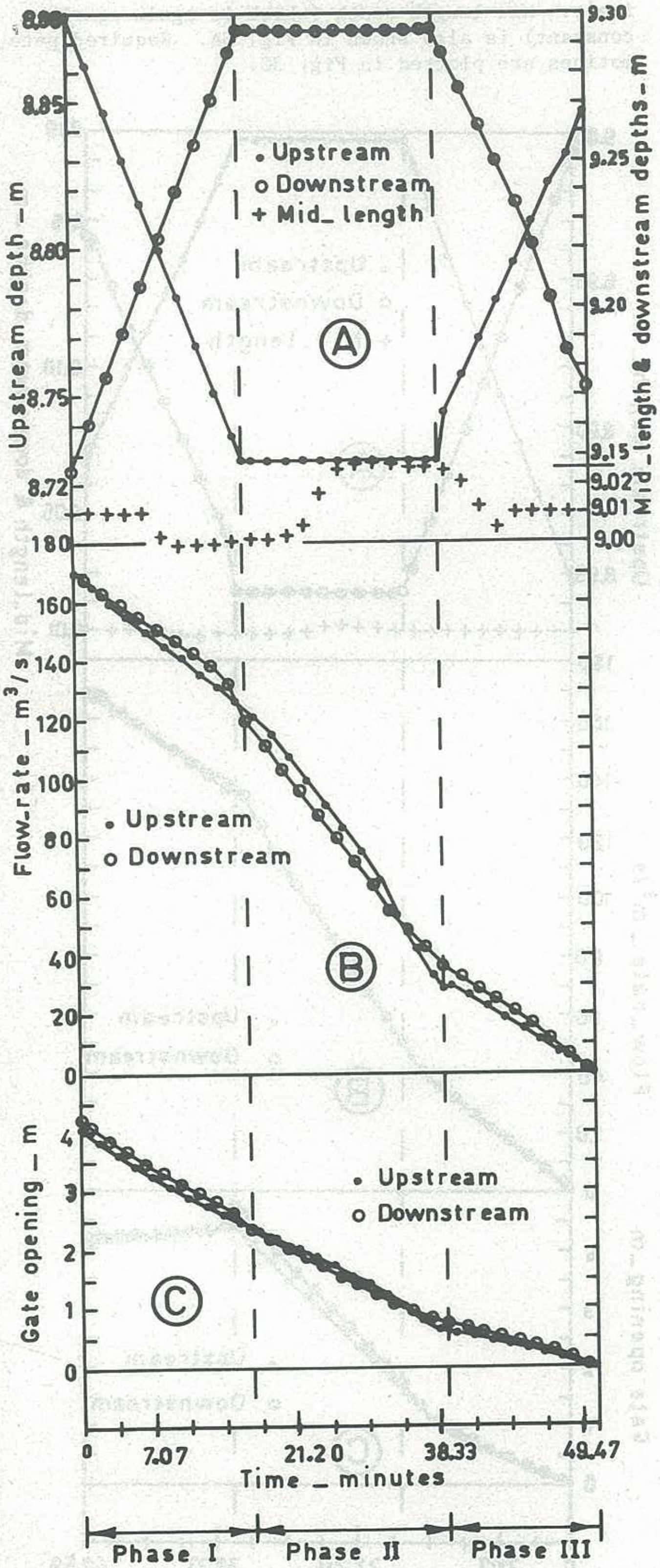

Figure 3 Flow Arrest, 169.92 m^3/s to zero. Maximum depth change, 0.15 m, imposed linearly in 1015 seconds (Phase I). A:- Imposed depth variations. B:- Resulting variation in flow-rate. C:- Stroking-designed gate motions.

Dewey and Madsen (3) report gate movements of 1 ft/min (≃ 0.3 m/min) on the California Aqueduct. The "stroking-designed" gate motions above, for both flow initiation and flow arrest require rates of gate movement considerably lower than this except for one brief period during flow initiation when a rate of 0.42 m/min is required of the downstream gate (Fig. 2C). With this brief exception, perhaps, the designed gate motions should therefore be readily achievable in practice.

For the case in which both flow-rate and depth are controlled at a particular channel section, Bodley and Wylie (1) showed that a two-speed linear approximation to the "stroking-designed" gate motions brought about the desired flow changes in an almost identical manner to that prescribed. The most significant effect of the approximations was that small residual disturbances remained in the channel after the gate motions were completed, whereas with the "stroking-designed" motions no such residuals exist. The gate motions presented here (Figs. 2C and 3C) are made up of approximately straight line segments suggesting that again two-speed linear motions would very satisfactorily approximate the "stroking-designed" motions.

7 CONCLUSIONS

It has been shown that the motions of control gates at the two ends of a channel pool can be designed to bring about a desired change of flow through the pool while maintaining water depths and, indirectly, rates of change of water depth within specified limits.

As with other "stroking-designed" gate motions (1, 7) when the gate motions are completed, the specified, final steady flow exists everywhere in the pool, with no residual disturbances from the transient.

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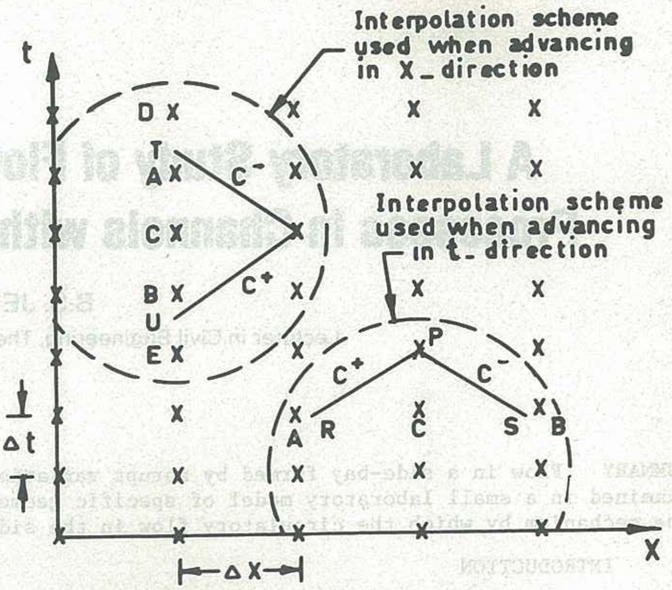

Figure 4 Grid of Points in the xt-Plane and Interpolation Schemes Employed in Gate Stroking Program for Computations Advancing in t Direction and x Direction.

APPENDIX I Grid Size and Interpolation Schemes

A1.1 Gate Stroking Program

A rectangular grid was used with Δx = 453.39 m and Δt prescribed such that for computations advancing in the t direction, (see Fig. 1) both characteristics through the current grid point, P, intersected the previous time line within one distance increment either side of P, i.e., the points R and S in Fig. 4. It was desirable to use this same grid of points for those computations which advanced in the x direction (see Fig. 1). The intersections of the characteristics on the previous distance line now occurred beyond one time increment either side of P, i.e., the points T and U in Fig. 4. Interpolation was therefore performed between values at B and E for the intersection point, U, and between values at A and D for the intersection point, T. Such a scheme has previously been utilized by Vardy (6) for pipeline transients.

First order interpolation was used throughout. Where standard interpolation was used, i.e., for computation advancing in the t direction, the slopes of the characteristics were based on values of V and C at the central point, C (refer to Fig. 4) of the previous time line. Where Vardy's interpolation scheme was used, i.e., for computation advancing in the x-direction, slopes of the characteristics were based on values of V and C at the grid point closest to the points of intersection of the characteristics with the previous distance line, i.e., the points A and B in Fig. 4.

A1.2 Analysis Program

Computation in the Analysis Program advanced always in the t direction; thus there was no need for two interpolation schemes as in the Stroking Program. Standard interpolation was used, Δx was again set equal to 453.39 m and Δt was calculated afresh for each advance in time, such that the flattest characteristic (positive or negative) in that particular line of the computation intersected the previous time line exactly in a grid point. This choice of Δt ensured that all other characteristics in that line intersected the previous time line as close as possible to the relevant grid points and, therefore, that interpolation involved the smallest possible distance from the points at which conditions were known.