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Cross Flows in the Flow from an Elongated Orifice

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SUMMARY An approximate analysis was made of the cross flows produced in the free jet formed by the flow of liquid through a plane, elongated, orifice. The analysis required that the width of the orifice be an order of magnitude less than its span, that the lips of the orifice should follow a smoothly varying curve, and the ends of the orifice should be sharply pointed. It was found that substantial residual cross flows remained in the jet far downstream of the orifice, and these were such as to cause the jet to converge in the plane of the orifice span.

1 INTRODUCTION

Nozzles which produce fan shaped liquid sprays employ a variety of differing internal configurations.[1,2] A common configuration is shown in fig.1 It is formed by taking an axisymmetric duct and, at the crown of the duct, making a vee-shaped "slash" with a grinding wheel, or similar tool. The resulting nozzle opening is an elongated slit, of varying width which, in principle at least, has sharp ends. The contour of the plane of the slit is curved, with the curvature determined by the interpenetration between the "slash" surface and the original duct contour. It is desirable to develop methods for predicting the mass flow distribution in the spray produced by such nozzles, as this will make it possible to design them to generate given spray distributions.

The flow distribution in the spray fan is determined by the direction of the flow as it leaves the nozzle and therefore in predicting the spray distribution, it is necessary to consider the factors which govern the flow direction. As the width of the nozzle orifice changes along its length, the dimensions of the pressure field associated with acceleration of the liquid through the nozzle must also change. This implies that there will be substantial crosswise pressure gradients at the nozzle, tending to impart a component of velocity tangential to the local plane of the orifice.

A theoretical analysis is undertaken to explore this effect for a plane orifice. This is shown in fig.2.

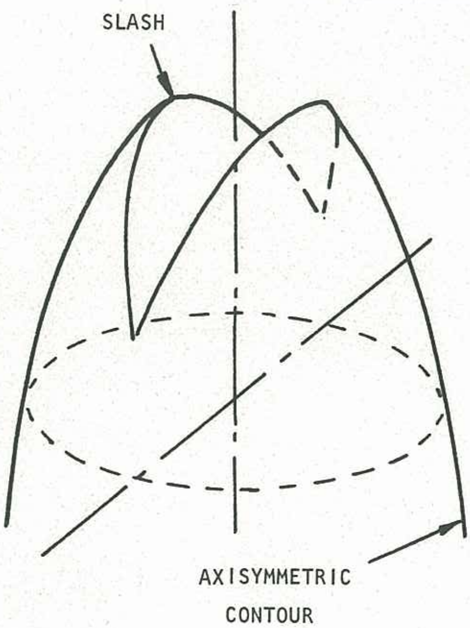

Figure 1 Spray nozzle configuration

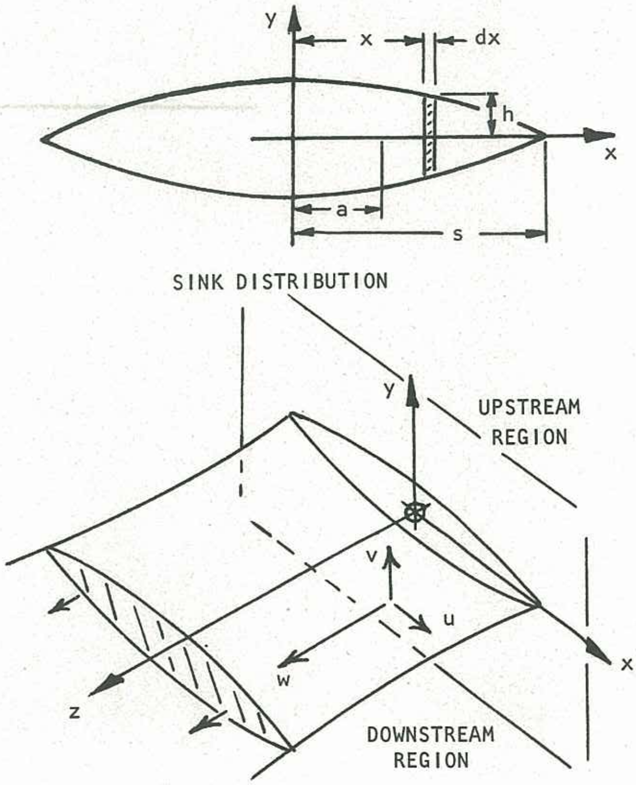

Figure 2 Flow through plane orifice

The analysis requires that the width of the orifice be an order of magnitude less than its span, that the orifice lips should follow a smoothly varying curve, and that the ends of the orifice should be sharply pointed. The orifice is formed in an infinitely thin flat plate, and co-ordinate axes x, y, z are chosen, with corresponding velocity components u, v, w as shown. It is convenient to consider the flow field as two adjoining regions, respectively upstream and downstream of the plane of the orifice, and to analyse each region independently. The two regions are joined by matching velocities at the orifice.

2 THE UPSTREAM REGION

The flow in the upstream region is analysed by treating the flow through the orifice as a distribution of elemental line sinks, aligned parallel to the "y" axis. One such sink is shown in the figure, located at a distance, x, from the centre of the orifice, and occupying an interval, dx, of the "x" axis. The aim of the analysis is to calculate the cross flow velocity, u, at a point on the "x" axis which is a distance "a" from the centre of the orifice.

The length of each sink is taken as the width of the orifice, 2h, at the station "x" at which the sink is located. It is assumed that the sink strength distribution is uniform over the length of each sink. An analysis of the effect of a linear distribution of strength over the length of the sink indicates that the error in velocity at "a", resulting from the assumption of a uniform sink strength distribution, is of $0(h/(x-a))^2$. Provided that the width of the orifice varies slowly with x, it can then be shown that an error of this magnitude has a negligible effect on the crossflow velocity, when the total effect of all the sinks is taken into account.

The crossflow velocity on the "x" axis, at a, is taken as the crossflow velocity over the width of the orifice. Analysis showed that, for a single line sink, the error involved in making this assumption was again of $0(h/(x-a))^2$, and that the effect of the error on the total crossflow velocity due to the complete sink distribution, again was negligible.

The contribution of the elemental line sink at x to the crossflow velocity at "a" may be written as

$$du = \frac{W_o}{\pi} \frac{h\,dx}{(x-a)\left\{(x-a)^2+h^2\right\}^{\frac{1}{2}}} \tag{1}$$

where $W_o = \psi_{wo}/2h$ is the mean velocity through the orifice, with $\psi_{wo} = \int_{-h}^{+h} w\,dy$

Upon integration, eq.(1) yields the total crossflow velocity, u_o as

$$u_o = \frac{W_o}{\pi} \int_{-s}^{s} \frac{h\,dx}{(x-a)\left\{(x-a)^2+h^2\right\}^{\frac{1}{2}}} \tag{2}$$

where s is the semispan of the orifice. Putting $h=f(x)$, and expanding $f(x)$ as a Taylor series about a, i.e.

$$f(x) = f(a) + (x-a)\,f'(a) + \frac{(x-a)}{2!}\,f''(a) + \ldots\ldots.,$$

eq.(2) becomes

$$u_o/W_o = \pi^{-1} \int_{-s}^{s} \frac{f(a)\,dx}{(x-a)\left\{(x-a)^2+h^2\right\}^{\frac{1}{2}}} + \int_{-s}^{s} \frac{f'(a)\,dx}{\left\{(x-a)^2+h^2\right\}^{\frac{1}{2}}} + \ldots.$$

To perform the integrations in this series, it is noted that, in the denominators of the integrands, h makes a significant contribution only when $(x-a) \sim h$. Since h varies only slowly with x, this implies that, for the integration, it is sufficiently accurate to put $h = \text{const}\ (=f(a))$. Then, integrating term by term, and taking $|h/(s-a)| \ll 1$, the velocity is

$$\begin{aligned} u_o/W_o = \pi^{-1} \Big\{ & -2a\,f(a)/(s^2-a^2) \\ & + f'(a)\,\ln(4(s^2-a^2)/h^2) - a\,f''(a) \\ & + 0.167\,f'''(a)\left[s^2+a^2-0.5h^2\ln(4(s^2-a^2)/h^2)\right] + \ldots \Big\} \end{aligned} \tag{3}$$

3 THE DOWNSTREAM REGION

The flow in the downstream region is analysed by noting that the fluid is assumed to be inviscid, and that the flow is irrotational. Thus the vorticity is zero everywhere, and this fact may be exploited to write

$$\frac{\partial u}{\partial x} = \frac{\partial w}{\partial x}$$

or

$$\int_{-g}^{+g} \frac{\partial u}{\partial z}\,dy = \int_{-g}^{+g} \frac{\partial w}{\partial x}\,dy \tag{4}$$

where g is the semi width of the jet in the "y" direction.

Now

$$\frac{\partial}{\partial x}\left(\int_{-g}^{+g} w\,dy\right) = \int_{-g}^{+g} \frac{\partial w}{\partial x}\,dy + 2w_g \frac{\partial g}{\partial x},$$

and

$$\frac{\partial}{\partial z}\left(\int_{-g}^{+g} u\,dy\right) = \int_{-g}^{+g} \frac{\partial u}{\partial z}\,dy + 2u_g \frac{\partial g}{\partial z},$$

where the subscript "g" signifies velocities at the surface of the jet. Thus, writing

$$\int_{-g}^{+g} u\,dy = \psi_u, \text{ and } \int_{-g}^{+g} w\,dy = \psi_w,$$

eq.(4) becomes

$$\frac{\partial}{\partial z}(\psi_u) - 2u_g\frac{\partial g}{\partial z} = \frac{\partial}{\partial x}(\psi_w) - 2w_g\frac{\partial g}{\partial x},$$

and, integrating with respect to z, this becomes

$$\psi_u - \psi_{uo} = \int_0^z \frac{\partial}{\partial x}(\psi_w)\,dz - 2\int_0^z w_g\frac{\partial g}{\partial x}\,dz + 2\int_0^z u_g\frac{\partial g}{\partial z}\,dz \tag{5}$$

where ψ_{uo} is the value of ψ_u at the orifice.

To obtain an expression for the last term on the right hand side of eq.(5), note that $u \ll w$, and therefore the resultant velocity at any point may be written as

$$q = \sqrt{w^2+u^2}.$$

Then, since the surface of the jet is a constant pressure surface, q is constant there, and therefore

$$\left[\frac{\partial q^2}{\partial x}\right]_g = -\left[\frac{\partial q^2}{\partial y}\right]_g \frac{\partial g}{\partial x} \tag{6}$$

where $\left[\frac{\partial}{\partial x}\right]_g$ and $\left[\frac{\partial}{\partial y}\right]_g$ signify the value of the derivatives at the surface.
Eq.(6) may be rewritten as

$$w_g \left[\frac{\partial w}{\partial x}\right]_g + u_g \left[\frac{\partial v}{\partial x}\right]_g = -\left\{w_g \left[\frac{\partial w}{\partial y}\right]_g + u_g \left[\frac{\partial v}{\partial y}\right]_g\right\} \frac{\partial g}{\partial x}. \tag{7}$$

Because the vorticity is zero everywhere

$$\left[\frac{\partial w}{\partial x}\right]_g = \left[\frac{\partial u}{\partial z}\right]_g, \quad \left[\frac{\partial v}{\partial x}\right]_g = \left[\frac{\partial u}{\partial y}\right]_g$$

$$\text{and} \quad \left[\frac{\partial w}{\partial y}\right]_g = \left[\frac{\partial v}{\partial z}\right]_g,$$

and, using these relations, eq.(7) becomes

$$\left[\frac{\partial u}{\partial s}\right]_g = -\left[\frac{\partial v}{\partial s}\right]_g \frac{\partial g}{\partial x}, \tag{8}$$

where s is the distance measured along the surface streamline, and

$$W_\infty \left[\frac{\partial}{\partial s}\right]_g = w_g \left[\frac{\partial}{\partial z}\right]_g + u_g \left[\frac{\partial}{\partial y}\right]_g,$$

where W_∞ is the velocity of the jet far downstream, which is equal to q at the surface of the jet. Now, v is the "y" component of q, and so

$$v_g = W_\infty \sin\theta$$

where $\tan\theta = \partial g/\partial z$ is the slope of the surface in the "y-z" plane. Substituting into eq.(8), and integrating, a relation for u_g is obtained, i.e.

$$u_g = u_{og} - W_\infty \int_o^s \frac{\partial g}{\partial x} \frac{\partial(\sin\theta)}{\partial s} ds, \tag{9}$$

Passing far downstream in the jet, to where $g = g_\infty$, and using

$$u_D = (\psi_u - \psi_{uo})/2g_\infty$$

to denote the mean crossflow velocity imparted to the flow as it passes along the jet, eq.(5) may be written

$$u_D = (2g_\infty)^{-1}\left\{\int_o^\infty \frac{\partial}{\partial x}(\psi_w)dz - 2\int_o^\infty w_g \frac{\partial g}{\partial x} dz + 2\int_o^\infty u_g \frac{\partial g}{\partial z} dz\right\}, \tag{10}$$

where u_g is given by eq(9).

In order to evaluate this expression, the contour of the jet boundary must be specified. Since the width of the orifice changes slowly with x, it is assumed that the boundary of the jet in any "y-z" plane is identical with that of a two-dimensional flow from a slit, of half width equal to the half width, h, of the orifice at the corresponding value of x. The contour of the boundary of a two dimensional jet is given by [3]

$$g = g_o - \frac{2b}{\pi}(1 + \sin\theta) \tag{11)(i}$$

$$z = \frac{2b}{\pi}\left\{\tanh^{-1}(\cos\theta) - \cos\theta\right\} \tag{11)(ii}$$

where $b = (g_o - g_\infty)$. Note that $-\frac{\pi}{2}<\theta<o$, $\theta = -\frac{\pi}{2}$ when $z = o$, and $\theta \to o$ as $z \to \infty$.

Eq.(10) may now be developed by writing

$$I = \int_o^\infty \frac{\partial}{\partial x}(\psi_w)dz - 2\int_o^\infty w_g \frac{\partial g}{\partial x} dz,$$

and then putting

$$\psi_w = 2W_\infty g_\infty, w_g = W_\infty \cos\theta, \frac{\partial g}{\partial x} = \frac{\partial g_o}{\partial x}\frac{g}{g_o},$$

and changing the variable of integration in the second term by substituting $dz = dg.\tan\theta$, to obtain

$$I = 2W_\infty \frac{\partial g_o}{\partial x}\frac{1}{g_o}\left[g_\infty z - \int_{g_o}^{g}\left\{(\sin\theta)^{-1} - \sin\theta\right\} g\,dg\right]_{z=o}^{z\to\infty}$$

Substituting for $\sin\theta$ and z from eqs.(11)(i) and (11)(ii) respectively, and performing the required integration, yields

$$I = 2W_\infty \frac{\partial g_o}{\partial x}\frac{1}{g_o}\left\{g_\infty \frac{2b}{\pi}(\ln 2 - 1) - \frac{2b}{\pi}(g_o - g_\infty) + \frac{\pi}{2b}\frac{1}{2}(g_\infty^2 - g_o^2) - \frac{\pi}{2b}\frac{1}{3}(g_\infty^3 - g_o^3)\right\} \tag{12}$$

The last term on the right hand side of eq.(10) may be obtained by using eq.(11)(i) to substitute for $\sin\theta$ in eq.(9), leading to

$$u_g = u_{og} + W_\infty \frac{\partial g_o}{\partial x}\frac{1}{g_o}\frac{\pi}{2b}(g^2 - g_o^2),$$

and hence, to

$$2\int_o^\infty u_g \frac{\partial g}{\partial z} dz = 2u_{og}(g_\infty - g_o) + W_\infty \frac{\partial g_o}{\partial x}\frac{1}{g_o}\frac{\pi}{2b}\left\{\frac{1}{3}(g_\infty^3 - g^3) - g_o^2(g_\infty - g_o)\right\}. \tag{13}$$

Substituting from eqs.(12) and (13) into eq.(10), and noting that

$$g_o/g_\infty = 1 + 2\pi^{-1},$$

eq.(10) simplifies to

$$u_D/W_\infty = 0.186\ \frac{\partial g_o}{\partial x} - 0.635\ u_{og}/W_\infty. \qquad (14)$$

By obtaining u_{og} from eq.(3), and noting that $W_\infty = (1 + 2\pi^{-1})\ W_o$, this expression can be evaluated to yield u_D.

4 RESULTS AND DISCUSSION

Calculations have been performed for a family of orifices of sinusoidal shape, and the results are presented in fig.3.

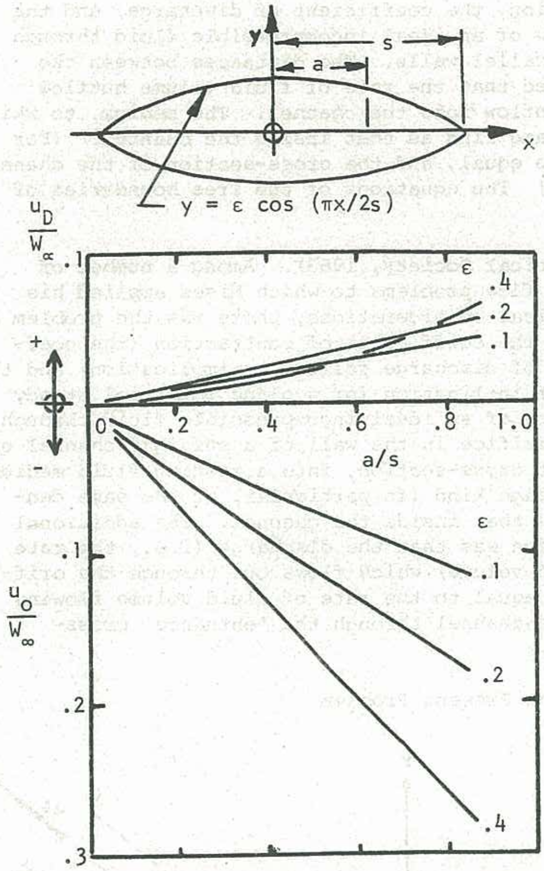

Figure 3 Cross flow velocities in orifice flow

It can be seen that u_D, the mean crossflow velocity imparted to the flow downstream of the orifice, is much smaller in magnitude than u_o, the crossflow velocity induced upstream of the orifice. Since u_o is negative, and the net downstream crossflow is the sum of u_D and u_o, this implies that the flow will retain a negative crossflow component far downstream of the orifice and therefore, the jet will converge in a plane parallel to the major axis of the orifice. Of course, as the convergence develops, the assumption of quasi two dimensional flow in the "y-z" planes will become increasingly invalid, and so the present analysis then will not apply.

It may be noted that curves have been presented for values of ε up to 0.4. For this value, the width of the orifice is not small compared with its span, and the assumptions of the analysis are not valid. However, the curves provide a qualitative indication that relatively large crossflows can be expected under these conditions.

Although the analysis has been developed for a plane orifice it seems plausible to apply it to the curved orifice of fig.1, provided that the nozzle width is small, not only in relation to its length, but also in relation to the radius of curvature of the surfaces forming the interior of the nozzle duct. Under such circumstances, the analysis may be expected to provide a useful first approximation in predicting nozzle spray distributions.

5 REFERENCES

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