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Obtaining Basic Aerodynamics from Simple Trajectory Measurements by Parameter Estimation

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SUMMARY Free flight testing of projectiles generally involves costly and complex experiments. A new, relatively cheap and simple method has been developed for measuring, very accurately, the attitude and position of a vehicle in free flight. The vehicle is launched at night from a compressed air gun and the trajectory data is obtained from ballistic camera records of flashing lights in the nose and tail of the vehicle. The use of parameter estimation techniques in the data analysis has made it possible to extract the basic aerodynamic coefficients of the test vehicle from the trajectory data. Initial trials have yielded most promising results, with aerodynamic forces and static and dynamic stabilities estimated with root mean square errors of only a few per cent. Details are given of the data analysis procedure and an example is included showing the type and quality of the results obtained.

1 INTRODUCTION

Free flight testing of projectile designs is generally necessary to confirm that accurate estimates have been made of effects due to the change in size from wind tunnel model to the full scale vehicle and of effects arising from the dynamic behaviour of the missile. During recent years parameter estimation techniques have been developed to provide superior methods for analysis of data from both free flight trials and dynamic wind tunnel tests (Waterfall, 1970; Chapman and Kirk, 1970). The advantages of these techniques over the older methods (Shinbrot, 1954) are that they require measurements of relatively few variables characterising the motion of the vehicle and that results can be obtained in the presence of much higher noise levels. These two characteristics, together with the high precision of position data (Pope, 1976a), have made it possible to extract aerodynamic parameters from data which was originally intended to provide position and velocity data only. In the initial trials using the flashing light measurement technique (Pope, 1976a), the position of the flashing light was measured with a root mean square error of 0.01 metres. The high degree of accuracy attained with these trajectory measurements led to the much more ambitious project of obtaining both static and dynamic stability data for a model, at small angles of attack.

2 DATA COLLECTION

2.1 Range Layout

The range consists of a flat rectangular grassed area approximately 1200 metres long by 600 metres wide. The main features are indicated in Figure 1. A compressed air gun with a 6 metres long, 384 mm diameter barrel is located in one corner and fires approximately along the diagonal. The principal range instrumentation is three WRE K/F Mark III ballistic cameras sited as indicated in figure 1, together with reference light arrays which supply position fixes for analysing the ballistic

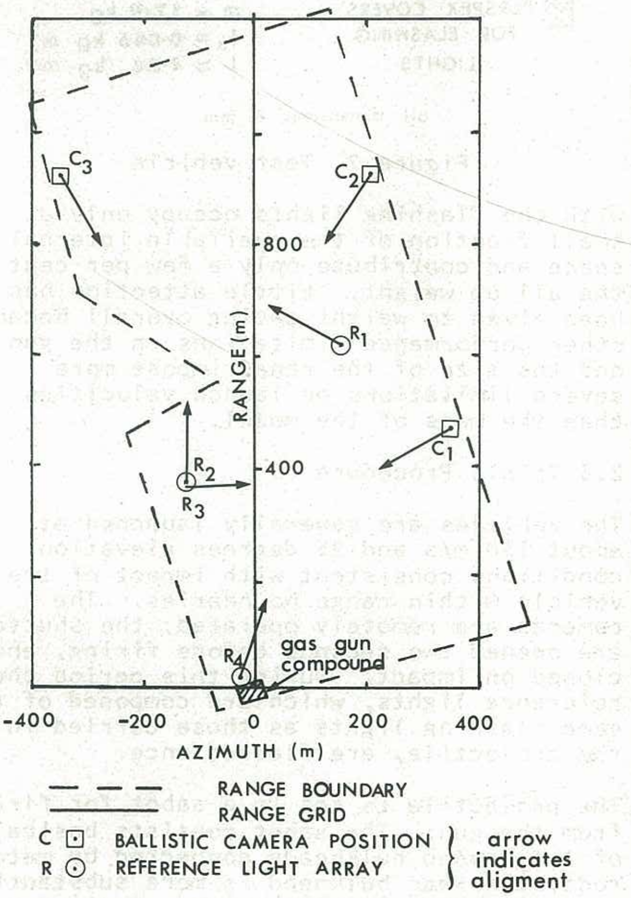

Figure 1 Range layout

camera records. In addition, a telemetry receiving station can be used to record data from sensors, such as magnetometers, carried on the vehicle. The cameras have a 52 degrees circular field of view with a measured accuracy of 0.05 milliradians (r.m.s) and are aligned to include the gas gun near one edge of the photographic plate. At least two accurately positioned reference light arrays are included within the field of view of each camera. With the present camera positioning and alignment only the first four seconds of flight can be covered by three cameras. However, this is quite adequate because the data analysis relies on analysing vehicle oscillations induced by an initial disturbance and all significant pitch oscillations have damped out by by four seconds after launch.

2.2 Test Vehicle

Typically a projectile is about one metre long, simply constructed of wood and metal, with one Xenon stroboscopic flash lamp near the nose and another near the tail, both under perspex covers. Figure 2 presents a sketch of the model used in the trial for which results are presented below. The electronics and power supplies associated

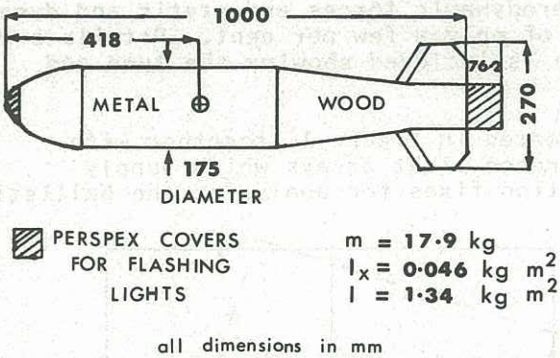

Figure 2 Test vehicle

with the flashing lights occupy only a small fraction of the available internal space and contribute only a few per cent of the all up weight. Little attention has been given to weight saving overall because other performance limitations on the gun and the size of the range impose more severe limitations on launch velocities than the mass of the model.

2.3 Trials Procedure

The vehicles are generally launched at about 130 m/s and 25 degrees elevation, conditions consistent with impact of the vehicle within range boundaries. The cameras are remotely operated; the shutters are opened two seconds before firing, and closed on impact. During this period the reference lights, which are composed of the same flashing lights as those carried in the projectile, are flashed once.

The projectile is set in a sabot for firing from the gun. The sabot consists basically of two wooden bulkheads connected by metal rods; the rear bulkhead is more substantial to support the driving loads. Various degrees of support may be required between the bulkheads, depending on the shape of the projectile and its attitude in the sabot. An initial angle of about 10 degrees is used with the projectile shown in Figure 2. Immediately sabot and model leave the gun muzzle they begin to separate. The sabot falls behind rapidly owing to its much higher drag to weight ratio. As the model emerges, flow is established over the forebody only, resulting in aerodynamic moments which are destabilising. Consequently the initial angle of attack can easily grow to fifteen or twenty degrees before flow becomes established over the fins and the projectile becomes aerodynamically stable. This initial disturbance of the body initiates a pitch oscillation which can be analysed to obtain the required aerodynamic coefficients.

Separation of the projectile from the sabot initiates a timer which controls the flashing of the lights. The start of the flashing is delayed by 0.15 seconds from the initiation to ensure that the projectile has cleared background street lighting. This makes it easier to identify the first flash and thereby synchronise the flashes recorded on each camera plate. The primary requirement for the timer is that its frequency should be very stable throughout the flight of the projectile because it is the only source of timing for data points and any instability in the flash frequency would introduce serious noise into the data. A flash frequency of between 30 and 40 Hz provides an adequate sampling rate without cluttering camera plates with too many images. In order to obtain as clear an image as possible a very short duration flash is used, about 150 micro-seconds.

A Zeiss comparator is used to measure the position of each flash on each camera plate relative to the observed reference lights. The position of each flash is then computed in range axes, using a least squares solution of the triangulation problem posed by the data from all three cameras. The missile nose and tail positions so obtained are then used to estimate both the position of the vehicle centre of gravity and the elevation and azimuth angles which describe the direction of the longitudinal axis or attitude of the vehicle. The nose and tail positions can also be used to estimate the length of the vehicle and, since the length of the missile is a known constant, such derivations can be used as a guide to the reliability of the data. The velocity of the centre of gravity is obtained from the position data, using a simple central difference numerical differentiation procedure (Lanczos, 1957). The three components of the velocity of the centre of gravity of the missile in range axes, $\dot{x}$, $\dot{y}$ and $\dot{z}$, together with the angles of azimuth and elevation, ψ and θ respectively, provide the basic data for analysis using parameter estimation. The range axes system used has OX downrange, OY to the right and OZ vertically downwards, thus forming a right handed system.

3 DATA ANALYSIS

To apply parameter estimation methods to the data, a parametric form of mathematical model is needed to describe the motion of the vehicle. Following Kolk (1964), the general equations of motion in non-rolling body axes with origin at the centre of gravity for a four finned axisymmetric rigid body are,

$$\begin{aligned}
&\dot{u} + qw - rv = X/m - g\sin\theta,\\
&\dot{v} + ru - pw = Y/m,\\
&\dot{w} + pv - qu = Z/m + g\cos\theta,\\
&I(\dot{q} - pr) = M, \qquad\qquad (1)\\
&I(\dot{r} + pq) = N,\\
&\dot{\theta} = q,\\
&\dot{\psi} = r\sec\theta.
\end{aligned}$$

The vector (u, v, w) is velocity in rotating body axes. This can be transformed to velocity in inertial range axes, $(\dot{x}, \dot{y}, \dot{z})$ using the transformation matrix of direction cosines which is a function of the angles ψ and θ. The axes rotation rate is given by (p, q, r) where q, r are pitch and yaw rates of the body and $p = -r\tan\theta$ is a residual roll rate which maintains the plane GX'Z' of body axes vertical where, GX' is the body longitudinal axis. The equation governing axial spin

$$I_x\dot{P} = L$$

where P is body spin rate, is uncoupled from the other equations and has been ignored because no measurements were made of axial spin in the initial trials. However, methods of measuring axial spin are presently being investigated. The aerodynamic forces are (X, Y, Z) and the aerodynamic moments are (L, M, N).

The unknown parameters which are to be found in the analysis fall into two classes, initial conditions and aerodynamic parameters. The initial conditions are

$$p_1 = \dot{x}_0,\ p_2 = \dot{y}_0,\ p_3 = \dot{z}_0,\ p_4 = \psi_0,\ p_5 = \theta_0,\ p_6 = q_0,\ p_7 = r_0. \qquad (2)$$

The other parameters give aerodynamic forces and moments in the form

$$\begin{aligned}
&X = QSp_8,\ Y = QSp_9 w/u,\ Z = QSp_9\ w/u,\\
&M = QSd\left[p_{10}(w/u) + p_{11}(qd/2u)\right] \qquad (3)\\
&N = QSd\left[-p_{10}(v/u) + p_{11}(rd/2u)\right],
\end{aligned}$$

where $Q = \rho V^2/2$ is dynamic pressure, ρ is air density, V is true air velocity, S is cross-sectional area and d is body diameter. The parameters p_8, p_9, p_{10} and p_{11}, essentially represent non-dimensional aerodynamic derivatives; p_8 is axial force coefficient, usually represented as C_x; p_9 is normal force derivative usually represented as $C_{z\alpha}$; p_{10} is pitching moment derivative, usually represented as $C_{m\alpha}$ and p_{11} is pitch damping derivative, usually represented as C_{mq}.

A parameter estimation algorithm is used to find values for the parameters p_i such that the model outputs for $\dot{x}$, $\dot{y}$, $\dot{z}$, ψ and θ match the measured values in some optimum sense. The measure of optimality commonly chosen is to minimise the sum of squares of the difference between each measured value and the corresponding prediction from the mathematical model. An iterative technique which is called a modified Newton-Raphson method or method of differential corrections can be developed (Waterfall, 1970; Chapman and Kirk, 1970) which achieves this minimisation by repeated numerical solution of the equations of motion and adjustment of the parameter values.

The algorithm works in the following way. Initial estimates of the parameter values are used as input to a set of 84 simultaneous, ordinary, first order differential equations comprised of the seven equations of motion (1) together with 77 equations defining partial derivatives $\partial x/\partial p_i$, $\partial y/\partial p_i$ and so on, which can be obtained from equations (1). The solutions of these equations are used to construct a simultaneous, linear system of normal equations. The solutions of the linear equations are used to update parameter values (Pope 1976b) and the whole process repeated. Only a few iterations are generally necessary to obtain convergence if the initial parameter estimates are such that model outputs are a reasonable imitation of the measured data.

4 EXPERIMENTAL RESULTS

The general quality of the data is shown by the r.m.s. noise levels given in Table I. It is clear that nose trajectory data were of much higher quality than tail trajectory data. Estimates of accuracies which could

TABLE I

ESTIMATED R.M.S. NOISE LEVELS IN EXPERIMENTAL DATA

Variable (units)	Nose Trajectory	Tail Trajectory	Centre of Gravity
x(m)	0.013	0.038	-
y(m)	0.010	0.048	-
z(m)	0.018	0.065	-
$\dot{x}$(m s^{-1})	0.47	1.23	0.47
$\dot{y}$(m s^{-1})	0.49	1.11	0.45
$\dot{z}$(m s^{-1})	0.32	0.86	0.40
ψ(rad)	-	-	0.045
θ(rad)	-	-	0.071

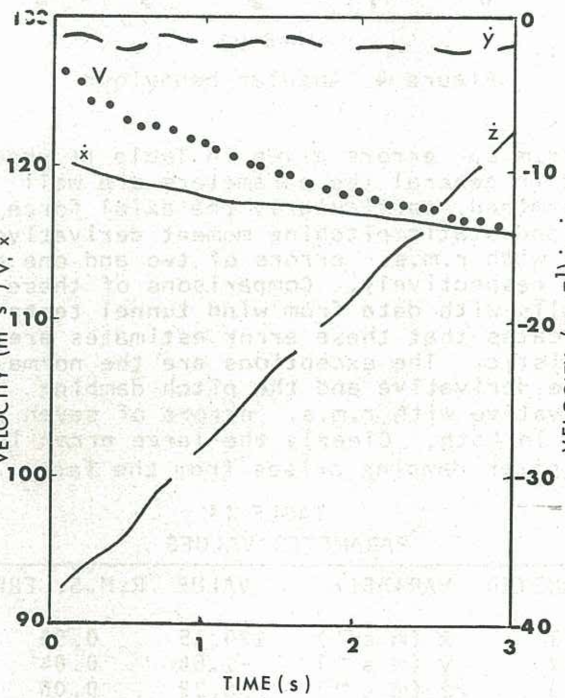

Figure 3 Velocity components

be achieved indicate that the nose trajectory data is near optimum. The problem with the tail trajectory seems to be due to a slightly erratic flash rate. Subsequent trials indicate that noise levels in velocity and attitude can be reduced by a factor of two with little difficulty.

The results of the data analysis are shown in Table II and Figures 3 and 4. Figure 3 gives the velocity history of the vehicle, but only the model outputs are shown because the differences between model outputs and measured values are too small to show effectively on the scale used. Figure 4 gives results for the angular data. It is apparent from this figure that there is some deficiency in the model because there is a definite bias error in fitting experimental data points. The bias error is probably caused by a small non-zero trim angle of attack arising from body misalignments, which has not been allowed for in the mathematical model.

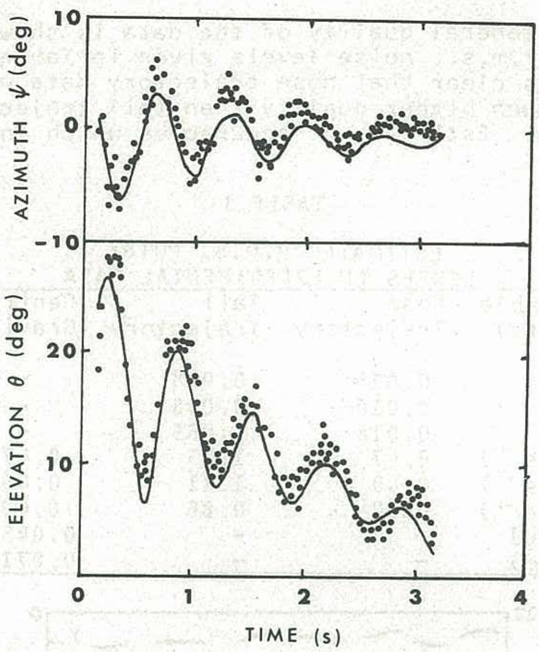

Figure 4 Angular behaviour

The r.m.s. errors given in Table II show that in general the parameters are well determined, particularly the axial force, C_x, and static pitching moment derivative, $C_{m\alpha}$, with r.m.s. errors of two and one per cent respectively. Comparisons of these results with data from wind tunnel tests indicates that these error estimates are realistic. The exceptions are the normal force derivative and the pitch damping derivative with r.m.s. errors of seven per cent in both. Clearly the large error in the pitch damping arises from the fact

TABLE II

PARAMETER VALUES

PARAMETER	VARIABLE	VALUE	R.M.S. ERROR
1	$\dot{x}$ (m s^{-1})	120.15	0.06
2	$\dot{y}$ (m s^{-1})	-1.64	0.04
3	$\dot{z}$ (m s^{-1})	-36.29	0.06
4	ψ (rad)	0.034	0.007
5	θ (rad)	0.373	0.008
6	q (rad s^{-1})	1.73	0.084
7	r (rad s^{-1})	-0.97	0.060
8	C_x	-0.100	0.002
9	$C_{z\alpha}$	-3.54	0.24
10	$C_{m\alpha}$	-3.29	0.03
11	C_{mq}	-58.0	4.2

that it makes a relatively small contribution to the total pitching moment so that the value obtained for the pitch damping derivative is proportionately less accurate than that extracted for the static pitching moment derivative. However, the accuracy of the pitch damping derivative compares favourably with results from other measurement methods. The poorly determined value of the normal force derivative is basically caused by the insensitivity of the motion to normal force. The effects of normal force are small because of the small amplitude of oscillation of the angle of attack, α. In future trials the error in normal force will be reduced by choosing muzzle velocity and flash frequency to minimise these errors. However, the best that can be expected is an r.m.s. error of about three per cent. On the other hand, with the substantial improvements which can be expected in the angular data, the accuracy of both pitching moment derivatives should improve considerably.

5 CONCLUSIONS

A free flight testing technique has been presented, together with results from an initial test. The technique is simple and relatively cheap, and the output from the experiment amply repays the resources used in the trial and data analysis. The results from the initial test show that the method is feasible and can produce accurate estimates of aerodynamic data. Further developments are planned. In particular, work is proceeding on the measurement of vehicle spin and the inclusion of spin in the data analysis.

6 ACKNOWLEDGEMENT

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