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Effect of Circulation on Longitudinal Dispersion in Open Channel

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SUMMARY The paper describes an experimental study of the longitudinal dispersion in an open channel with artificially induced circulation. The experiments were carried out in a laboratory flume 0.4m wide and 15m long. The circulation was induced with the help of a rotating shaft placed along the flow at the centroid of the flow section. The longitudinal dispersion coefficient has been obtained using neutrally buoyant salt solution as the tracer. The variation of the longitudinal dispersion coefficient with parameters quantifying the induced circulation has been reported. Using the experimental data an equation relating the dispersion coefficient with parameters which can be easily evaluated for any channel has been proposed.

1 INTRODUCTION

From the point of view of water quality management and pollution control it is very important to investigate the dispersion characteristics of a stream into which increasing quantities of industrial, municipal and agricultural wastes are disposed of. The effect of various parameters affecting the dispersion process thus needs a careful study.

A number of studies covering the various aspects of dispersion in open channels are available. The effect of different types of roughness on longitudinal dispersion coefficient have been investigated by Fischer (1966), McQuivy (1970), Miller and Richardson (1974), Pande and Pathak (1975), El Hadi and Davar (1976). The characteristics of transverse mixing in open channel flows have been studied by Okoye (1970), Holley et al. (1972) and Holley and Abraham (1974).

Another important aspect has been investigated by Fukoka and Sayre (1973) who studied the effect of sinuousity of the channel on the longitudinal dispersion coefficient. Sooky (1969) investigated theoretically the effect of triangular and circular arc cross-sectional shapes on the dispersion coefficient. The effect of cross-sectional shapes, namely, rectangular, triangular and trapezoidal, was experimentally investigated by the authors (1977). In these studies the band or the cross-sectional shape introduced a secondary circulation in the flow which modifies the dispersion process. Obviously, in these studies it was not possible to control or quantify this circulation precisely so as to evaluate its effect on the dispersion coefficient.

In the present investigation, in order to study the effect of secondary circulation on the longitudinal dispersion coefficient, the circulation was artificially induced with the help of a rotating shaft placed along the flow at the centroid of the flow section. The value of the induced circulation could thus be controlled and quantified. The experimental data so collected has been analysed and the effect of parameters quantifying the induced circulation on the longitudinal dispersion coefficient has been reported.

2 BASIC EQUATIONS

The diffusion of a dissolved substance in a turbulent flow is governed by the following convective diffusion equation

$$\frac{\partial c}{\partial t} + u_i \frac{\partial c}{\partial x_i} = D_m \frac{\partial^2 c}{\partial x_i \partial x_i} + \frac{\partial}{\partial x_i}\left(e_{x_i} \frac{\partial c}{\partial x_i}\right)$$

$$i = 1,\ 2,\ 3 \tag{1}$$

where c is the concentration of the dissolved substance, t the time, u_i are the velocity components, D_m the molecular diffusion coefficient and e_{x_i} are the local turbulent diffusion coefficients.

According to Taylor (1954), since the primary mechanism for dispersion in shear flow is the variation of convective velocity within the cross-section, the entire process could be described by the following one-dimensional dispersion equation

$$\frac{\partial \bar{C}}{\partial t} + \bar{U} \frac{\partial \bar{C}}{\partial x} = D_L \frac{\partial^2 \bar{C}}{\partial x^2} \tag{2}$$

where D_L is the bulk cross-sectional coefficient generally known as the longitudinal dispersion coefficient, $\bar{C}$ and $\bar{U}$ are the average cross-sectional concentration and velocity respectively. Equation (2) is applicable only after the initial mixing period, that is, when diffusion is complete across the flow cross-section. The longitudinal dispersion coefficient D_L can be calculated using the method of moments according to which

$$D_L = (\bar{U}^3/2)\ d\sigma_t^2/dx \tag{3}$$

where σ_t^2 is the variance given by

$$\sigma_t^2 = \int_0^\infty t^2 \bar{C}\ dt \Big/ \int_0^\infty \bar{C}\ dt - \bar{t}^2 \tag{4}$$

in which the mean travel time $\bar{t}$ is given by

$$\bar{t} = \int_0^\infty t \bar{C}\ dt \Big/ \int_0^\infty \bar{C}\ dt \tag{5}$$

The mean velocity $\bar{U}$ can be calculated by

$$\bar{U} = (d\bar{t} / dx)^{-1} \quad (6)$$

3 EXPERIMENTAL PROGRAMME

The experiments were carried out in a 0.4m wide and 15m long tilting bed flume with glass walls and steel bottom. The bed of the flume was roughened with 25mm glass balls packed at an areal density 0.785. A schematic diagram of the flume along with various other details are shown in Figure 1. The water to the flume was supplied from a constant level overhead tank through a pipeline fitted with a sluice valve for regulation purpose. The return drain from the flume to the sump was provided with a 0.9m wide calibrated sharp crested weir for discharge measurement.

In order to introduce artificial circulation an aluminium shaft of 19mm diameter was mounted horizontally along the flow at the centroid of the flow section. This shaft was rotated with the help of a variable shunt/series type D.C. motor and a pulley arrangement placed at the upstream entrance section of the flume. The shaft could be rotated at speeds up to 350 rpm.

For experimental determination of longitudinal dispersion coefficient the tracer used was sodium chloride made neutrally buoyant with denatured spirit (methanol). The tracer was fed instantaneously into the flow as a line source along the width of the flume at the upstream entrance section by rapidly rotating the semicircular tilting trough containing the tracer. The concentration versus time curves were recorded at three stations, located 3m, 4m and 5m downstream of the injection point, using a set of four conductivity probes in parallel at a station, associated electronic circuit and an 'Encardiorite' strip chart recorder. From these $\bar{C}$ versus t records, the longitudinal dispersion coefficient D_L was calculated by the method of moments.

A total of twenty four runs were taken. Velocity profiles along four verticals across the channel width were also taken at station 2, and the kinetic energy correction factor evaluated from these measurements for each run.

4 ANALYSIS AND DISCUSSION OF RESULTS

The variables affecting the dispersion coefficient D_L in an open channel evidently consist of mean velocity $\bar{U}$, circulation Γ, hydraulic radius R, mean boundary shear stress τ_0, areal concentration of roughness elements λ, dynamic viscosity μ, mass density ρ and the acceleration due to gravity g. It can be shown by dimensional analysis that

$$D_L / R\bar{U} = f(\Gamma/R\bar{U},\ \bar{U}/u_*,\ \lambda,\ Re,\ Fr.) \quad (7)$$

In the present investigation areal concentration is kept constant and the values of Re (from 2.1×10^4 to 6.8×10^4) and Fr (from 0.28 to 0.35) are also within a narrow range. It has been shown by Sooky (1969) and El-Hadi (1976) that the effect of Reynolds number on the longitudinal dispersion coefficient is insignificant. Therefore, excluding Re, Fr and λ from further consideration, the Eq.(7) reduces to

$$D_L/R\bar{U} = f(\Gamma/R\bar{U},\ \bar{U}/u*) \quad (8)$$

The dispersion coefficient $D_L/R\bar{U}$ was plotted against the circulation parameter $\Gamma/R\bar{U}$ with $U/u*$ $(=C/\sqrt{g})$, as the third parameter in Figure 2. It can be observed from this figure that, for a given $U/u*$, as the value of circulation parameter $\Gamma/R\bar{U}$ increases the value of dimensionless dispersion coefficient first decreases and then it starts increasing. The physical implication of this observation is that as circulation increases the non-uniformity already present in the flow is first smoothed out causing a reduction in $D_L/R\bar{U}$ and then as the circulation increases further, non-uniformity in the form of helical flow fully penetrates into the entire cross-section thereby again increasing the value of $DL/R\bar{U}$.

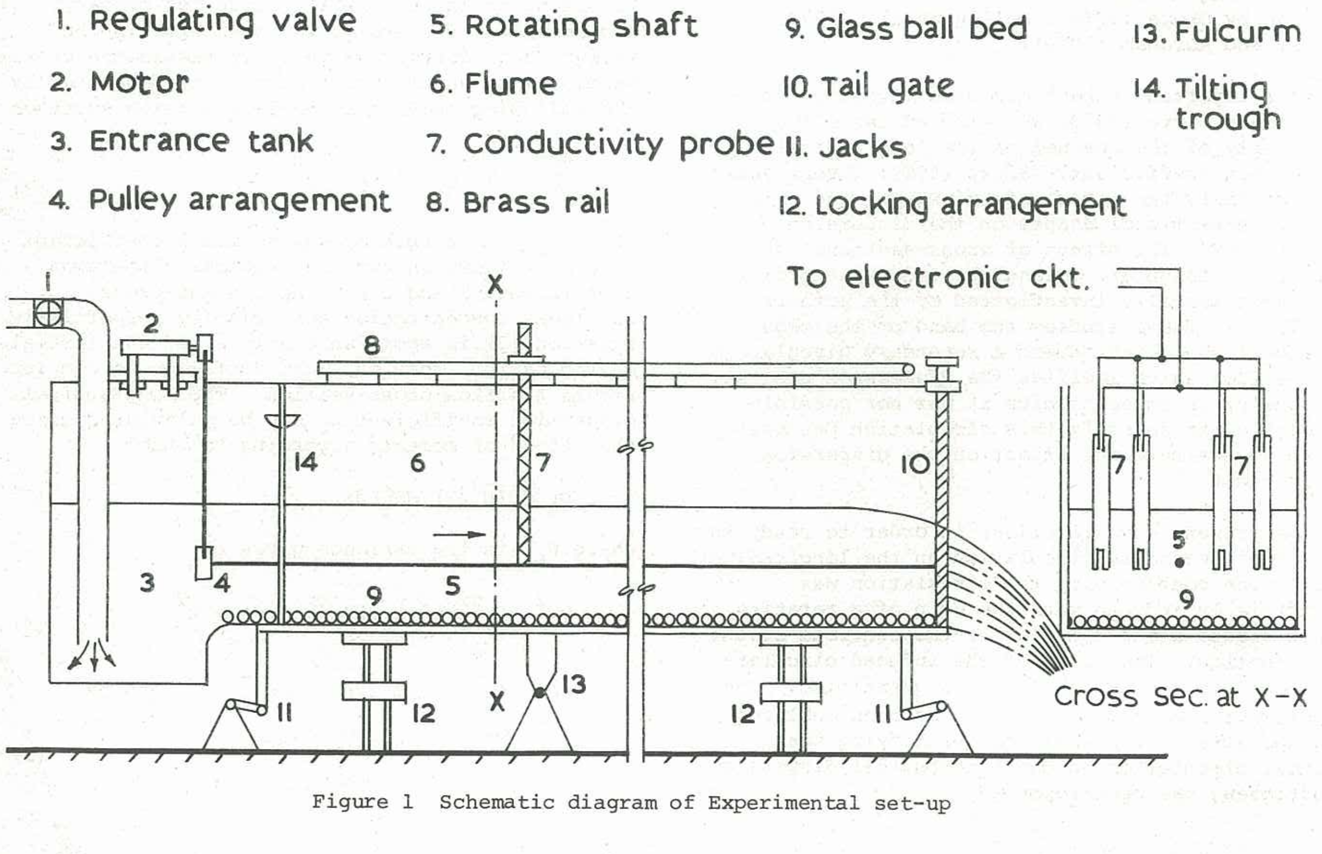

Figure 1 Schematic diagram of Experimental set-up

Further it may be noted from the same figure that, for the same value of circulation parameter Γ/RU, as the value of $\bar{U}/u_*$ or $C/\sqrt{g}$ increases, the value of $D_L/R\bar{U}$ decreases and that this very trend persists for the entire range of $\Gamma/R\bar{U}$. This quite understandable as increase in $\bar{U}/u_*$ implies a smoother channel for which the dispersion coefficient is expected to decrease.

In order to represent the nonuniformity of velocity distribution at a cross-section, caused by the secondary circulation, the energy correction factor was calculated from the velocity measurements at each value of the shaft speed. Figure 3 shows the plot of α versus $\Gamma/R\bar{U}$ with $\bar{U}/u_*$ as the third parameter. For a constant U/u_*, the value of α first decreases with an increase in $\Gamma/R\bar{U}$ and then increases with subsequent increase in $\Gamma/R\bar{U}$. This plot thus confirms the physical argument given earlier regarding a decrease at first and subsequent increase of $D_L/R\bar{U}$ in response to a continuous increase in circulation parameter $\Gamma/R\bar{U}$. Computations have shown that standard deviation σ as a measure of non-uniformity displays the same behavioural trend as α with increasing value of $\Gamma/R\bar{U}$.

Figure 4 also shows that for the same value of circulation parameter the value of α decreases with increasing $\bar{U}/u_*$.

With a view to evolve a practically useful relationship, $D_L/R\bar{U}$ was sought to be related to parameters such as $\bar{U}/u_*$ and α which can be easily computed for any open channel flow. The plot of $D_L/R\bar{U}$ versus $\alpha/(\bar{U}/u_*)^{0.3}$, shown in Figure 4 exhibits a promising relationship which can be expressed by the following equation

$$D_L/R\bar{U} = 11.87 \quad \alpha/(\bar{U}/u_*)^{0.3} - 4.97 \qquad (9)$$

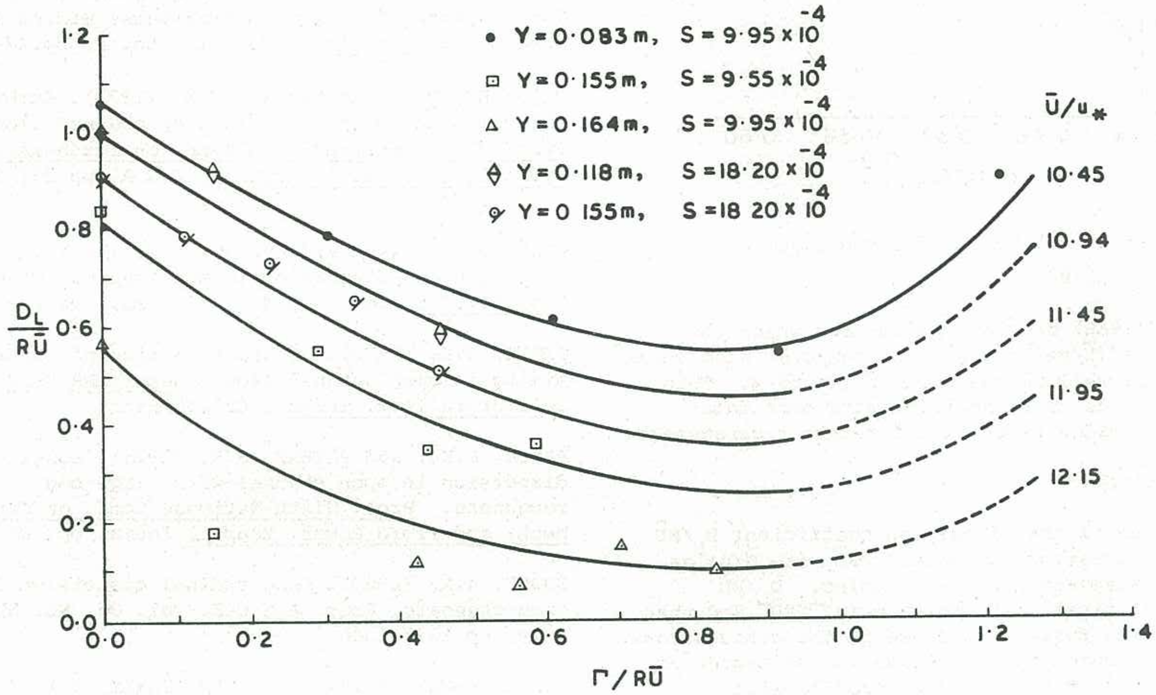

Figure 2 Variation of $D_L/R\bar{U}$ with $\Gamma/R\bar{U}$

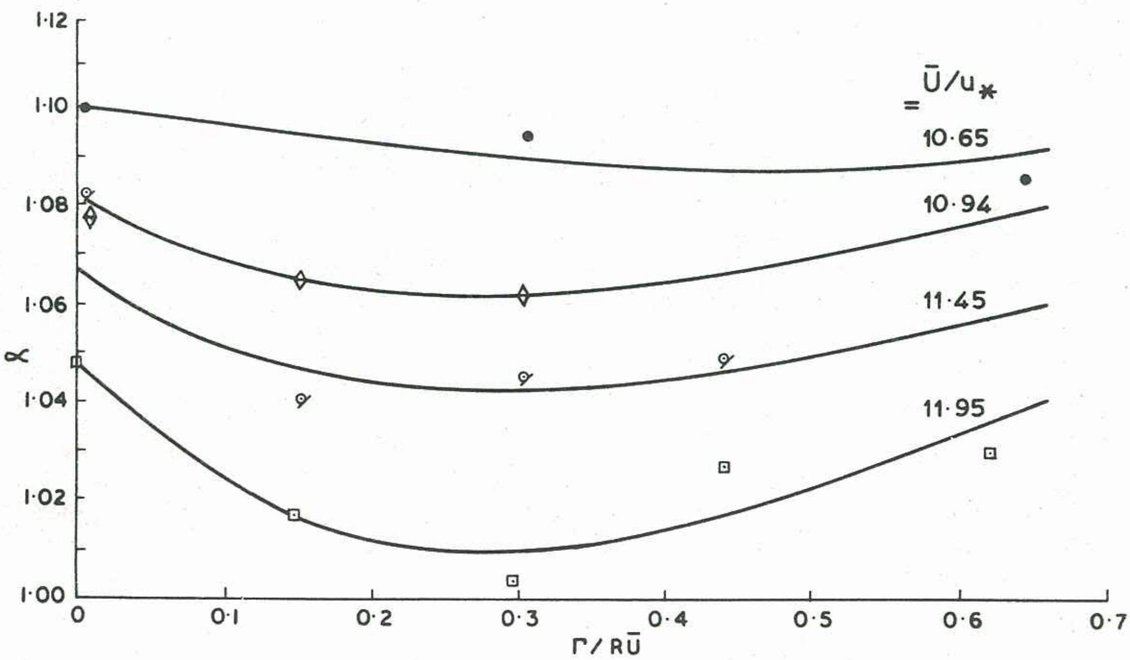

Figure 3 Variation of α with /RŪ

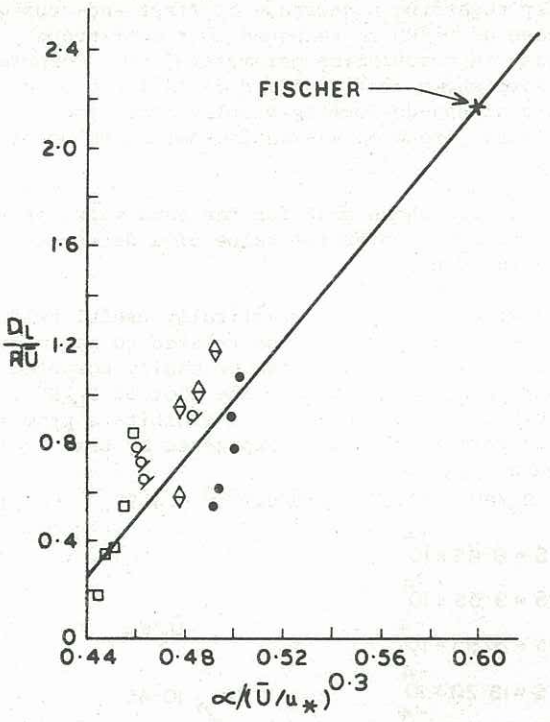

Figure 4 Variation of $D_L/R\bar{U}$ with $\alpha/(\bar{U}/u_*)^{0.3}$

Fisher's (1966) one data points for which the parameter $\alpha/(\bar{U}/u_*)^{0.3}$ could be computed also seems to fit very well in the plot of Figure 4. This, however, needs to be verified with more data covering a wider range of the relevant parameters.

5 CONCLUSIONS

The variation of the dispersion coefficient $D_L/R\bar{U}$ with the circulation parameter $\Gamma/R\bar{U}$ with $\bar{U}/u_*$ as the third parameter has been studied. $D_L/R\bar{U}$ decreases at first with increase in $\Gamma/R\bar{U}$ and then increases with further increase in the circulation. This is attributed to the change in the degree of non-uniformity of the velocity profile with increasing circulation. The parameters $D_L/R\bar{U}$ and $\alpha/(\bar{U}/u_*)^{0.3}$ show a promising relationship of practical utility.

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