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Shock Waves Generated by Spark Discharge

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SUMMARY Shock waves have been generated in air by laboratory spark discharges. The relationship between this process and the generation of thunder by lightning has been critically assessed. Pressure profiles were recorded for shocks generated by sparks of length L = 0.185 m. Results were obtained for spark discharges of energy from 1250 J to 6000 J and at radii within the range, $6.65 \leq R/L \leq 21.78$. Peak shock over pressure results showed good repeatability and closely followed the $R^{-1}(\ln R)^{-\frac{1}{2}}$ dependence predicted by weak spherical shock theory. The acoustic efficiency of the spark discharge was found to be from 40 to 18% and varied inversely with the square of input energy.

1 INTRODUCTION

Thunder is often used as a comparator for the subjective assessment of explosions, gun-fire and sonic boom and has some damage potential itself. An understanding of thunder generation and propagation is clearly necessary for quantitative treatment of these aspects. The work described here is concerned with the efficiency of thunder generation by lightning. In this investigation lightning was simulated by laboratory spark discharges. To establish the validity of this simulation, a brief summary is given below of cloud-to-ground lightning which is the cause of the highest intensity thunder experienced at the ground. This is followed by a brief description of thunder features and a review of theoretical models for thunder propagation. Finally a description is given of the experiment and the results discussed.

2 LIGHTNING FEATURES

Cloud-to-ground lightning consists of an upward propagating stroke from the ground to the negative region of the cloud (Berger, 1967). This discharge has been described in detail by Uman (1969). It commences with a weakly luminous pre-discharge "stepped leader" which propagates downwards and is eventually met by "streamers" which are relatively small discharges originating at the ground. In this way a weakly conducting path is formed along which the first major discharge (stroke) occurs. This "return stroke" effects a large transfer of negative charge in the lower portion of the cloud and the discharge ceases. The pre-discharge leader-streamer processes result in a relatively long time-to-peak for the current. Berger (1967) has reported current measurements which show peak current values of about 4×10^4 A reached in about 10 μs with current duration of about 50 μs.

Usually there are several more strokes in a lightning flash as charge redistribution takes place in the cloud, but interest here is confined to the first return stroke since this stroke dissipates the most energy, and in some cases is the only stroke that comprises the lightning flash. Since it also propagates through an atmosphere with an immediate history of only weak electrical disturbance, the first return stroke would be the stroke that is most closely approximated by laboratory spark discharges.

During lightning there is intense heating of the lightning channel with mean channel temperatures of 24000-28000 K being achieved (Prueitte, 1963). This intense heating results in the formation of an outwardly propagating cylindrical shock wave. Because of the tortuosity of the lightning channel, with straight portions ranging from < 5 m to > 100 m, at distances large compared with the straight length of channel, the shock wave effectively propagates as a spherical shock of strength dependent on the length of straight channel source. For this reason the current theory of thunder propagation is a combination of cylindrical and spherical shock wave theory (Few, 1969). This theory requires as one initial condition a value for the energy input to the shock formation process. However, there is great uncertainty about the total energy dissipated in a lightning stroke and also the proportion of this energy that appears as acoustic energy.

Attempts at evaluating the energy dissipated during lightning have been made using electrical and spectroscopic methods. Using average values for cloud charge and breakdown voltage, Uman (1969) has estimated energy dissipation to be of the order of 10^5 J m^{-1} per stroke, but because of the uncertainty of both the charge and the voltage, the energy could vary from 10^{-4} to 10^{-8} J m^{-1}. However, the figure of 10^5 J m^{-1} is comparable with that of 2.3×10^5 J m^{-1} deduced by Krider et.al. (1968) from comparisons of the total radiant energy emitted by a single lightning stroke with that from a 4 metre air spark of known electrical energy input. However, the detector used in Krider's work had a spectral bandwidth of only 0.4 - 1.1 μm and there is significant radiation outside this band (Raether, 1964). This fact was implicit in Krider's results since only 0.8 per cent of the electrical input appeared as radiation in this bandwidth. Krider's lightning energy figure quoted above was based on the assumption that this "radiative efficiency" was the same for both laboratory spark and lightning stroke. The seriousness of this assumption can be seen from fig.1, where the radiation emissive power is plotted for temperatures of 24000 K and 30000 K which represent the likely limits to temperature within the channel during discharge, as discussed above. Although actual radiation from the channel would depend on the opacity and emissivity of the gas, it seems probable from fig.1 that there are large amounts of radiation outside the 0.4 - 1.1 μm band. Furthermore, because of the likely range of average channel temperature, it seems highly unlikely that the "radiative efficiency" in the wavelength band of 0.4 - 1.1 μm is the same for lightning stroke and spark discharge.

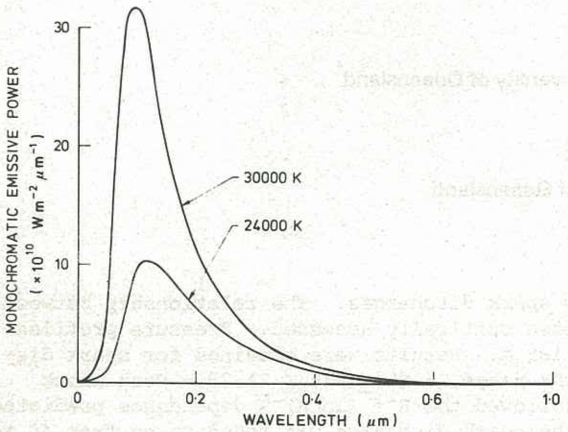

Figure 1 Blackbody emissive power as a function of temperature

Using Krider's low radiative efficiency in the 0.4 - 1.1 μm band together with estimates of energy stored in the channel in molecular excitation, Uman (1969) reached the conclusion that the acoustic efficiency of lightning must be near 100%. But in the light of the above argument the acoustic efficiency is probably quite low. This conclusion is consistent with the relatively low values inferred from the experimental results given below.

4 SHOCK WAVE THEORY

As discussed above, the current theory describing thunder propagation uses a combination of cylindrical and spherical shock-wave theory (Few, 1969). To provide grounds for comparison with the experimental results, the principal results of the theoretical treatment will be briefly summarized.

4.1 Strong Shock Regime

The analysis for this regime was initially developed by Taylor (1950) for the spherical case and later extended to the cylindrical system by Lin (1954). Assuming a perfect gas, strong shock conditions, instantaneous energy release and isentropic flow behind the shock, it was found that the peak shock over-pressure could be written,

$$\frac{\Delta P}{P_o} = \frac{2\gamma}{\gamma+1}\left[\left(\frac{2}{\alpha+2}\right)^2 x^{-\alpha}\right] \quad (1)$$

where $\alpha = 1$ for the cylindrical case, $\alpha = 2$ for the spherical case, γ is the ratio of specific heats, ΔP, the peak over-pressure, P_o, the atmospheric pressure, x, the non-dimensional radius R/R_o and R_o the characteristic radius given by

$$R_o = \left[\left(\frac{\alpha+2}{2}\right)^2 \frac{1}{B\gamma}\,\frac{E_o}{P_o}\right]^{\frac{1}{2}} \quad (2)$$

where B is an energy integral factor, and E_o the energy per unit length ($\alpha = 1$) or the total energy ($\alpha = 2$).

4.2 Intermediate and weak shock regime

The behaviour of weak cylindrical and spherical shock waves has been investigated by Bethe (1944) and Whitham (1952) using small perturbation theory. They showed that for this case,

$$\frac{\Delta P}{P_o} \propto \begin{cases} R^{-1}\,(\ln R)^{-\frac{1}{2}} & \text{spherical case} \\ R^{-3/4} & \text{cylindrical case} \end{cases} \quad (3)$$

Hence simple analytical relationships are available for both strong and very weak shocks.

More recently, numerical solutions have been obtained for the complete flow field. For convenience, analytical approximations have been obtained to these solutions by Jones (1968) and Plooster (1968) for the spherical and cylindrical modes respectively. Based on best fit and correct limit methods, the peak over pressure was found to be given by

$$\frac{\Delta P}{R_o} = \frac{2\gamma}{\gamma+1}\left(\frac{2}{\alpha+2}\right)^2 \left\{ C_{c,s}\left[\left(1 + \frac{D_{c,s}}{\beta}\,x^{\alpha}\right)^{3/8} - 1\right]\right\}^{-1} \quad (4)$$

where $C_{c,s}$ and $D_{c,s}$ and β are constants. β is a correction term for real gas effects and is unity for perfect gas flow. As far as this work is concerned the important result is that the non-dimensional radius, x, is the only independent variable in all expressions for the peak shock over-pressure.

5 EXPERIMENT

As discussed above in Section 2, laboratory spark discharges most closely approximate the first return stroke of a lightning discharge. Pressure signatures have previously been obtained from 4 m spark discharges by Uman et.al. (1970) but these results showed poor reproducibility and were subject to shock reflection and diffraction effects. The experiment described here was designed to obtain pressure signatures devoid of these difficulties.

5.1 Generation of the laboratory spark

The experimental sparks were produced using a sphere-to-sphere gap (fig.2). The two identical spheres were positioned vertically, were of 0.150 m diameter and were 0.185 m apart. The discharge energy was supplied by a 900 kV voltage impulse generator. This consisted of a bank of 6 × 0.12 μF capacitors which were charged in parallel to a preset voltage level that could be measured electrostatically to within 3%. The capacitor set was then rearranged in series to discharge across the gap on the application of a slight over-voltage. The circuit was determined so that a 1 × 50 μs positive voltage impulse (1 μs to peak and 50 μs to half value point) could be applied to the gap. The current resulting from this impulse was not measured but in the similar experiment by Uman et.al. (1970) it was found that voltage impulses of this kind led to discharge currents which peaked within about 4 μs and had a duration of about 10 μs. Both these values are significantly shorter than the corresponding figures for lightning discharge which are about 10 μs and 50 μs respectively. But all of these time scales are very short compared with shock propagation times for distances large compared with the channel diameter. Hence it was assumed in this work, as elsewhere, that energy was added instantaneously.

The electrical energy input to the discharge was estimated from the energy stored in the capacitors prior to discharge allowing for a 15% resistive loss during discharge. This loss figure is based on previous experience with the impulse generator. The energy input was varied by adjusting the amplitude of the voltage impulse applied across the sphere gap, but the pulse shape (1 × 50 μs) was unchanged.

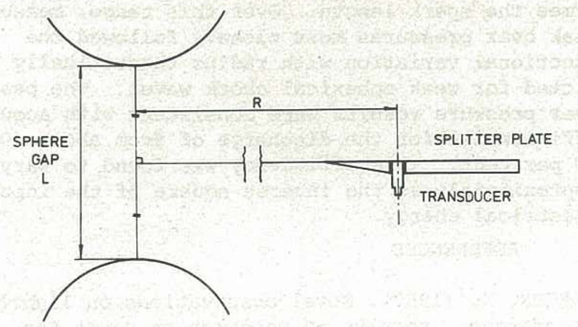

Figure 2 Experimental arrangement

5.2 Pressure Recording

Pressure profiles of the shock waves generated by the spark were measured with a piezo-electric pressure transducer (Kistler model 701A) mounted on a splitter plate aligned normal to the direction of shock propagation (fig.2). Mounted in this way the transducer recorded the undisturbed static pressure profile. The dimensions of the plate were chosen to ensure that any diffraction effects from the edges would not interfere with the spark induced shock. The pressure profiles were recorded photographically from oscilloscope traces. They were recorded at up to 4 different radii from the spark channel for 4 different values of energy input. To establish repeatability, each test was run 10 times. Mean values and standard derivation for the peak over pressure are shown in Table 1. The reproducibility of the results can be seen to be good. The standard deviations are comparable to the resolution for the transducer used (40 Pa).

TABLE I

PEAK OVER PRESSURE RESULTS ($\times 10^5$ Pa)

Energy	Distance			
(J)	1.23m	1.79m	2.91m	4.03m
1250	.02267 ±.00061	.01360 ±.00065	.00696 ±.00027	.00484 ±.00021
2500	.02581 ±.00080	.01523 ±.00095	.00796 ±.00013	.00557 ±.00023
4400	.02871 ±.00076	.01683 ±.00041	.00906 ±.00013	.00633 ±.00045
6000	.02900 ±.00089	.01785 ±.00022	.00955 ±.00011	

5.3 Discussion of Results

From section 4, shock wave theory indicated a functional relationship between over-pressure and non-dimensional radius of the form,

$$\frac{\Delta P}{P_o} = f\ (x), \qquad (5)$$

the characteristic radius in x being defined by eq. (2) with $\alpha = 1$ for the cylindrical case and $\alpha = 2$ for the spherical case. Using the total energy supplied to the channel to calculate the appropriate characteristic radius, the over pressure results were plotted in the form of eq.(5) using both the cylindrical and spherical formulations for R_o and hence x. The result is shown in fig.3 and it can be seen that in neither case did the experimental results collapse on to one curve as indicated from theory, but agreement was best for the spherical formulation.

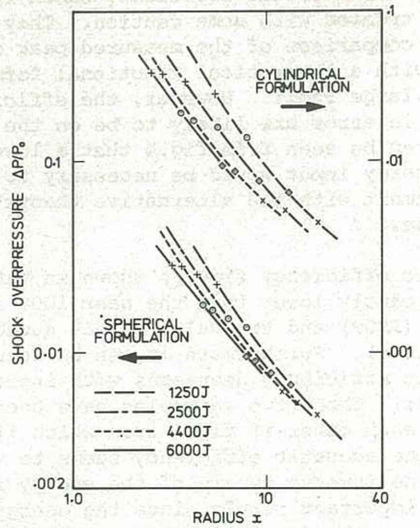

Figure 3 Over pressure as a function of non-dimensional radius

The experimental results for peak over-pressure were compared with theoretical predictions of over-pressure variation with radius. Sample results are shown in fig.4 for the case of 2500 J energy dissipation. In a "first cut" attempt to allow for the acoustic efficiency of shock formation, the theoretical curves shown in fig.4 are based on a total energy value of 1000 J. It can be seen that the functional variation of the over pressure with radius most closely followed the form given by eq. (3) for spherical waves. The value of energy dissipation that led to best fit between eq.(3) and the experimental results was found to be 700 J, which implied an acoustic efficiency of 28 per cent for this case. Similar procedures were followed for the results corresponding to the other 3 energy inputs and the results are summarized in Table II.

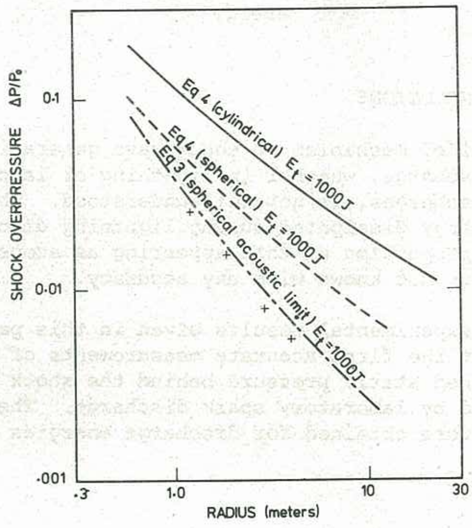

Figure 4 Theoretical and measured over pressure variations

TABLE II

ACOUSTIC EFFICIENCY

Energy Input (J)	1250	2500	4400	6000
Energy for best fit with eq. (J)	500	700	1000	1100
Implied acoustic efficiency (%)	40	28	23	18

The figures for acoustic efficiency shown in Table II must be treated with some caution. They are based on a comparison of the measured peak over pressures with a theoretical functional form appropriate for large radii. However, the efficiency figures if in error are likely to be on the high side - it can be seen from fig.4 that a lower acoustic energy input would be necessary to bring close agreement with the alternative theoretical formulations.

The acoustic efficiency figures shown in Table II are significantly lower than the near 100% suggested by Uman (1969) and the value of 66% suggested by Plooster (1968). Furthermore it can be seen that the acoustic efficiency decreases with increasing energy input. These two variables have been plotted against each other if fig.5 from which it can be seen that the acoustic efficiency seems to vary almost as the inverse square of the energy input. This is an important result since the energy dissipation in the first lightning return stroke is likely to be significantly higher than the values achieved in this experiment (see section 3).

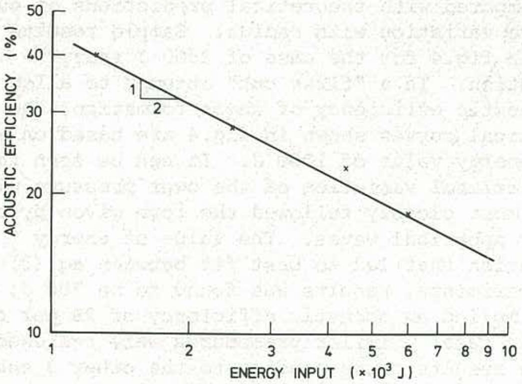

Figure 5 Acoustic efficiency as a function of input energy

6 CONCLUSIONS

The detailed mechanism of shock wave generation by spark discharge, whether in lightning or laboratory spark discharges, is not well understood. The total energy dissipated during lightning discharge and the proportion of that appearing as acoustic energy are not known with any accuracy.

The new experimental results given in this paper represent the first accurate measurements of the undisturbed static pressure behind the shock wave generated by laboratory spark discharge. These results were obtained for discharge energies from 1250 J to 6000 J and at radii from 6.65 to 21.78 times the spark length. Over this range, measured peak over pressures most closely followed the functional variation with radius theoretically predicted for weak spherical shock waves. The peak over pressure results were consistent with acoustic efficiencies for the discharge of from about 40 to 18 per cent. This efficiency was found to vary approximately as the inverse square of the input electrical energy.

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