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Sand Sausages for Beach Defence Work

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SUMMARY An extension of the sand-bag type of armour unit for breakwaters, which is coming into prominence at present, is that of a sand sausage of large dimensions. Materials are becoming available which can withstand loads of saturated sand when the tube is submerged in the sea. This concept has been applied for small sized tubes but when extended to bigger structures the shape is non-circular. A theoretical analysis is presented for the profile and stresses of the skin. Experimental verification of the cross-section proved successful.

1 INTRODUCTION

The normal rubble-mound construction for groynes and breakwaters is extremely expensive and will become more so as raw materials become scarcer and more distant from the coast. Costs of transport are rising as they are for access roads and other infrastructure. The structures themselves must be designed large enough to carry heavy traffic and are therefore overdesigned for marine purposes. These economic trends point to the need for utilising materials that are readily available at the coast, namely beach sand. It can be pumped into position with the help of seawater or can be transmitted through pipelines as a cement mortar or concrete. In all cases the sedimentary mixture requires to be held in place by formwork.

It is the nature of this formwork which is the subject of this paper, namely flexible skin structures. They have been promoted recently (1) (2) by the concept of multiple small bags filled with either sand, mortar or concrete for breakwater construction. Materials available on the market are strong in all directions and can be either sewn, glued or welded to provide containers of varying shape. They can also withstand ultra-violet rays from the sun, are inert to most chemical compounds and can withstand wear of sand-laden water jets as occasioned in breaking waves.

Such armour units can be fabricated to any size depending upon the strength of the fabric and its jointing capabilities. The larger the unit the less likely are the highest waves to dislodge it. The concept analysed herein is for very large containers which could be termed "sausages" since their length is very much larger than their cross-sectional dimensions. Materials such as polyethylene are now available in widths of 10 metres, thickness of nearly 3 mm and lengths of 100 metres or more. It provides a long-term hoop strength of 3600 kgs per metre length and hence can be welded into reasonably large sausage-like structures. When filled with saturated sand or mortar the weight is sufficient to withstand the forces of the highest waves possible in the depths where waves can be broken over the structure as it serves as a breakwater. If scouring should occur at the seawards edge or at the extremities, the structure will sink into the hole without undue strain on the flexible membrane. Some of these phenomena require further research. Multiple sausages laid parallel or across each other will compress and form a firm contact one to the other.

Sand tubes have already been employed in coastal defense (3). These have been in the order of 1 metre diameter and fabricated from polyethylene or rubber tubing. They have been placed parallel to the beach line and also perpendicular to it. Either application the authors disagree with, since the former is equivalent to a seawall and the latter to a groyne, both concepts of coastal defense which are seldom successful. However the sand sausage is readily applied to offshore breakwaters, to create a tombolo and finally become a headland. The intermediate bay (4) (5) is sculptured by the waves to some maximum indentation, dependent upon the approach direction of the persistant swell. The equilibrium shape can be predicted from the tools developed in model studies and observations of prototype bays (6).

The design condition is for the flexible tube to be filled with saturated sand whilst submerged in water of equal depth to the pressure head applied. This loading condition is equivalent to the tube being filled with water when in the atmosphere, that is, a specific gravity of contents equal to 2. In practice dry sand has a specific weight twice that of water and hence the said loading stress implies dewatering of the sausage when filling is complete. There are certain conditions to be met in the pumping process and final sealing but these will not be discussed here since the main purpose of the paper is to present a theoretical analysis of the stable shape and an experimental verification of it.

2 THEORETICAL SOLUTION

Two loading conditions should be investigated, that of fully and of partially filled sausage. It will be realized readily that when pumping such a tube full of sand by dredging methods it will be first filled with water, since a maximum concentration of 30% sediment is likely. Water must pass into and out of the container whilst the sand is accreting. The maximum pressure that can be maintained inside the balloon-like structure is very modest as can be computed by the hoop tension formula for pipes. It is only a matter of one or two metres of water head. The fully loaded condition is treated in Reference (7)

The likely cross-section is depicted in Fig. 1, where the variables are shown. The final applied pressure can be denoted by an equivalent water head b_1. The co-ordinate system is x horizontally from the centre-line and y vertically downwards from the maximum head level which is b_1 from the base of the water-filled sausage in air. This implies a submergence to this level of the sand filled structure in the sea.

A segment of the total circumference s is considered as ds, it being part of the lower section of the membrane ADB. At this level the pressure p acting over ds whose width is unity, applies a force pds as shown in Fig. 2. The elemental length has a radius of curvature R which is dependent upon the loading at that zone. The hoop tension T will be shown to remain constant around the whole sausage circumference where the membrane is free. This does not apply over the contact width L′ of the structure with the floor, where friction will produce zero hoop tension some small distance within the extremity. (see Figure 1.)

In the analysis it will be assumed that the moment and shearing forces at either end of the segment ds are zero and hence the only forces are those of tension in line with the membrane. The equations for static equilibrium are thus

$$T_x - (T_x + \frac{dT_x}{ds} ds) - p_x ds = 0 \qquad (1)$$

in the horizontal direction, and

$$T_y - (T_y + \frac{dT_y}{ds} ds) - p_y ds = 0 \qquad (2)$$

in the vertical direction.

These equations when simplified and reduced accordingly to Fig. 2 yield

$$\frac{d}{ds}(T \frac{dx}{ds}) + p \frac{dy}{ds} = 0 \qquad (3)$$

$$\frac{d}{ds}(T \frac{dy}{ds}) - p \frac{dx}{ds} = 0 \qquad (4)$$

The pressure distribution of material within the sausage will be assumed to be hydrostatic. Thus, the pressure p acting on the segment ADB is expressed as γy. Substituting this into Eq. (3) and integrating for the boundary condition that $\frac{dx}{ds} = 0$ at $y = b_3$ gives

$$T \frac{dx}{ds} = \frac{\gamma}{2} (b_3^2 - y^2) \qquad (5)$$

Reducing Eq. (4) by Eq. (5) and expanding and setting $\frac{dy}{dx} = z$ gives

$$\frac{2ydy}{b_3^2 - y^2} = \frac{zdz}{1 + z^2} \qquad (6)$$

Integrating Eq. (6) for the boundary condition that $\frac{dy}{dx} = 0$ at $y = b_1$ results in

$$\frac{dy}{dx} = \pm \sqrt{\frac{(b_1^2 - b_3^2)^2}{(y^2 - b_3^2)^2} - 1} \text{ for } b_1 \geq y \geq b_3 > o \qquad (7)$$

Eq. (7), as previously noted, serves the segment ADB of Fig. 1. The segment ACB is served by

$$\frac{dy}{dx} = \pm \sqrt{\frac{(b_3^2 - b_2^2)^2}{(b_3^2 - y^2)^2} - 1} \quad \text{for } b_3 \geq y \geq b_2 > o \qquad (8)$$

Integration of Eqs. (7) and (3) provides the general solution of a fully filled submerged sausage.

In the lower section $b_3 > b_1 - H/2$ (see Fig. 1), which implies that $2 b_3^2 > b_1^2$, so that

$$x = \pm \{b_1[E(k) - E(k ,\phi)] - \frac{b_3^2}{b_1} [K(k) - F(k ,\phi)]\}$$

for $\sqrt{2}\, b_3 > b_1 > o$ where (9)

$F(k,\phi)$ and $E(k,\phi)$ are the general elliptic integrals of the first and second kind respectively, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind respectively,

k is the modulus of $F(k,\phi)$ and $E(k,\phi)$ expressed by

$$k = \frac{\sqrt{2(b_1^2 - b_3^2)}}{b_1}, \text{ and}$$

ϕ is the amplitude of $F(k,\phi)$ and $E(k,\phi)$ expressed

$$\text{by } \sin \phi = \frac{\sqrt{b_1^2 - y^2}}{\sqrt{2(b_1^2 - b_3^2)}}$$

In the upper section of the sausage

$$x = \pm \{ \frac{b_3^2 - b_2^2}{\sqrt{2b_3^2 - b_2^2}} [F(k,\phi) - K(k)] - \sqrt{2b_3^2 - b_2^2}$$

$$\cdot[E(k,\phi) - E(k)]\} \quad \text{for } \sqrt{2}b_3 > b_2 > o \qquad (10)$$

$$\text{where } k = \frac{\sqrt{2(b_3^2 - b_2^2)}}{\sqrt{2b_3^2 - b_2^2}} \quad \text{and } \sin \phi = \frac{\sqrt{(2b_3^2 - b_2^2)(y^2 - b_2^2)}}{\sqrt{2y^2(b_3^2 - b_2^2)}}$$

3 APPLICATION OF THEORETICAL SOLUTION

In practice the circumference of the tubular membrane is known and no elastic extension is contemplated in the analysis. This should not introduce great errors if the stresses are kept well within the yielding values for the materials. For a given circumference s only the pressure head b_1 will determine the resultant shape. According to Fig. 1, this circumference can be expressed as

$$s = \frac{2(b_1^2 - b_3^2)}{b_1} [K(k_1) - F(k_1,\phi_1)] + \frac{2(b_3^2 - b_2^2)}{\sqrt{2b_3^2 - b_2^2}}$$

$$\cdot[F(k_2,\phi_2) - K(k_2)] + L' \qquad (11)$$

where $b_1 < \sqrt{2}b_3$, $b_2 < \sqrt{2}b_3$, $b_1 \geq y \geq b_2 > o$

$$\left.\begin{aligned} k_1 &= \frac{\sqrt{2(b_1^2 - b_3^2)}}{b_1}, \quad k_2 = \frac{\sqrt{2(b_3^2 - b_2^2)}}{\sqrt{2b_3^2 - b_2^2}} \\ \sin \phi_1 &= \sqrt{\frac{b_1^2 - y^2}{2(b_1^2 - b_3^2)}}, \quad \sin \phi_2 = \sqrt{\frac{(2b_3^2 - b_2^2)(y^2 - b_2^2)}{2y^2(b_3^2 - b_2^2)}} \end{aligned}\right\} (12)$$

and L′ = bottom contact width (see Fig. 1)

From a given value of b_1 (total equivalent water head) and circumference s the values of b_2, b_3 and L′ can be determined from Eqs. (9), (10) and (11). Thus the characteristics of the stable shape

of maximum height (H), maximum width (L) and contact width (L´) can be decided.

To derive the hoop tension Eqs. (3) and (4) may be transformed, using Fig. 2 to

$$d(T\frac{dx}{ds}) = d(T\cos\theta) = \cos\theta dT - T\sin\theta d\theta = p\sin\theta \cdot R \cdot d\theta \quad (13)$$

$$d(T\frac{dy}{ds}) = -d(T\sin\theta) = -\sin\theta dt - T\cos\theta d\theta = p\cos\theta \cdot R \cdot d\theta \quad (14)$$

of which the solutions of dT and T are expressed as

$$T = -pR = -\gamma y R \quad (15)$$

$$\text{and } dT = 0,\ T = \text{constant} \quad (16)$$

From Eq.(16) it is seen that the hoop tension of a sand sausage under full load is constant. The radius of curvature (R) at any point is given by

$$R = [1 + (\frac{dy}{dx})^2]^{3/2} / \frac{d^2y}{dx^2} = (1 + z^2)^{3/2} / \frac{dz}{dx} \quad (17)$$

from Eq. (6) $\frac{dz}{dy}$ can be rewritten

$$\frac{dz}{dy} = \frac{1 + z^2}{b_3^2 - y^2} \frac{2y}{z} \quad (18)$$

of which further transformation is given by

$$\frac{dz}{dx} = -2y \frac{1 + z^2}{y^2 - b_3^2} \quad (19)$$

Substituting Eqs. (7) and (19) into Eq. (17), values of R and T can be finally derived as

$$R = -\frac{b_1^2 - b_3^2}{2y} \quad (20)$$

$$T = \frac{\gamma}{2}(b_1^2 - b_3^2) \quad (21)$$

The hoop tension is thus equal to the net weight of water exerted against the sausage from the bed to the level where the tangent is vertical (see Fig. 1).

The stable shape must be continuous at the intersection of the upper and lower sections. Thus, the bottom contact width L´ and value b_2 may be determined from Eqs. (9) and (10) for an assumed b_3 value. The theoretical circumference s_1 can be computed from Eq. (1) which is then checked with the known s. If s_1 does not equal s, the computation is repeated until b_3 is found exactly. With b_3, known the stable shape can be derived from Eqs.(9) and (10) with the aid of the computer.

Limiting values of parameters can be assessed as follows:

(a) If the dimensionless water head (b_1/D) approaches infinity, the sausage tends to a circular shape, the ratios H/D and L/D become unity and L´/s tends to zero. Note that the diameter (D) of a circular sausage equivalent in area to the actual is given by s/π.

(b) When b_1/D vanishes the sausage flattens, so that the ratios H/D becomes zero, L/D tends to $\pi/2$ and L´/s becomes ½.

The theoretical solutions are presented in Fig. 3 as continuous curves, showing limits to where $b_1/D = 3$ or $b_1/s = 3/\pi = 0.96$

4 EXPERIMENTAL VERIFICATION

A manufacturer kindly supplied extra long plastic bags with sealing at both ends, the dimensions of which are listed in Table I. These were then filled with water by connecting two tubes to the top of the skin, one to induct water and the other to accurately measure the pressure head being applied. The straight joint at the extremities caused deformation of the shape, but at the centre of the 2 metres length the shape was comparable with the theoretical solution.

The bags were placed on a horizontal sheet of perspex between two tables. This was marked in 1 cm squares in order that the contact width L´ could be measured and a plane surface was available from which to record L and H plus other points around the circumference (see Fig. 4.) A measuring tape was slung around the sausage in order to record the instantaneous value of the circumference which could change under constant load due to creep of the PVC material.

TABLE I
VARIABLES OF THE FULLY FILLED SAUSAGE

Circumference s(cm)	Equivalent Diameter D(cm)	Water head b_1(cm)
72.0	22.92	3.0 - 9.8
81.4	25.92	22.0 - 52.8
92.8	29.40	17.9 - 50.7
101.6	32.34	5.5 - 37.9

When the water-head b_1 circumference s and assumed b_3 are inserted into the computer programme, the variables L, L´ and H can be obtained. These are compared with the measurements made on the models by the data in Fig. 3. It is seen that for higher pressure heads the ratio H/D is slightly less than the theoretical value. Also the contact width L´ appears to be greater than the theoretical for large b_1/D This could be due to the theoretical solution being based upon the original circumference. The theoretical and actual shapes for s = 92.8 cm are shown in Figs. 5 and 6 for b_1 values of 17.9 cm and 39.4 cm respectively. In both cases the experimental shapes are slightly flatter in character than the theoretical but the agreement is still quite acceptable.

5 CONCLUSIONS

1. The concept of flexible membrane bags containing sand, mortar or concrete must come into greater use in the future because of their economy in size of structure for marine purposes and construction costs.
2. The large sand filled sausage can serve as a core for a breakwater utilising smaller bags filled with a cemented mixture or be used alone for permanent or temporary protection.
3. The shape parameters of a flexible membrane can be derived for a given pressure head and circumference, such as height, breadth and width of contact with the bed.
4. The variables in (3) above have been verified experimentally without allowance for elastic elongation of the material.
5. The skin tension per unit length of structure is equal to the net pressure of material exerted on a vertical plane from the bed to the maximum width of the sausage and is the same around the complete circumference.

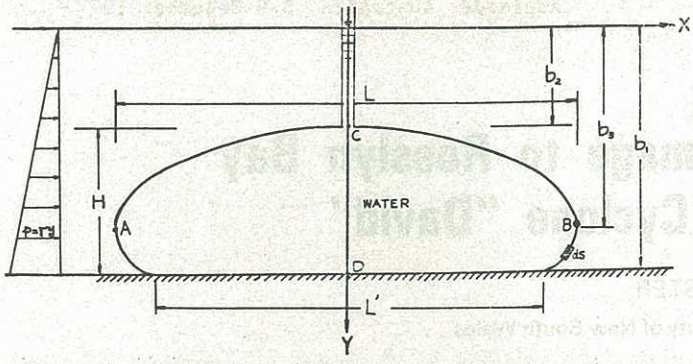

Figure 1 Definition sketch of equivalent sand sausage

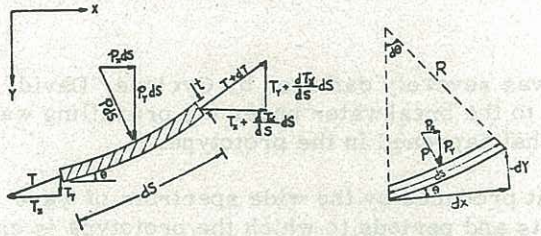

Figure 2 Force diagrams for segment of membrane

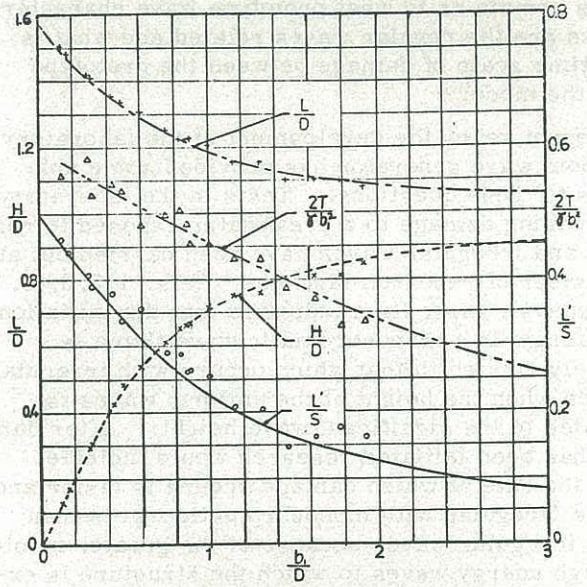

Figure 3 Dimensionless parameters from theory and experiment

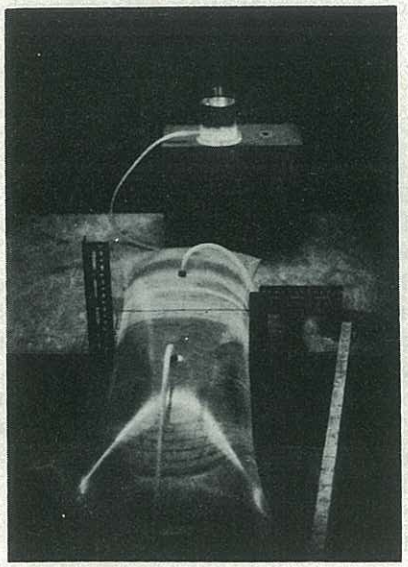

Figure 4 Experimental equipment for testing water filled sausage

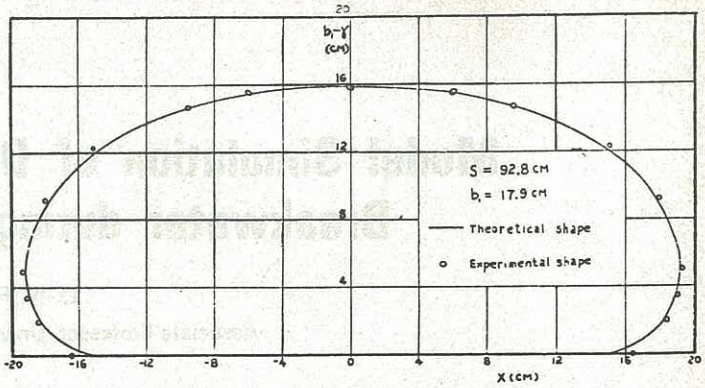

Figure 5 Comparison of measured and predicted shapes for a pressure head b_1=17.9cm

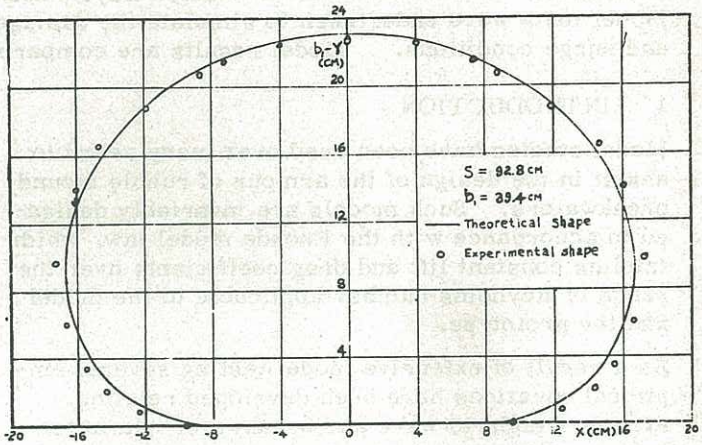

Figure 6 Comparison of measured and predicted shapes for a given pressure b_1=39.4cm

6 ACKNOWLEDGEMENTS

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7 REFERENCES

1. PORRAZ, M. "Textile forms slash cost of coastal zone structures" Ocean Industry II 1976, 61-66.
2. MARKS, W.D. & F.A. CLINTON "Michigan Demonstration Erosion Control Program" Shore and Beach, Oct. 1974, 11-16.
3. ARMSTRONG, J.M. "Demonstration Project for Low-Cost Shore Protection" Proc. 15th Conf. Coastal Eng. IV, 1976 (in Press).
4. SILVESTER, R. Stabilisation of Sedimentary Coastlines" Nature 188 (4749), 1960, pp.467-469.
5. SILVESTER, R. "Growth of Crenulate Shape Bays to Equilibrium" Proc. ASCE 96(WW2), 1970, pp.275-287.
6. SILVESTER, R. "Coastal Engineering Vol.II" Elsevier Publish. Co., 1974.
7. MONONOBE, N. Hydraulics Iwanami Book Co., Japan.