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Flow in a Channel with a Symmetric Sudden Expansion

A. KUMAR and K. S. YAJNIK
Scientists, Aerodynamics Division, National Aeronautical Laboratory, Bangalore, India

SUMMARY The effect of expansion ratio and entry condition on various flow characteristics of a symmetric sudden expansion are examined. Equation for large Reynolds number is integrated by using an expansion in eigenfunctions of the Poiseuille flow development. Results include streamlines, the distribution of centreline velocity and pressure, the reattachment length, the development length and the pressure recovery coefficient.

1 INTRODUCTION

Flow in a channel with a sudden expansion is a basic prototype of internal separated flows which arise in many engineering applications. This paper focuses attention on laminar flow that is steady, two-dimensional, and symmetrical.

Solution of the Navier-Stokes equations for such flows by using a finite-difference scheme is given by Hung and Macagno (1966), Morihara (1972), and Durst et al. (1974). Kumar and Yajnik (1976a) have considered internal separated flow at large Reynolds number (see also Yajnik and Kumar, 1972). They show that the appropriate governing equation (except possible for small subregions) in non-dimensional form is

$$\psi_y \ \psi_{yyx} - \psi_x \ \psi_{yyy} = \psi_{yyyy} \qquad (1)$$

where ψ is the streamfunction, and x and y refer to the streamwise and the transverse coordinates (see Fig.1). x in (1) has been scaled by the Reynolds number R, which is based on channel half-width and average velocity downstream of the expansion. They have also developed a method to solve (1) which uses an expansion in the eigenfunctions of the Poiseuille flow development. The problem is thus reduced to solving a set of first order ordinary differential equations.

This paper examines the effects of expansion ratio and entry condition on various flow characteristics by using the above method.

2 BRIEF REVIEW OF THE METHOD

Let ψ be expressed as

$$\psi = \tfrac{1}{2}(3y - y^3) + \sum_m a_m(x)\ \phi_m(y) \qquad (2)$$

where ϕ_m s are the eigenfunctions of

$$\phi^{iv} + \lambda[3/2(1 - y^2)\ \phi'' + 3\phi] = 0 \qquad (3)$$

with $\phi = \phi' = 0$ at $y = \pm 1$. The adjoint problem is given by

$$\theta^{iv} + \lambda[3/2(1 - y^2)\ \theta'' - 6y\ \theta'] = 0 \qquad (4)$$

with $\theta = \theta' = 0$ at $y = \pm 1$. The biorthogonality relation for the present problem is

$$\int_{-1}^{1} \phi_m''\ \theta_n''\ dy = 0, \quad m \neq n. \qquad (5)$$

As the flow is symmetrical, only those eigenfunctions are used in (2) for which ϕ' is an even function of y. The following equations are obtained by substitution of (2) in (1), multiplication by θ_m, and integration.

$$a_m' + \lambda_m a_m = \sum_p \sum_q c_{mpq}\ a_p'\ a_q \qquad (6)$$

where

$$c_{mpq} = \lambda_m G_m^{-1} \int_{-1}^{1} \theta_m'(\phi_p\ \phi_q'' - \phi_p'\ \phi_q')\ dy \qquad (7)$$

$$G_m = \int_{-1}^{1} \phi_m''\ \theta_m''\ dy \qquad (8)$$

The initial values $a_{mo} = a_m(0)$ are

$$a_{mo} = G_m^{-1} \int_{-1}^{1} \theta_m^{iv}\ [\psi(0,y) - \tfrac{1}{2}(3y - y^3)]\ dy \qquad (9)$$

The problem is thus reduced to solving (6) with (9). Flow variables such as ψ and pressure p can then be determined. Note that p does not depend on y. In a calculation with N eigenfunctions, the range of summation in (2) and (6) is from 1 to N.

3 ENTRY CONDITION

Two extreme entry conditions are considered, namely, parabolic and uniform velocity profiles, corresponding to a long and a short inlet. Intermediate conditions with non-zero boundary-layer thickness can also be considered. a_{mo} is given by

$$a_{mo} = -6\ [h\ \theta_m'(h) - \theta_m(h)]/(G_m h^3)$$

(parabolic entry) (10a)

$$= -2\ \theta_m''(h)/(G_m h), \quad \text{(uniform entry)} \qquad (10b)$$

where 1/h is the expansion ratio.

4 RESULTS

Experience suggested that satisfactory results are obtained even when only a few eigenfunctions are used (Kumar and Yajnik, 1976 a and b). N was restricted to five for the present calculations.

4.1 Streamlines

Fig. 1 shows streamlines for h=0.5 with parabolic entry along with those obtained by Hung and Macagno (1966). Quantitative comparison of flow tangential injection for two sizes of slot. The diffusers which have axial injection clearly have a much better performance, the peak C_p being higher and occurring at a lower rate of injection. With the 0.35 mm slot, the peak C_p is about 0.08 higher, and with the 0.75 mm slot it is about 0.05. With no injection, the performance with the tangential slot is again worse than with the axial one possibly due to the small downstream-facing step present in the tangential slot geometry, as shown in Figure 4. One effect of the step is to shorten the diffuser slightly but a more important consequence is thought to be that it encourages flow separation and counteracts the benefits due to injection. In most other respects (e.g. regarding the effects of slot size) the axial and tangential modes of injection produce similar behaviour.

Further data relating to the discharging flow were obtained from velocity measurements for axial and tangential injection and for differenct values of k. For the axial slot, an improvement in velocity profile occurs with increasing injection and the recirculation region which is present initially becomes smaller and ultimately disappears when k is about 2, a similar value to the optimum for maximum pressure recovery. The profiles do not reveal any deteriorating symmetry due to injection and so the results for the axial slot do not support the tentative conclusion to teh contrary ehich was drawn in the preliminary work with large slot.

Turning to the velocity profiles for tangential injection, these reveal that the flow is now less stable than for axial injection and there is a tendency for any asymmetry already present to be am amplified by the injection. The present profiles for small tangential slots are in fact similar to those for large axial ones form which the above tentative conclusion was made.

It may be concluded from the comparison between the axial and tangential modes of injection that in every aspect that has been examined the axial mode is better. The slot geometry is less complex, the pressure recovery is greater and is achieved with less injection, and the discharging flow is more stable and symmetrical.

Regarding the general merits of secondary injection, it is concluded that, by causing the flow to remain attached to the conical wall, injection yields considerable improvements in both the quality of the discharging flow and the magnitude of the pressure recovery. C_p is more than doubled by optimum injection in spite of the fact that C_p is defined so as to take account of the additional kinetic energy and pressure energy of the injected fluid. The slot width should be as large as possible, consistent with there being an appropriate secondary supply available, and for design purposes it is recommended that the size should be estimated from the empirical expression

$$0.14c = (\dot{m}_s/\dot{m}_1)^2 .$$

Reduction of the cone angle from 30° to 20° offers a small improvement in the maximum C_p at the expense of an increase of over 50% in the cone length.

4 ACKNOWLEDGEMENTS

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Figure 5 Effects of injection rate and slot size on Pressure Recovery Coefficient (Axial injection and 30° cone angle)

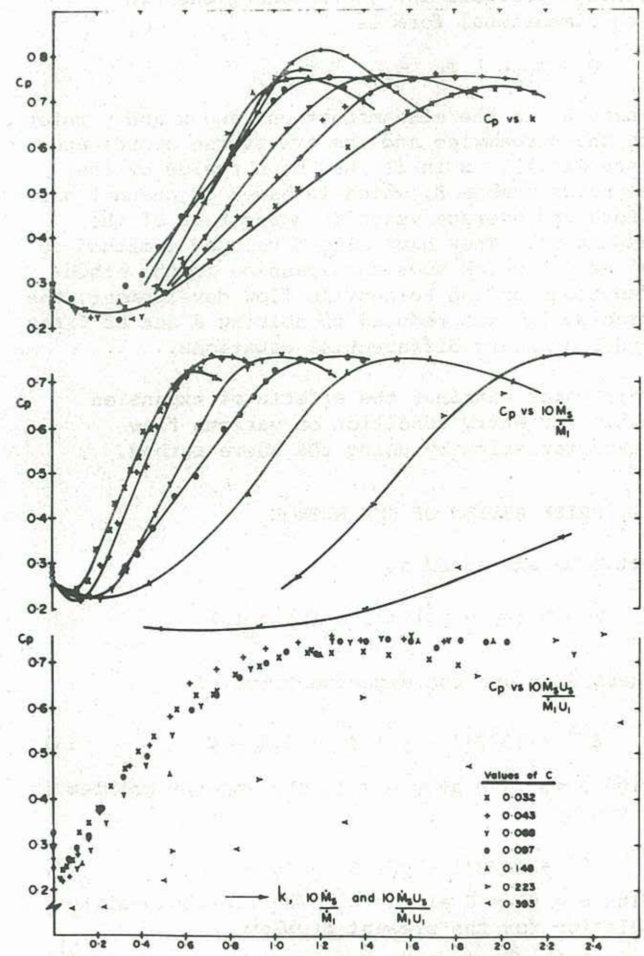
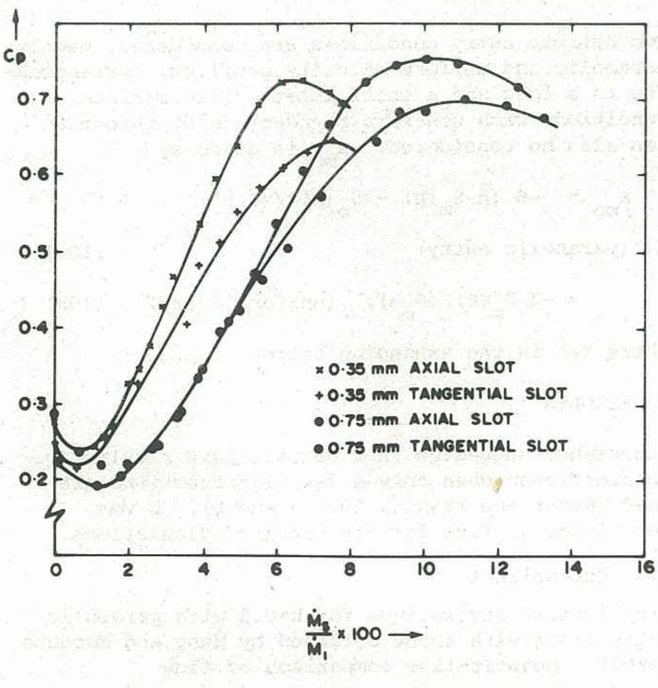


Figure 6 Comparison of axial and tangential injection for two sizes of slot.



characteristics is given in Table I. x_r and x_e are streamwise coordinates of the point of reattachment and the centre of eddy respectively. Recirculation in the eddy is given by $(\psi_e - 1)$. Thus the present results are in good agreement, qualitatively as well as quantitatively, with earlier results.

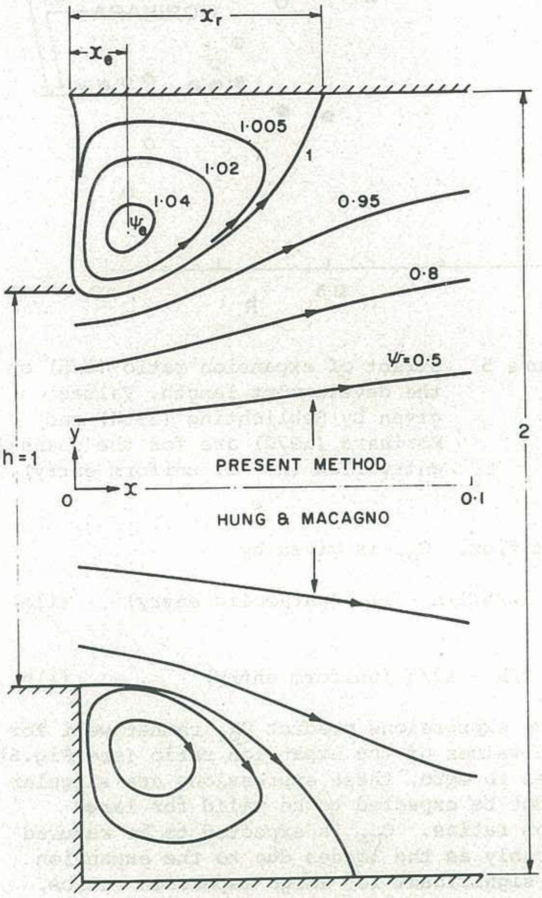


Figure 1 Streamlines obtained by the present method and given by Hung and Macagno (1966). h = 0.5, parabolic entry. N = 3; R= 46.6 (for Hung and Macagno)

TABLE I

COMPARISON OF THE PRESENT CALCULATION WITH THAT OF HUNG AND MACAGNO (1966)

	x_r	x_e	(ψ_e-1)
Present Calculation (N= 3)	0.064	0.014	0.045
Hung and Macagno	0.066	0.013	0.052

4.2 Centreline Velocity Distribution

Fig. 2 shows centreline velocity distribution u_c. For the parabolic entry, u_c decreases monotonically to the Poiseuille flow value. A different behaviour is observed for h = 0.75 with uniform entry. The incoming flow decelerates at first due to its mixing with the 'dead' fluid. Later as the flow develops, u_c increases to attain $u_c(\infty)$ as it has fallen below its final value. Similar behaviour also occurs for h = 0.6. 0.7 and 0.8(not shown in Fig.2). However, if $u_c(0)$ is much larger than $u_c(\infty)$, e.g. h = 0.5, $u_c(x)$ does not fall below $u_c(\infty)$ and the above behaviour is not observed. The above 'dip' in u_c is not seen for h = 0.9 and also 0.95 (not shown in Fig. 2), as it is possibly too small to be detected with N = 5.

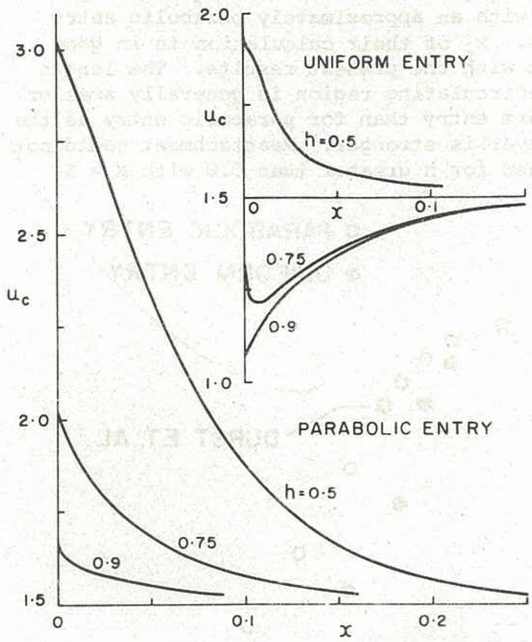


Figure 2 Centreline velocity distribution with uniform and parabolic entry. N = 5. h = 0.5, 0.75 and 0.9

4.3 Pressure Distribution

Figure 3 shows the pressure variation. Asymptotes to these pressure distributions are $p = -3x + P_r$, where P_r is called the recovery pressure. The reattachment point A is a short distance upstream of the pressure maximum. The behaviour close to x = 0 for h = 0.5 and 0.9 with uniform entry probably calls for larger values of N.

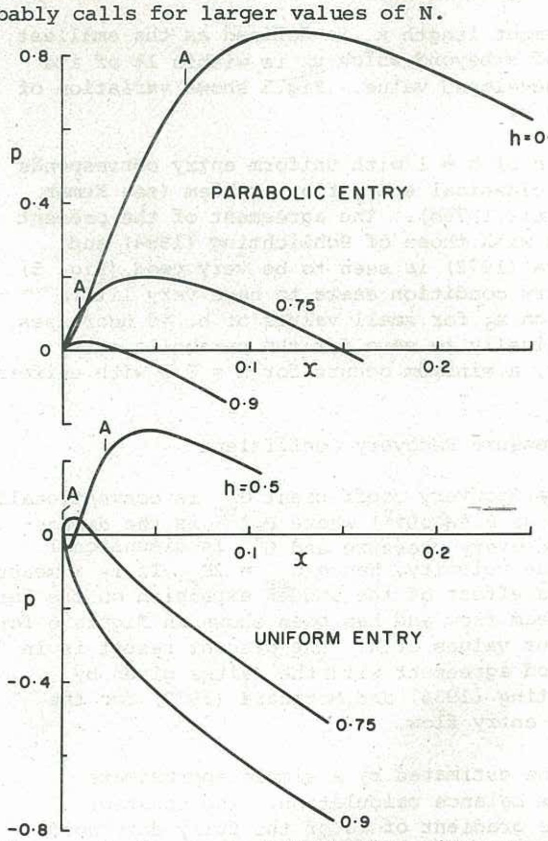


Figure 3 Pressure distribution with parabolic and uniform entry. N=5. h = 0.5,0.75 and 0.9. A denotes reattachment point

4.4 Reattachment Length

Fig. 4 shows the variation of reattachment length x_r with h. Durst et al. (1974) have calculated the flow using the Navier-Stokes equations for h = 1/3, with an approximately parabolic entry condition. x_r of their calculation is in good agreement with the present results. The length of the recirculating region is generally smaller for uniform entry than for parabolic entry as the mixing layer is stronger. Reattachment could not be obtained for h greater than 0.8 with N = 5.

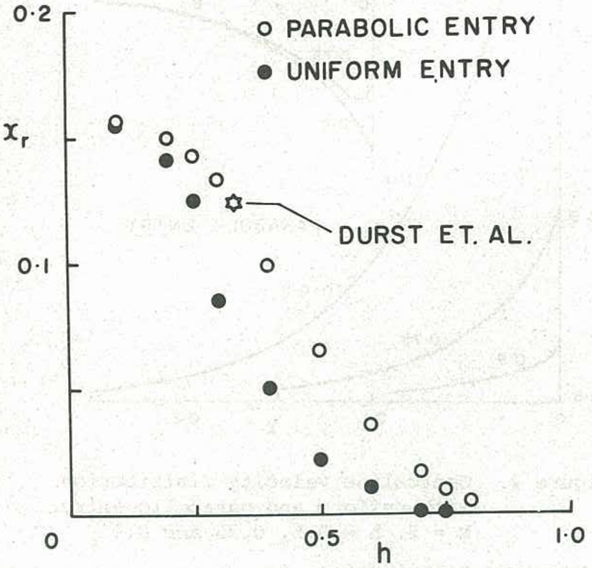


Figure 4 Effect of expansion ratio (1/h) on the reattachment length. Value given by Durst et al. (1974) is for h = 1/3, R = 18.67.

4.5 Development Length

Development length x_d is defined as the smallest value of x beyond which u_c is within 1% of its fully developed value. Fig.5 shows variation of x_d with h.

The case of h = 1 with uniform entry corresponds to the classical entry flow problem (see Kumar and Yajnik,1976b). The agreement of the present results with those of Schlichting (1934) and Morihara (1972) is seen to be very good (Fig. 5). The entry condition seems to have very little effect on x_d for small values of h. x_d decreases monotonically to zero for the parabolic entry. However, a minimum occurs for h ≃ 0.5 with uniform entry.

4.6 Pressure Recovery Coefficient

Pressure Recovery coefficient C_{pr} is conventionally defined as $P_r^*/(\frac{1}{2}\rho U^{*2})$ where P_r^* is the dimensional recovery pressure and U^* is dimensional reference velocity. Hence $C_{pr} = 2P_r$. It is a measure of gross effect of the sudden expansion on the far downstream flow and has been shown in Figure 6 for different values of h. The present result is in very good agreement with the values given by Schlichting (1934) and Morihara (1972) for the channel entry flow.

P_r can be estimated by a simple approximate momentum balance calculation. The constant pressure gradient of -3 of the fully developed flow, and the direct viscous effect are disregarded. Pressure acting on a sufficiently downstream section then is P_r and is due to the change in

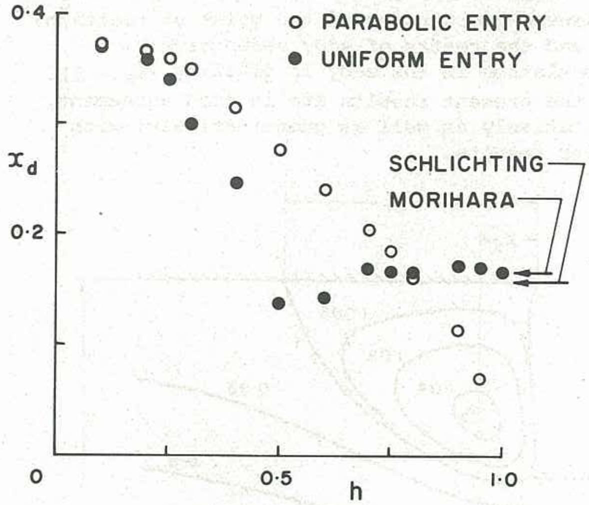


Figure 5 Effect of expansion ratio (1/h) on the development length. Values given by Schlichting (1934) and Morihara (1972) are for the channel entry flow (h = 1, uniform entry).

momentum flux. C_{pr} is given by

$$C_{pr} = 12/5(1/h - 1), \text{ (parabolic entry)} \qquad (11a)$$

$$= 2/h - 12/5 \text{ (uniform entry)} \qquad (11b)$$

The above expressions predict C_{pr} rather well for moderate values of the expansion ratio (see Fig.6). As h goes to zero, these expressions are singular and cannot be expected to be valid for large expansion ratios. C_{pr} is expected to be reduced considerably as the losses due to the expansion will be significant for large expansion ratios. The present calculation suggest that C_{pr} may be approximately taken equal to 4 irrespective of the entry condition for h ≤ 1/3.

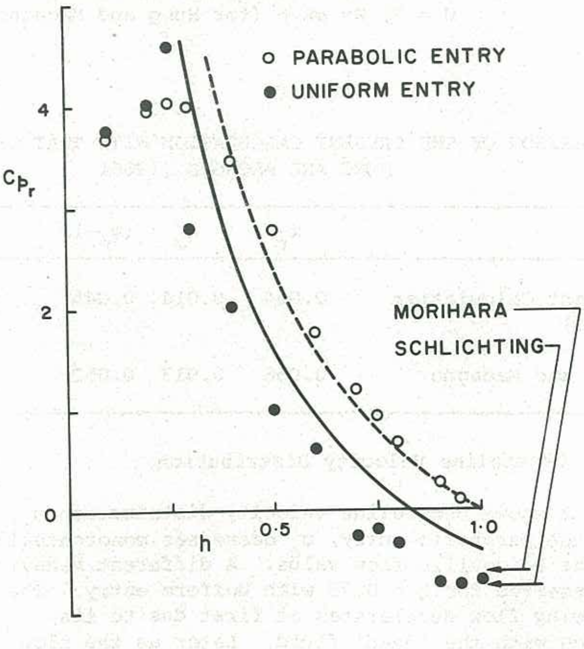


Figure 6 Effect of expansion ration (1/h) on the pressure recovery coefficient ---, (11a); ___(11b).Values given by Schlichting (1934) and Morihara (1972) are for the channel entry flow.

CONCLUDING REMARKS

A simple method requiring solution of a few ordinary differential equations is used to solve the present problem. The calculated streamline pattern and other flow characteristics compare well with the solution of the Navier-Stokes equations for an expansion ratio where results are available.

The centreline velocity decreases monotonically to its fully developed value for parabolic entry. However, it is not so for a certain range of h when the entry flow is uniform. Also there is a region beyond reattachment where pressure tends to be approximately constant.

Reattachment with uniform entry occurs in general earlier than with parabolic entry.

Development length monotonically decreases to zero as h tends to one for the parabolic entry. While with uniform entry, it has a minimum for the expansion ratio of about 2.

Pressure recovery coefficient is in general smaller for uniform entry than for parabolic entry. Its approximate evaluation for moderate values of the expansion ratio is given. The pressure recovery coefficient may be taken to be 4 irrespective of the entry condition for ratios of expansion greater than 3.

The present calculation is based on a large Reynolds number limit equation. The favourable comparison of the present calculation with that of Durst et al. (1974) for R = 18.67 shows that the results are applicable for moderately large R.

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