

Laminar Boundary Layer and Heat Transfer over an Extending Film

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SUMMARY Boundary layer and heat transfer over an extending film of polymer melt are treated by the integral approximation. It is found that the heat transfer coefficient can either increase or decrease along the film. Physical explanations are given.

1 INTRODUCTION

Polymer processing has presented some interesting fluid mechanics problems. One of these is the calculation of the fiber diameter in the spinning process or of the film thickness in the film casting process. One would, for the first approximation, consider the polymer melt as an isothermal, highly viscous (low Reynolds number) Newtonian fluid and obtain a very simple solution. For a polymer film with the thickness $H(X)$, it is simply $H(X) = H_0 \exp(-AX)$, (Yeow, 1974), where $A = q/(4\mu Q)$, q is the tension on the film, μ is the coefficient of viscosity and Q is the volumetric flow rate per unit width. However, the coefficient of viscosity μ of the polymer melt is a highly sensitive function of the temperature. For example, the coefficient of viscosity of a low density polyethylene melt behaves as

$$\mu(T) = 48.5 \exp\left(\frac{5670}{T}\right) \quad (1)$$

where T is in degree Kelvin and μ is in gm/cm-sec. One immediate question to be answered is how the heat transfer between the polymer melt and the surrounding air affects the variation of the film thickness.

For polymer spinning, Lamonte and Han (1972) have tried to calculate the radius of the fiber by taking into account the air drag and heat transfer effects. But in his theory, the heat transfer coefficient and the air drag are provided by the empirical formulas (Kase, 1967) in which the only parameter reflecting the interactions between the polymer melt and the cooling air is the local fiber velocity. It is dubious how the effective boundary layer thickness and the local velocity are related to each other in such a simple algebraic way. For blown film process, Han and Park (1975) have used an empirical formula for the heat transfer rate which implies a constant heat transfer coefficient along the film surface. Again, there is no rationale behind his formula except that the temperature variation so determined is claimed to be in good agreement with the experimentally measured results.

The best way to resolve this question is to formulate the problem in such a way that the development of the boundary layer and the evolution of the film or fiber are determined simultaneously. Acierno et al (1971) have treated the boundary layer and heat transfer around the polymer fiber by using a Pohlhausen type integral approximation. The resulting heat transfer coefficient is then employed in the calculation of the fiber radius. However, they have neglected the effect of the variation of the fiber velocity along the fiber on the growth of the

boundary layer. The coupling between the boundary layer and the fiber is not complete. As a result, their conclusion that the Nusselt number decreases along the fiber, is in contradiction with the empirical formula given by Kase and Natsuo (1967).

Here, we shall study the boundary layer over an extending film by an integral approximation taking into account the interaction between the variations of the properties of the film and the growth of the boundary layer. Similar technique can be applied to the fiber case.

2 FORMULATION OF THE PROBLEM

Let us consider a two dimensional polymer film being pulled by a tension q at one end. Figure (1) shows the schematic definition of the problem

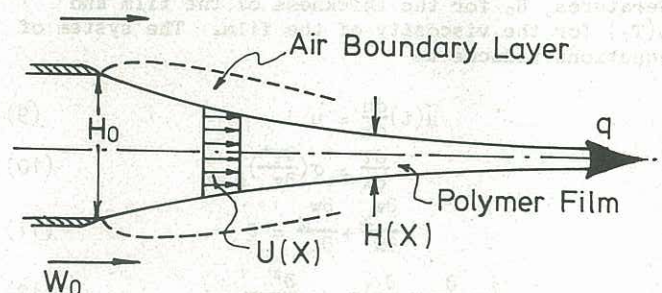


Figure 1 Schematic definition of the problem

The viscosity of the fluid is so high that the inertia of the film, the air drag as well as its own weight are small in comparison with the tension q . The continuity and the momentum equations can be written as

$$HU = Q \quad (2)$$

$$4\mu(T)H \frac{dU}{dX} = q \quad (3)$$

where U is the velocity of the film and $\mu(T)$ is assumed to be of the form of the equation (1). Now, the temperature of the film may vary along the film due to the heat transfer at the air-liquid interface. The following equation describes the variation of the temperature of the film.

$$MQC \frac{dT}{dX} = 2\kappa \left(\frac{\partial T^*}{\partial Y} \right)_{Y=0} \quad (4)$$

where M and C are the density and the heat capacity of the polymer melt, κ and T^* are thermal conductivity and the temperature of the air respectively, and Y stands for the coordinate perpendicular to the film surface. For air, the usual boundary layer assumptions will be made and the governing equations become

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0 \quad (5)$$

$$(w_x \frac{\partial}{\partial x} + w_y \frac{\partial}{\partial y}) w_x = \nu \frac{\partial^2 w_x}{\partial y^2} \quad (6)$$

$$(w_x \frac{\partial}{\partial x} + w_y \frac{\partial}{\partial y}) T^* = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^2} \quad (7)$$

where (w_x, w_y) are the components of the velocity of the air, ρ and ν are the density and the kinematic viscosity of the air respectively. The thickness of the film is so small that the induced pressure gradient along the film surface is assumed negligible. The boundary conditions are

$$\begin{aligned} H(0) &= H_0; T(0) = T_0; w_x(0, Y) = w_0 \\ w_x(X, \infty) &= w_0; w_x(X, 0) = U(X); w_y(X, 0) = 0 \\ T^*(0, Y) &= T_0^*; T^*(X, 0) = T(X) \end{aligned} \quad (8)$$

where w_0 and T_0^* are the free stream velocity and the free stream temperature of the air respectively, H_0 and T_0 are the thickness and temperature of the film at the exit. Since the boundary layer equations are parabolic, the equations (2) to (8) constitute a mixed initial-boundary value problem.

The variables are then non-dimensionalized by the following reference quantities: w_0 for the velocities, $4\mu(T_0)Q/q$ for the length, T_0^* for the temperatures, H_0 for the thickness of the film and $\mu(T_0)$ for the viscosity of the film. The system of equations reduces to

$$\mu(t) \frac{du}{dx} = u \quad (9)$$

$$\frac{dt}{dx} = \sigma \left(\frac{\partial t^*}{\partial z} \right)_{z=0} \quad (10)$$

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial z} = 0 \quad (11)$$

$$(w_x \frac{\partial}{\partial x} + w_y \frac{\partial}{\partial z}) w_x = \frac{\partial^2 w_x}{\partial z^2} \quad (12)$$

$$(w_x \frac{\partial}{\partial x} + w_y \frac{\partial}{\partial z}) t^* = \frac{1}{Pr} \frac{\partial^2 t^*}{\partial z^2} \quad (13)$$

$$\left. \begin{aligned} u(0) &= u_0; t(0) = t_0 \\ w_x(0, z) &= 1; w_x(x, \infty) = 1 \\ w_x(x, 0) &= u(x); w_y(x, 0) = 0 \\ t^*(0, z) &= 1; t^*(x, \infty) = 1 \\ t^*(x, 0) &= t(x) \end{aligned} \right\} \quad (14)$$

where the variables in small letters now stand for the non-dimensional variables for the corresponding dimensional variables in capital letters. $\mu(t)$ is the nondimensional viscosity of the polymer melt, and the other dimensionless parameters are Pr the Prandtl number for the air, $\sigma = \frac{2\kappa}{MQC_p} R_e^{\frac{1}{2}}$ and the Reynolds number defined as

$$R_e = \frac{4\mu(T_0)QW_0}{\nu q} \quad (15)$$

In the equations (9) to (14), the standard boundary layer coordinate stretching

$$z = R_e^{\frac{1}{2}} y \quad (16)$$

is already employed.

In order to make an engineering estimation of the heat transfer rate at the interface, the integral approximation seems to be the most powerful and effective tool in hands. Let us integrate the equations (12) and (13) across the boundary layer and manipulate the resulting equations by the well known procedure.

$$\frac{d}{dx} \int_0^\infty (1-w_x) w_x dz = \left(\frac{\partial w_x}{\partial z} \right)_{z=0} \quad (17)$$

$$\frac{d}{dx} \int_0^\infty (1-t^*) w_x dz = \frac{1}{Pr} \left(\frac{\partial t^*}{\partial z} \right)_{z=0} \quad (18)$$

The velocity profile and the temperature profile across the boundary layer are assumed to be third order polynomials.

$$w_x = u + a_1 \eta + a_2 \eta^2 + a_3 \eta^3 \quad (19)$$

$$t^* = t + b_1 \eta + b_2 \eta^2 + b_3 \eta^3 \quad (20)$$

where $\eta = z/\delta(x)$ and $\delta(x)$ is the boundary layer thickness to be determined. The thermal boundary layer and the momentum boundary layer are considered of the same thickness, but the coefficient b_1 which represents the non-dimensional thermal gradient at the interface is left as a variable and will be determined through the solution of the resulting ordinary equations. Since the Prandtl number of the air is of the order one, the treatment given above seems to be reasonable. Any difference between the detail thermal and velocity profiles will project itself on the coefficient b_1 .

By satisfying the boundary conditions at $\eta = 1$ and determining the remaining coefficients in the equations (19) and (20), the equation (17) and (18) can be reduced to the following

$$\frac{d}{dx} [\delta(3+4u)(1-u)] = \frac{8u}{\delta}(1-u) \quad (21)$$

$$\begin{aligned} \frac{db_1}{dx} = -\frac{1}{\delta(2u+5)} \left\{ \frac{8u}{Pr} \delta + [b_1(2u+5) + (18u+24)(t-1)] \frac{d\delta}{dx} \right. \\ \left. + \delta[2b_1+18(t-1)] \frac{du}{dx} + \delta[18u+24] \frac{dt}{dx} \right\} \end{aligned} \quad (22)$$

The equation (10) can be written as

$$\frac{dt}{dx} = \sigma \left(\frac{b_1}{\delta} \right) \quad (23)$$

The equations (9), (21), (22) and (23) describe the evolution of the variables u , t , δ and b_1 . They can now be easily integrated numerically.

3 NUMERICAL SOLUTIONS

It is found that $u = 1$ is a singularity of the equation (21). This seems to be the inherent difficulty of the integral approximation in treating this type of boundary layer. Since the velocity profile is parameterized by a third order polynomial, the boundary layer must disappear as the velocity at the interface approaches that of the free stream air. There is no way for the boundary layer thickness δ

to adjust itself to a definite value. For this difficulty, we shall examine two extreme cases, namely the case of a strong air flow for which $u < 1$ everywhere and the case of a weak airflow i.e. $u > 1$. The transition case can only be treated by solving the exact system of parabolic partial differential equations.

At the leading edge, δ and b_1 are found to behave as

$$\delta \sim \left(\frac{168x}{3+4u_0} \right)^{\frac{1}{2}} \quad (24)$$

$$b_1 \sim - \left\{ \frac{6(t_0-1)(3u_0+4)p_r}{p_r(2u_0+5)+(3+4u_0)} \right\} \quad (25)$$

The above relations are used in the numerical calculation in order to stay away from the leading edge singularity.

The Nusselt number Nu based on the characteristic length $4\mu(T_0)Q/q$ is related to the Reynolds number and the variables δ , b_1 and t by the following relation

$$\left(\frac{Nu}{Re^{\frac{1}{2}}} \right) = \left(\frac{b_1}{\delta} \right) \frac{1}{t-1} \quad (26)$$

Figure (2) and (3) show some typical variations of $Nu Re^{\frac{1}{2}}$ and the film velocity u along the film.

Figure (4) shows the growth of the boundary layer thickness.

4 CONCLUSIONS AND DISCUSSIONS

The results of the numerical solution show that for the strong air flow, the Nusselt number is essentially that of a flat plate solution when the surface velocity is small in comparison to the free stream velocity. But when the surface velocity, which increases along the film surface, reaches some significant portion of the free stream velocity,

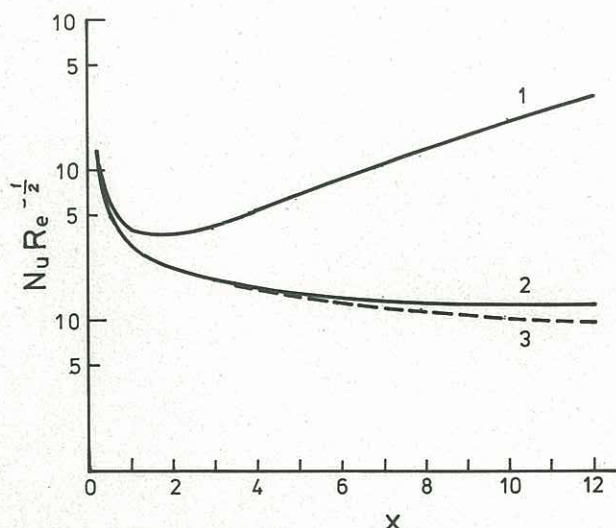


Figure 2 $Nu Re^{\frac{1}{2}}$ vs. x

- 1: $U_0 = 2.2\text{cm/sec}$, $W_0 = 2.0\text{cm/sec}$, $Q = 0.11\text{cm}^2/\text{sec}$, $H_0 = 0.05\text{cm}$, $T_0 = 450^\circ\text{K}$, $t_0 = 300^\circ\text{K}$, $4\mu(T_0)Q/q = 5\text{cm}$.
2: Same as 1 except $W_0 = 200\text{cm/sec}$.
3: Flow over a flat plate.

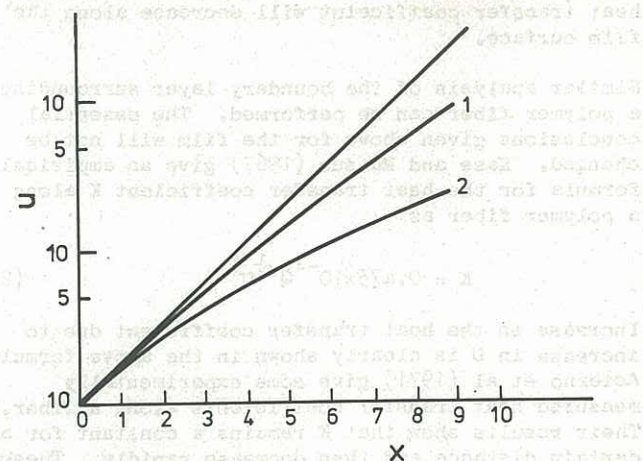


Figure 3 Variation of the velocity of the film
Case 1, 2: Same as Fig. 2.

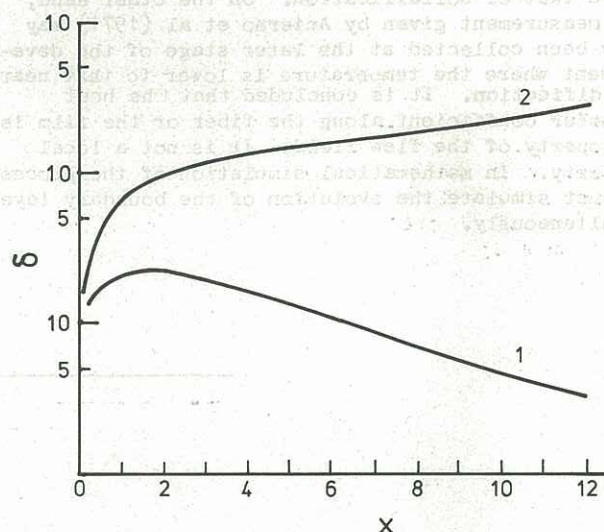


Figure 4 The variations of the Boundary layer thickness

Case 1, 2: Same as Fig. 2.

city, the Nusselt number has a considerable increase over that of the flat plate solution. However, the most interesting case is the weak air stream. The Nusselt number increases along the film surface after the leading edge effect has faded. Physically, this can be explained as follows. When the surface velocity exceeds that of the air stream, the corresponding displacement thickness is negative. Furthermore, the magnitude of this negative displacement thickness is increasing along the film surface due to the increasing surface velocity. The boundary layer is continuously supplied by the cool air outside. The vorticity and the thermal energy are convected inward by the negative vertical velocity and the growth of the boundary layer is suppressed. Its thickness can even become thinner downstream. This suction effect and the diffusion mechanism of the vorticity and the thermal energy are competing against each other. When the temperature of the film is far above that of the solidification, the surface velocity increases rapidly and the suction effect is dominant. The heat transfer coefficient increases accordingly. However, it is expected that the diffusion mechanism will take over when the temperature is lowered to

that near the solidification. In this case, the heat transfer coefficient will decrease along the film surface.

Similar analysis of the boundary layer surrounding a polymer fiber can be performed. The essential conclusions given above for the film will not be changed. Kase and Matsuo (1967) give an empirical formula for the heat transfer coefficient K along a polymer fiber as

$$K = 0.473 \times 10^{-4} Q^{-\frac{1}{3}} U^{\frac{2}{3}} \quad (27)$$

Increase in the heat transfer coefficient due to increase in U is clearly shown in the above formula. Acierno et al (1971) give some experimentally measured heat transfer coefficients along a fiber. Their results show that K remains a constant for a certain distance and then decrease rapidly. These seemingly anomalous and contradictory observed phenomena may possibly be explained by the present theory. The formula given by Kase and Matsuo (1967) may have been based on the data collected at the early stage of the evolution of the boundary layer where the temperature of the fiber is still far above that of solidification. On the other hand, the measurement given by Acierno et al (1971) may have been collected at the later stage of the development where the temperature is lower to that near solidification. It is concluded that the heat transfer coefficient along the fiber or the film is a property of the flow field. It is not a local property. In mathematical simulation of the process, we must simulate the evolution of the boundary layer simultaneously.

5 ACKNOWLEDGMENTS

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