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Characteristics of the Interface at the Base of the Ocean Mixed Layer

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SUMMARY The region at the base of the wind driven ocean mixed layer appears to be an example of a stable sheared layer where the eddy diffusivity of density is dependent upon a non-dimensional group that characterises inertia , viscous and buoyancy forces. The relationship between this group and eddy diffusivity is determined empirically from two laboratory experiments. Measurements in the Tasman Sea have been used to estimate the value of the non dimensional group at the base of the wind driven mixed layer.

1 INTRODUCTION

At the base of the ocean mixed layer the downward diffusion of heat and momentum is inhibited by a thin region of large stability so that the annual cycle of heating and cooling of the ocean is confined to the top few hundred meters. The stabilizing effect of the oceans on the climate is limited by the insulating nature of this region. It is during the summer months, when heat addition at the sea surface inhibits the occurrence of convective mixing, that erosion of the thermocline appears to be dominated by wind mixing processes. In this paper we are concerned with this situation, where the wind stress at the sea surface produces a mixed layer current which, in turn, creates a shear across the base of the mixed layer.

How should the sheared stable layer at the base of the mixed layer be paramaterized? Kitaigorodski (1960) proposed that the rate of entrainment adjusts so that a fraction of the mean work of the wind stress at the ocean surface balances the work needed to mix heat downward across the base of the mixed layer.

Pollard, Rhines and Thompson (1973) have suggested that diffusion across the interface occurs when the Froude number of the layer rises above unity. The characteristic length scale in the Froude number is the depth of the mixed layer D, the velocity scale is the average velocity of the mixed layer relative to the deeper water while the buoyancy jump, $g\,\frac{\Delta\rho}{\rho}$, completes the dimensionless number. This parameterization has been used in a mixed layer model, Thompson (1976, 1977) without great success. Kato and Phillips (1969) have characterised the mixing in terms of a Froude number using the surface friction velocity in place of the average velocity.

All the above appear to be gross characterizations of the interface whereas scaling variables more closely related to the sheared interface itself seem more relevant. The gradient Richardson number is one such parameter which is fairly successful in describing the change in diffusion coefficients under conditions with low Richardson numbers. One example of such a situation is the experiment of Arya and Plate (1969) in a turbulent boundary layer. The Richardson number was measured within the heated boundary layer of a wind tunnel.

We can define the thermal diffusivity, K, by

$$H = \rho C_p K \frac{\partial T}{\partial z}$$

where H is the heat flux and C_p and ρ are the specific heat and density respectively.

The measurements of Arya and Plate were made in the inner 15% of the boundary layer where the characteristic velocity is $u_* = \sqrt{\frac{\tau}{\rho}}$, τ is the wall stress and the characteristic length is the distance from the wall, Z. Their results are replotted in Fig 1 where the inverse of the thermal eddy diffusivity is plotted against the local gradient Richardson number,

$$R_i = g\frac{\partial\rho}{\partial z} / \left(\frac{\partial U}{\partial z}\right)^2$$

As is well known, the eddy diffusivity decreases with increasing Richardson number.

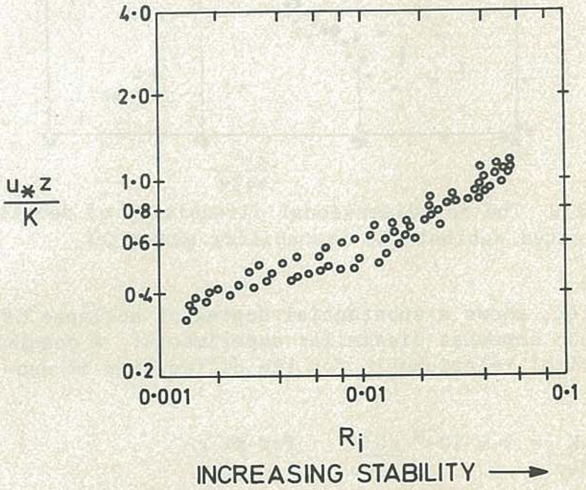

Figure 1 The thermal eddy diffusivity as a function of Richardson number

When the density gradient becomes large, so that the turbulence is almost completely damped, one suspects that the Richardson number may not be the best parameter to characterise the diffusion of heat. In this situation the shear region has been observed (e.g. Lofquist (1960)) to be wave like with occasional instabilities contributing to the mixing. Keulegan, (1949) has suggested that a criterion for the stability of interfacial waves is

$$\frac{U^3\rho}{\nu g\Delta\rho}$$

where ρ and ν are the density and viscosity of one or the other of the fluids and the Reynolds number is above some modest value.

2 DESCRIPTION OF LOFQUIST & MOORE & LONG EXPERIMENTS.

Lofquist (1960) produced a stratified shear flow by having salt water flow under a layer of stationary fresh water. Moore & Long (1971) produced a sheared flow by injecting tangentially either salt water or cooled water into the bottom of a "race track" type water tunnel while fresh water was injected in the other direction at the roof of the tunnel. The diffusivity (of either salt or heat) at the point of maximum density gradient was determined from the tabulated values of "entrainment" presented by both sets of authors.

One requires an appropriate length and velocity scale for the diffusivity and we have chosen the velocity jump across the sheared interface, U, and the length scale $L\rho$ defined as

$$L\rho = \Delta\rho \Big/ \left(\frac{\partial\rho}{\partial z}\right)_{max}$$

The value of $L\rho$ has been tabulated by Lofquist and was approximated by the "distance over which the velocity gradient has a constant value" in the Moore and Long experiment.

All the data from both experiments have been brought together by the interfacial stability parameter in Fig 2 which, despite the considerable

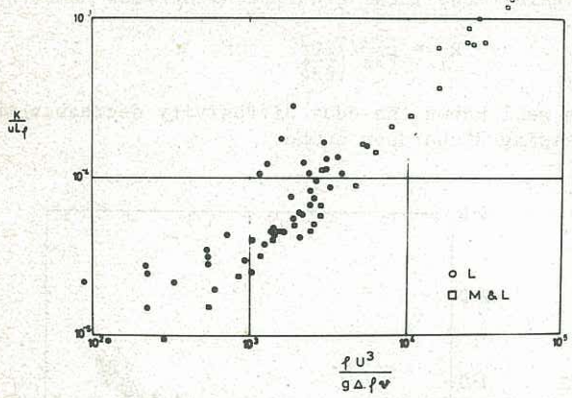

Figure 2 The non dimensional diffusivity of density plotted against the instability parameter

scatter, shows a substantial degree of collapse of the two somewhat dissimilar experiments. A possible empirical relationship for the diffusivity of density is

$$\frac{K}{UL\rho} = 3 \times 10^{-8} \frac{\rho U^3}{g\Delta\rho\nu} \qquad \text{for } K > \kappa$$

where κ is the molecular diffusivity appropriate to the phenomena causing the density gradient. The above relationship must be restricted to values of instability number that are not too large, since for a neutral density shear layer, Fiedler (1974) found that

$$\frac{K}{UL\rho} = .02$$

When we attempted to collapse the two experiments on gradient Richardson number, there was a substantial discrepancy between the two sets of data mainly because the characteristic length scales of the two experiments were different. The instability parameter has no characteristic length, being the ratio of the square of inertia forces to the product of the damping forces due to viscosity and buoyancy. If the eddy diffusivity is substantially greater than the molecular value, then the viscous forces are not important. Thus the non dimensional group $U^3\rho/g\Delta\rho\nu$, will not be a very appropriate scaling parameter and one would expect the Richardson number, ie the ratio of inertia forces to buoyancy forces, to be more appropriate.

There must be a transition for shear flows between the Richardson number region where the diffusivity of density is near the value of $.02UL\rho$ and the instability parameter region where eddy diffusivity is near the molecular value. In order to clarify this point appropriate measurements in shear layers at near neutral stabilities are required.

3 THE BASE OF THE OCEAN MIXED LAYER

Which parameter do we expect to be the most appropriate to the shear layer at the base of the ocean mixed layer? To estimate the non dimensional group $U^3\rho/g\Delta\rho\nu$ one needs to know the velocity and density jump at the base of the surface layer.

For the velocity jump we will use (for lack of direct measurements) the value of the Ekman drift predicted from the measured wind speed. We assume that either, the wind has been blowing for a long time compared with the inertial period or that the inertial oscillations of the layer are of the order of the Ekman drift.

The wind drift of our layer is

$$U_E = \frac{\tau}{\rho f D}$$

where τ is the surface wind stress
D is the mixed layer depth
and f is the Coriolus parameter (10^{-4}sec^{-1})

The surface stress is related to the wind by the expression

$$\tau = \rho_a C_{10} V_{10}^2$$

where a value of $C_{10} = 1.5 \times 10^{-3}$ was used with the 10 meter wind speed, V_{10}. For the density jump we will use measured temperature profiles and assume that density change can be calculated with the aid of the coefficient of expansion of water.

An initial survey of the thermal structure of the upper ocean was made for this purpose from HMAS KIMBLA over a 200km square box centered near 152°E, 36.5°S. This survey suggested that there was a large mesoscale eddy present, a cross section through which is shown in Fig 3. All the measurements reported were within this somewhat circular eddy so that an approximation to horizontal homogeneity was achieved.

Temperature traces were made from expendible bathythermograph (XBT) probes manufactured by Plessy. The manufacturer states that the accuracy is ±.2°C but one feels confident that greater resolution than this is achieved. The time constant of the thermistor used is stated to be 100 msec and the rate of fall of the standard probe about 6.5 m/sec, which gives a vertical resolution of 65 cm. In order to

TABLE 1

OCEAN MIXED LAYER MEASUREMENTS

Time	Depth	Length	Density	Velocity	Instability Parameter	Richardson Number
	D	$L\rho$	$\frac{g\Delta\rho}{\rho}$	U_E	$\frac{U^3\rho}{g\Delta\rho\nu}$	$\frac{g\Delta\rho L\rho}{\rho\Delta U^2}$
Zone K	m	m	cm/sec^2	cm/sec		
25 1530	55	3	.14	.21	7	950
25 1630	50	1.5	.04	.24	35	105
25 1730	55	2.0	.04	.21	23	180
25 1830	53	1.6	.10	.22	10	330
25 1930	55	1.7	.12	.21	7	460
25 2130	65	3.0	.26	.36	18	600
26 0930	70	2.0	.20	.43	39	220
26 1030	63	0.5	.08	.48	138	17
26 1130	68	3.0	.20	.45	45	300
26 1230	69	3.0	.18	.44	47	280
26 1330	71	2.5	.12	.43	67	160
26 1430	75	1.5	.04	.40	160	38
26 1530	73	1.0	.06	.42	124	34
26 1630	75	3.0	.20	.40	32	370
26 1730	72	2.0	.08	.67	380	35
26 1830	65	*	*	.73		
26 1930	65	1.0	.08	.73	490	15
27 0030	55	1.0	.10	1.2	1720	7
27 0130	65	2.0	.12	1.4	2300	12
27 0230	50		.16	1.8	3640	
27 0330	68	4.0	.30	4.0	21300	7
27 0430	75	3.0	.28	3.6	16700	
27 0530	72		.20	3.8	27500	
27 0730	60	5.0	.20	2.0	4000	25
27 0830	60	3.0	.24	1.5	1410	30
27 0930	63	3.0	.26	1.5	1300	35
27 1030	67	3.0	.14	1.4	1970	21
27 1130	75	4.0	.20	1.2	860	51
27 1230	72	3.0	.26	1.3	840	46
27 1330	55	4.0	.14	1.7	3500	19

*unable to define for this trace

ensure that this resolution was adequate some probes were modified to reduce their fall rate to about 1.3 m/sec and so increase the vertical resolution to 13 cm.

Two high resolution XBT traces are reproduced in Fig 4 to show the estimate of the temperature jump (which was the variation over the depth in which the gradient of temperature had dropped to half its maximum value). The length scale $L\rho$ was also estimated with the aid of this maximum gradient. There was considerable subjectivity in these estimates, only a few of the temperature profiles being as simple as that in Fig 4b. One is looking for some kind of "temporal average" of temperature jump and $L\rho$ for comparison with the laboratory experiments. If the spatial resolution of our sensor had been greater both the temperature jump and the length $L\rho$, as defined, may well have become smaller as we would have measured the gradients of "turbulent" fluctuations (if any were present). Since the resolution was about 10 times $L\rho$ for the slow XBT probes, the "average" value of $L\rho$ should not be greatly overestimated.

Thirty one XBT profiles, all at least an hour apart, were analysed and the value of the interface

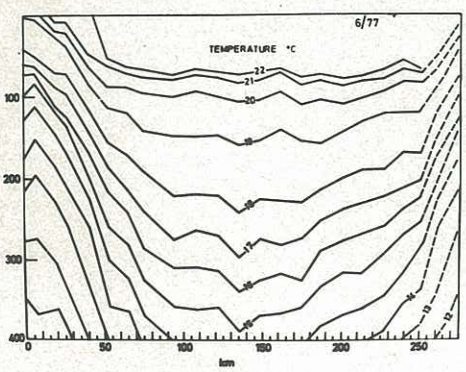

Figure 3 Temperature cross section thru the mesoscale eddy. The dotted isothermals are conjecture.

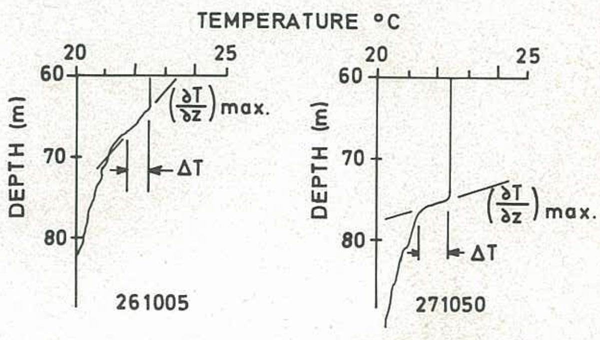

Figure 4 Two high resolution SBT traces showing the definitions of $L\rho$ and ΔT

stability parameter, as well as the Richardson number based on the length $L\rho$, is presented in Table 1. During these periods the research ship was within the homogeneous portion of the eddy and the wind speed varied from 2 m/sec to 12 m/sec. These estimated stability numbers and Richardson numbers must be considered only order of magnitude because of the large number of assumptions and approximations used. The most questionable assumption is that all the velocity difference between the ocean mixed layer and the deeper water occurs across the first stable region below the mixed layer. To obtain a resolution of this question direct measurement of the velocities are required.

The maximum value of the interface stability parameter was 3×10^4 while the lowest value of the Richardson number was 7. These order of magnitude results appear to place the base of the mixed layer in the situation where molecular influences should be taken into account. Thus, for our present ocean data, the parameter $U^3\rho/g\Delta\rho\nu$ appears to be more appropriate in describing the diffusivity of density than the Richardson number.

The values of the interfacial stability parameter and the Richardson number in Table 1 appear likely to be typical for moderate latitudes and reasonable wind stresses. We can say very little at present about high wind stress situations.

4 CONCLUSIONS

The diffusion of density across a stable layer between two streams moving with different velocities, when very stable, appears to depend upon a dimensionless group that characterises the instability of interfacial waves. This dimensionless group is the ratio of the square of the inertia forces to the product of buoyancy and viscous forces. Order of magnitude estimates of the base of the ocean mixed layer suggest that in many situations diffusion at the bottom of the wind mixed layer will be more accurately parametrised in terms of the instability parameter than in terms of Richardson number.

5 ACKNOWLEDGEMENTS

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