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A Quasi-Three-Dimensional Design of Axial Flow Compressor Blades by Use of Cascade Data

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SUMMARY This paper includes a method how to use experimental, two-dimensional cascade data for a quasi-three-dimensional design of axial flow compressor blades. A theoretical analysis has been performed by a suitable transformation of stream surface into a two-dimensional plane. Two parameters, which can be estimated from a through flow problem, are introduced to represent inclination of the stream surface and variation of axial velocity through the rotor. With these parameters, a camber and a stagger which are selected from the cascade data can be corrected easily for the stream surface inclination and the axial velocity variation.

1 INTRODUCTION

As increasing the pressure ratio of an axial flow compressor, a more reliable calculation is requested to the design of blade rows in which the annulus wall has frequently significant inclination to the axial direction and the axial velocity changes through the rotor. A method of the quasi-three-dimensional analysis proposed by Wu (1952) can be applied to such a design problem as well as a direct problem. A through flow in a meridional plane is determined by the streamline curvature method (Smith, 1966 or Novak, 1967) or the matrix method (Marsh, 1966), providing flow rate and specified enthalpy and loss distribution in the radial direction. A blade element is selected on an average stream surface which is formed by revolution of a meridional streamline about the rotating axis. Some calculating methods of the quasi-three-dimensional cascade flow (Katsanis, 1965, Wilkinson, 1970, Braembussche, 1973) may be available for the blade selection if iterating procedure is carried out by a big computer. As well known, however, the use of experimental cascade data for the retarded blade rows enables to work out a more reliable design than a theoretical calculation. Unfortunately, most of available cascade data are obtained in the two-dimensional flow and the use of them has been restricted to the case when the average stream surface is regarded as a cylindrical one. An expedient method how to use the two-dimensional cascade data in the case of inclined stream surface and varying axial velocity has been presented in this paper.

2 NOTATION

b : minute thickness of average stream surface (Fig.1)
c_{l0} : camber of NACA-65 series profile
f : circulation parameter ($=\tan\beta_1 - \tan\beta_2$)
H : total enthalpy
l : chord length
m : distance along meridional streamline
q_b : bound source on blade chord
r : radius from axis of rotation
s : co-ordinate along blade surface
t : spacing of cascade
u : speed of blade
v : velocity induced by μ and ζ
w : relative fluid velocity
x : distance along chordwise direction
X : axial coordinate in the transformed plane
y : distance normal to blade chord
y_c : y-coordinate of camber line
y_d : a half of blade thickness
Y : tangential coordinate in the transformed plane
z : distance along axial direction
β : relative flow angle
γ_b : bound vortices on blade surface or blade chord
ζ : distributed vortices on the transformed plane
θ : angular coordinate about axis of rotation
μ : distributed sources on the transformed plane
ν : stagger angle
ξ : axial-velocity variation parameter (AVP)
ρ : density of fluid
σ : solidity of cascade
ϕ : camber angle
Φ : local flow rate coefficient
χ : streamline inclination parameter (SIP)
ψ : stream function
Ψ : local enthalpy rise coefficient
ω : angular velocity

Subscript, Superscript and Prefix

b : bound vortices and sources
m : component in meridional direction
x : chordwise component
X : axial component on the transformed plane
y : component normal to chordwise direction
Y : tangential component of the transformed plane
ζ : distributed vortices on the transformed plane
μ : distributed sources on the transformed plane
θ : peripheral component
$_0$: cascade data in the case of $\mu=\zeta=0$
$_1$: inlet of cascade
$_2$: exit of cascade
∞ : vector mean velocity
$*$: reference radius
$-$: average
Δ : variation owing to streamline inclination and/or axial velocity change

3 ANALYSIS

In a quasi-three-dimensional design of an axial-flow compressor, meridional and peripheral velocities w_m and w_θ can be evaluated as the functions of radius r and the meridional streamlines are determined by a solution of through-flow problem, providing that the distributions of enthalpy $H(r)$ are specified before and behind the stages. Then, a blade-to-blade problem is to be solved on each average stream surface which is obtained by revolving the meridional streamline about the axis of rotation (See Fig.1). For the convenience of calculation, a local flow rate coefficient Φ and a local enthalpy rise coefficient Ψ are defined on each stream surface as follows:

$$\Phi = (r_1 w_{m1} + r_2 w_{m2})/(2r^* u^*) \tag{1}$$

$$\Psi = (H_2 - H_1)/(u^{*2}/2)$$
$$= \{u_2(u_2 + w_{\theta 2}) - u_1(u_1 + w_{\theta 1})\}/(u^{*2}/2) \tag{2}$$

For steady, isentropic flow on the surface, the differential equation is given by (Vavra 1961)

$$\frac{\partial^2\psi}{\partial m^2} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \{\frac{1}{r}\frac{\partial r}{\partial m} - \frac{1}{b\rho}\frac{\partial(b\rho)}{\partial m}\}\frac{\partial\psi}{\partial m}$$
$$- \frac{1}{\rho}\frac{\partial\rho}{r\partial\theta}\frac{\partial\psi}{r\partial\theta} = -2\omega b\rho\frac{\partial r}{\partial m} \tag{3}$$

where the stream function ψ is related to the relative velocity as

$$w_\theta = (1/b\rho)(\partial\psi/\partial m), \quad w_m = -(1/b\rho)(\partial\psi/r\partial\theta) \tag{4}$$

With the relations of

$$dX/dm = r^*/r, \quad dY/rd\theta = -r^*/r \tag{5}$$

(3), (4), (1) and (2) are transformed conformally into the XY plane (Fig.3) as follows:

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} = -2\omega b\rho(r/r^*)^2\frac{dr}{dm}$$
$$+ \frac{1}{b\rho}\{\frac{\partial(b\rho)}{\partial X}\frac{\partial\psi}{\partial X} + \frac{\partial(b\rho)}{\partial Y}\frac{\partial\psi}{\partial Y}\} \tag{6}$$

$$w_X = (r/r^*)w_m, \quad w_Y = -(r/r^*)w_\theta \tag{7}$$

$$\Phi = w_{\infty X}/u^* \tag{8}$$

$$\Psi = 2\Phi(f+\chi) \tag{9}$$

where, $w_{\infty X}$ $(=w_{X\infty}) = (w_{X1}+w_{X2})/2$, $f = (w_{Y1}-w_{Y2})/w_{\infty X}$ and $\chi = \{(r_2/r^*)^2 - (r_1/r^*)^2\}/\Phi$.

If the right hand side of (6) vanishes, the equation of motion is identical to that for irrotational, incompressible, two-dimensional cascade, and the blade element can be selected by use of cascade data so as to satisfy the equivalent velocity diagram as shown by dash-dotted lines in Fig.2. In practical cascade, however, the right-hand side is not zero. Therefore the value of f in such cascade may differ from that for the two-dimensional cascade. This effect can be evaluated by replacing the distributed vortices $\zeta(X)$ for the first term and sources $\mu(X,Y)$ for the second term

$$\zeta(X) = 2\omega(r/r^*)^2(dr/dm) \tag{10}$$

$$\mu(X,Y) = -\frac{1}{(b\rho)^2}\{\frac{\partial(b\rho)}{\partial X}\frac{\partial\psi}{\partial Y} - \frac{\partial(b\rho)}{\partial Y}\frac{\partial\psi}{\partial X}\} \tag{11}$$

According to a method of singularity, a flow through a cascade is built up by placing bound vortices γ_b along the blade surface. Taking a circulation along a circuit indicated by a dash-dotted line in Fig.3, the circulation parameter f is written by

$$f = \frac{w_{Y1}-w_{Y2}}{w_{\infty X}} = \frac{1}{tw_{\infty X}}\oint\gamma_b ds - \frac{1}{w_{\infty X}}\int_{X_1}^{X_2}\zeta dX \tag{12}$$

As the bound vortices γ_{b0}, which denote the bound vortices in the case of $\zeta=\mu=0$, change to $\gamma_{b0}+\Delta\gamma_{b\zeta}$ when $\zeta\neq 0$, $\mu=0$ and to $\gamma_{b0}+\Delta\gamma_{b\mu}$ when $\zeta=0$, $\mu\neq 0$ in order to satisfy the boundary condition on the blade surface, the first term of the right-hand side of (12) can be represented as $f_0+\Delta f_\zeta+\Delta f_\mu$ where f_0, Δf_ζ and Δf_μ correspond to γ_{b0}, $\Delta\gamma_\zeta$ and $\Delta\gamma_\mu$ respectively. The second term becomes χ by integration after substituting (5) and (10). Then, it follows that

$$f = f_0 + \Delta f_\zeta + \Delta f_\mu - \chi \tag{13}$$

Thus, the circulation parameter is to be different from f by $\Delta f_\zeta+\Delta f_\mu-\chi$, if the design camber ϕ_0 and stagger ν_0 are selected from the cascade data so as to satisfy the equivalent velocity diagram in Fig.2. Therefore, the camber and stagger should be corrected by $\Delta\phi$ and $\Delta\nu$ so as to hold the following relation

$$f = f_0(\phi_0+\Delta\phi, \nu_0+\Delta\nu) + \Delta f_\zeta(\phi_0+\Delta\phi, \nu_0+\Delta\nu)$$
$$+ \Delta f_\mu(\phi_0+\Delta\phi, \nu_0+\Delta\nu) - \chi \tag{14}$$

Once corrected geometry of cascade is obtained by (14), it should be transformed into $m-\overline{r\theta}$ surface by use of (5).

However, it may be advisable to transform $z-\overline{r^*\theta}$ surface when the influence of sweep in blades should be considered. Smith and Yeh (1963) proved that the correct cascade performances for the cascade lying on the $m-\overline{r\theta}$ surface were obtained by analysing the flow past a projected view of the cascade looking along the local bound vortices γ_b at the actual flow on the $m-\overline{r\theta}$ surface. This conclusion is certainly self-evident for untwisted linear cascade of infinite span and implies a technique for proceeding

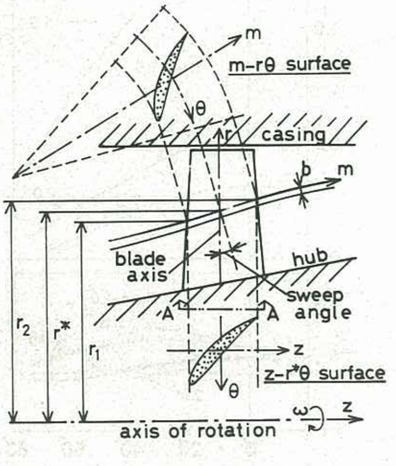

Figure 1 Meridional plane and average stream surface

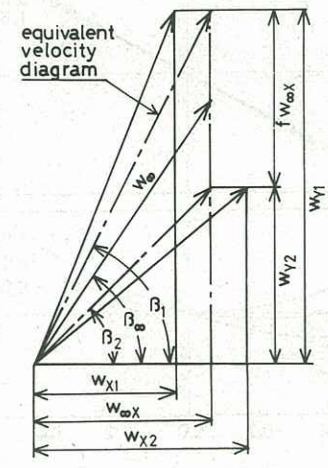

Figure 2 Velocity diagram on the XY plane

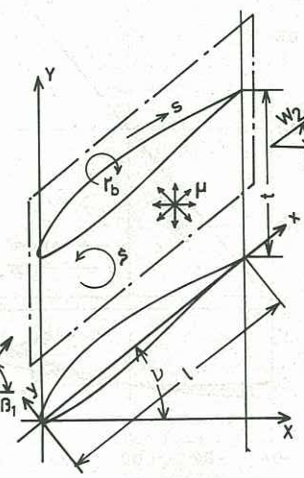

Figure 3 Flow field on the XY plane

even with cascade of blades twisted about a center line. Since the direction of blade axis is usually perpendicular to the rotating axis, the directions of vector $\vec{\gamma}_b$ are approximately identical to the radial direction. Therefore, the projection AA in Fig.1 should be analysed. For this reason, dm, $rd\theta$, w_m and w_θ in (4), (5) and (7) should be replaced by dz, $r^*d\theta$, w_z and $(r^*/r)w_\theta$ respectively when the sweep effect is taken into account.

4 CALCULATION OF CORRECTING VALUES

In a direct problem (prediction of performance), Δf_ζ can be evaluated exactly by solving Poisson's equation. But it is warrantable to obtain an approximate solution by replacing averaged vortices $\overline{\zeta}$ for ζ as the inclination of stream surface is not large in a conventional axial-flow machine (Inoue-Mori, 1971). On the otherhand, it is difficult to get an exact solution for Δf_μ since μ includes an unknown function ψ as shown in (11). Several approximate solutions have been proposed(Mani-Accosta, 1968, Shaalan-Horlock, 1968) and it may be reasonable to evaluate Δf_μ by assuming uniform distribution of source $\overline{\mu}$ (Kubota, 1959) when the variation of axial velocity is small.

In the present method, therefore, the solutions for the uniformly distributions of $\overline{\zeta}$ and $\overline{\mu}$ are applied to the design problem, since $\overline{\zeta}$ and $\overline{\mu}$ are given by the through-flow solution.

$$\overline{\zeta} = \frac{1}{X_2-X_1}\int_{X_1}^{X_2}\zeta dX = \frac{u^*}{X_2-X_1}\,\frac{r_2^2-r_1^2}{r^{*2}} \tag{15}$$

$$\overline{\mu} = (w_{X2} - w_{X1})/(X_2 - X_1) \tag{16}$$

Now, the following dimensionless parameters are defined as 'streamline inclination parameter (SIP)' χ and 'axial-velocity variation parameter (AVP)' ξ in relation to (15) and (16).

$$\chi = \overline{\zeta}(X_2-X_1)/w_{\infty X} = \{(r_2/r^*)^2 - (r_1/r^*)^2\}/\Phi \tag{17}$$

$$\xi = \overline{\mu}(X_2-X_1)/w_{\infty X} = (r_2 w_{m2} - r_1 w_{m1})/(r^* u^* \Phi) \tag{18}$$

Of course, the former is the same definition in (9), (13) and (14).

In order to solve the equation (14), Schlichting's three-terms method is adopted, in which a blade is replaced by bound vortices and sources along the chord line represented by 3-terms Glauert series (Scholz, 1965). The coefficients of Glauert series are decided so that the flow field induced by the bound vortices and sources satisfy the boundary conditions at three points on the blade chord. Here, velocities induced by the uniformly distributed vortices and sources should be superposed on the flowfield. Now, consider another Cartesian co-ordinate system in which the x-axis is taken in the chordwise direction as shown in Fig.3. On the blade chord, the x and y components of the induced velocities by $\overline{\zeta}$ and $\overline{\mu}$ can be represented as

$$\begin{aligned} v_{\zeta x}/w_{\infty x} &= \chi(x/l)\tan\nu/(1+\tan\nu\tan\beta_\infty)\\ v_{\zeta y}/w_{\infty y} &= \chi(x/l)/(1+\tan\nu\tan\beta_\infty)\\ v_{\mu x}/w_{\infty x} &= \xi(x/l)/(1+\tan\nu\tan\beta_\infty)\\ w_{\mu y}/w_{\infty y} &= -\xi(x/l)\tan\nu/(1+\tan\nu\tan\beta_\infty) \end{aligned} \tag{19}$$

Thus, the boundary conditions become as

$$\begin{aligned} \frac{dy_c}{dx} &= \frac{w_{\infty y}+w_{by}+v_{\zeta y}+v_{\mu y}}{w_{\infty x}+w_{bx}+v_{\zeta x}+v_{\mu x}}\\ \frac{dy_d}{dx} &= \frac{q_b}{w_{\infty x}+w_{bx}+v_{\zeta x}+v_{\mu x}} \end{aligned} \tag{20}$$

where y_c and y_d denote the y-coordinates of camber line and a half of the blade thickness; $w_{\infty x}$ and $w_{\infty y}$ are the x and y components of the vector meam velocity, and w_{bx} and w_{by} are the x and y components of velocities induced by the bound vortices and sources which are linear functions of the coefficients of Grauert series.

Firstly, the inlet flow angle β_1^* and the turning angle $\Delta\beta^*$ at impact-free condition are calculated for the cascade geometry (ϕ_0, ν_0) and $\chi = \xi = 0$. Next, adding χ and ξ, the camber angle ϕ and stagger ν are so varied as to keep the impact-free inlet and turning angles equal to the initial values by iterating calculations. A procedure to get rapid convergence has been presented (Ikui et al, 1977).

5 EXAMPLES OF CALCULATION

The above-described method is to be applied to the case when the cascade data have been prepared. In this study, thereupon, NACA 65-series compressor blades have been selected as calculating examples, since massive systematic data are widely used in practical design (Herrig et al., 1951). In these blades, ϕ is replaced by an equivalent camber angle which is defined as the center angle of a circular-

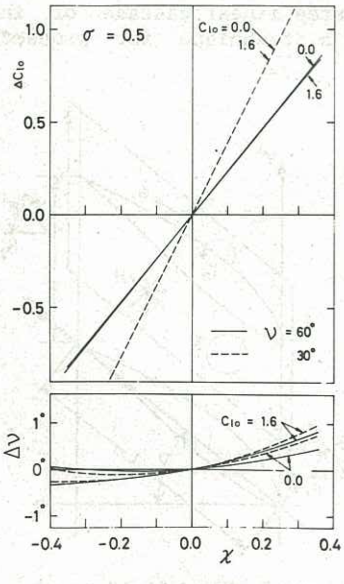

Figure 4 Correction for SIP

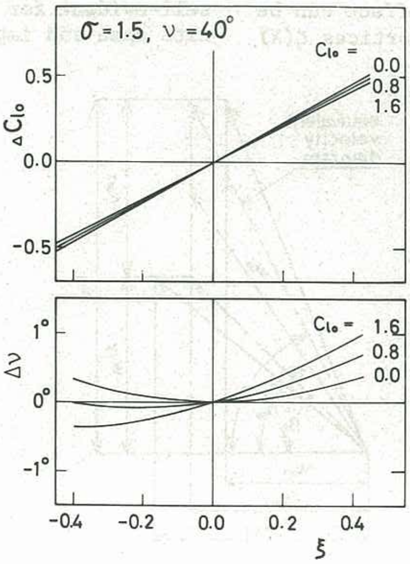

Figure 5 Correction for AVP

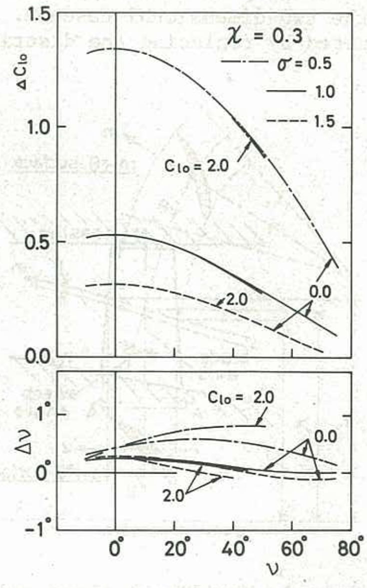

Figure 6 Correction for SIP=0.3

arc passing through the leading and trailing edges and the point of maximum camber at the mid-chord location. Hence, with the relation of

$$\phi = 4\tan^{-1}(0.1103\, c_{lo})$$

the correcting values of ϕ can be converted to those of c_{lo}.

Corrections of design camber and stagger are shown in Figs.4 to 8 for various conditions. Figs.4 to 7 indicates the correction when the design condition is affected by SIP or AVP individually. It is found that the correction of camber is significant since a variation of Δc_{lo}=0.04 corresponds to a difference of $\Delta\phi = 1°$. On the other hand, the values of $\Delta\nu$ are less than 1° in most cases. Δc_{lo} increases linearly with SIP and AVP (Figs.4 and 5). For positive SIP, Δc_{lo} reduces with solidity σ and stagger ν, but it changes little with camber c_{lo} (Fig.6). As to correction for positive AVP, Δc_{lo} increases with stagger ν except in the high stagger region, but changes little with solidity σ (Fig.7).

Fig.8 shows the modification of design camber and stagger in the case when both SIP and AVP must be considered simultaneously in the design. Dash lines indicate the sum of two-correcting values, of which one is calculated by considering SIP only and another AVP only. Namely, the reasonable correction may be obtained by the linear addition, when SIP and AVP are not so large (less than about 0.3).

Considering the above fact and the fact Δc_{lo} and $\Delta\nu$ have the linear relations with χ and ξ(Figs.4 and 5), the following relations are available.

$$\Delta c_{lo} = \frac{\partial(c_{lo})}{\partial\chi}\chi + \frac{\partial(c_{lo})}{\partial\xi}\xi$$
$$\Delta\nu = \frac{\partial\nu}{\partial\chi}\chi + \frac{\partial\nu}{\partial\xi}\xi \qquad (21)$$

If the gradients $\partial(c_{lo})/\partial\chi$, etc. are calculated beforehand for wide range of cascade geometry and presented in the form of the correction charts, it must be more convenient for the design. They have been prepared by the authors.

6 CONCLUSION

In this study, a quasi-three-dimensional design of axial flow compressor blades by use of cascade data is presented. And the method is given how to correct the camber and the stagger which are both selected from the two-dimensional cascade data, by considering the variation of the streamline inclination and the axial-velocity, before and behind the rotor. The authors have the correcting diagrams for NACA 65-series compressor blades.

7 ACKNOWLEDGEMENT

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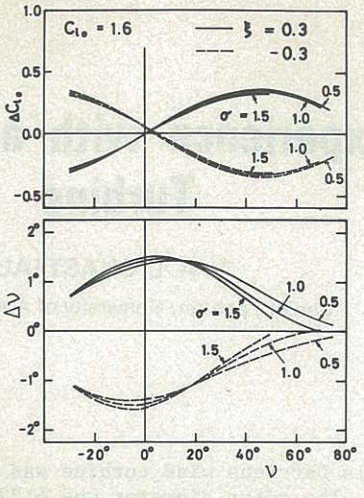

Figure 7 Correction for AVP = ± 0.3

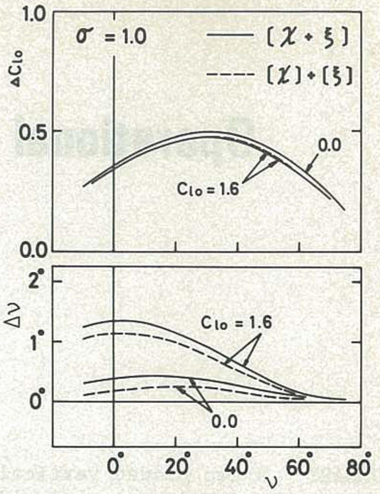

Figure 8 Correction for SIP = 0.2 and AVP = 0.2

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