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Unconfined Groundwater Recharge with Clogging

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SUMMARY Recharge with clogging from an infinitely long strip basin to a finite aquifer and the decay of the groundwater mound in such a domain is solved for both linear and non-linear Dupuit-Forchheimer theories. The comparison between the theoretical results and experimental data in the non-clogging case suggests that the non-linear model is more accurate when the time and initial infiltration rate are large. Investigation is also made in the optimal period over which the basin should operate in each recharge cycle. The results indicate that under such a recharge condition a maximum average rise in groundwater level is produced.

1 INTRODUCTION

The maintaining or raising of groundwater levels by artificial recharge provides a source of water for irrigation, industrial and domestic use and can also retard seawater encroachment in coastal areas. For confined aquifers artificial recharge is normally injected through wells whereas for unconfined aquifers it is usually more efficient to use surface water spreading through channels and pits. Only the unconfined situation is considered here.

In practice the artificial recharge process invariably results in clogging of the channel or pit due to air entrainment, the presence of suspended material, the growth of micro-organisms, or the chemical incompatability of the recharge water with the aquifer environment. As it poses a major problem in recharge operations, the nature of the problem and possible alleviation of its effects have been the subjects of previous investigations (Ripley and Saleem (1973) and Goss et al. (1973)). Clogging due specifically to suspended sediments, normally encountered when using floodwater for recharge, lends itself to simple mathematical formulation, and will be considered in this paper.

Analytical studies of the development of groundwater mounds in unconfined aquifers have been carried out by Dagan (1967), Hunt (1970), and Amar (1975a), in most cases involving some assumptions and simplifications. Numerical solutions have been proposed by Todsen (1971), Amar (1975a, 1975b). A comprehensive literature survey in both areas is given by Amar (1973).

The work described in this paper couples the model of clogging due to suspended solids with the one-dimensional model of an infinitely long strip recharge basin in a finite unconfined aquifer. The methods used for solving this problem include both linear and non-linear Dupuit-Forchheimer theory. The results in the non-clogging case are compared with the experimental data of Brock and Amar (1974).

2 NOTATION

a	Initial water table height
B	Aquifer width, $B' = B/L$
H	Elevation of water table above impervious stratum = $s + a$
K	Hydraulic conductivity
L	Basin width
P	Infiltration rate
P_o	Initial Infiltration rate, $p' = P_o L^2/Ka^2$
s	Elevation above initial water table, $s'' = s/ap'$
$\hat{s}''$	Average of s'' over the aquifer width during the recharge cycle
t	Time, $t' = tKa/\theta L^2$
t_r	Restoration time
t_u	Time at which recharge stops
x	Horizontal abscissa, $x' = x/L$
	All symbols with (') or (") are dimensionless
α	Clogging coefficient
β	$\alpha\nu\eta$
δ	Clogging parameter = $\beta\theta/\phi Ka$
η	Fraction of clogging substance retained on soil surface
θ	Porosity
ν	Concentration of clogging substance in the water
ϕ	Dummy variable = $(\pi/2B')^2$

3 THEORETICAL DEVELOPMENT

The assumptions are:

(i) The flow conforms with the Dupuit-Forchheimer approximation; and

(ii) The clogging substances produce a time-variant infiltration rate but do not alter the hydraulic properties (hydraulic conductivity and porosity) of the aquifer.

Berend (1970) derived the following equation to describe reduction of infiltration rate due to clogging based on observation and an assumption that clogging occurs through suspended solids in the water

$$P = P_o e^{-\alpha\nu\eta t} \qquad (1)$$

According to Dupuit-Forchheimer Theory the physical problem, schematically defined in Figure 1, is governed by the non-linear parabolic partial differential equation (Brock and Amar, 1974)

$$\frac{\partial}{\partial x}\left(H\frac{\partial H}{\partial x}\right) = \frac{\theta}{K}\frac{\partial H}{\partial t} - \frac{P(x,t)}{K} \qquad (2)$$

which is subjected to the following initial and boundary conditions:

$$P(x,t) = P_o\, e^{-\beta t}, \quad x \leqslant L \qquad (3)$$

$$P(x,t) = 0, \quad x > L \qquad (4)$$

$$H(x,o) = a \qquad (5)$$

$$H(B,t) = H(-B,t) = a \qquad (6)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=0} = 0 \qquad (7)$$

Linear Dupuit-Forchheimer model - (2) has no general analytical solution, but by writing H = s + a, and assuming that s/a << 1, it can be rewritten as

$$\frac{\partial s}{\partial t} = \frac{Ka}{\theta}\frac{\partial^2 s}{\partial x^2} + \frac{P(x,t)}{\theta}, \qquad (8)$$

which has the following non-dimensionalised solution:

$$s''(x',t') = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi\phi}\sin\left(\frac{2n-1}{2B'}\pi\right)\cos\left(\frac{2n-1}{2B'}\pi x'\right).$$

$$\left[\frac{e^{-\delta\phi t'} - e^{-(2n-1)^2\phi t'}}{(2n-1)^2 - \delta}\right] \qquad (9)$$

obtained by eigenfunction expansion.

At time t'_u when the recharge operation is stopped the solution for the decay of the groundwater mound from this point is obtained by separation of variables and given by

$$s''_D(x',t')\Big|_{t'>t'_u} = \sum_{n=1}^{\infty} s''_n(x',t'_u)\cdot e^{-(2n-1)^2\phi(t'-t'_u)} \qquad (10)$$

where s''_n is the n-th term of (9).

Non-linear Dupuit-Forchheimer model - Another solution to (2) is obtained by a numerical technique in which (2) is rewritten in dimensionless form as

$$\frac{\partial s''}{\partial t'} = \frac{\partial}{\partial x'}\left[(1 + p's'')\frac{\partial s''}{\partial x'}\right] + G\, e^{-\delta\phi t'} \qquad (11)$$

where $G = 1$, $x' \leqslant 1$
$= 0$, $x' > 1$
$= 0$ for all x' when recharge operation ceases.

The solution of the above non-linear parabolic partial differential equation in the case $\delta = 0$ is given by Amar (1973, 1975a) using explicit and 3-level implicit finite-difference schemes. For the general equation, details can be obtained from Mitchell (1969). In this paper the numerical solutions are derived from the 3-level implicit scheme shown by Douglas (1959) to be extremely stable with respect to round-off error.

Optimal Operation Period under Clogging Conditions- Under usual economic and operational conditions, where clogging is inevitable, a period of flooding of the basin t'_u, must be followed by a period of t'_r, for restoring infiltration to the initial rate during the interruption of recharge. The value of t'_r is generally independent of the flooding period and is generally known. Based on the concept of maximising the total amount of water percolating into the soil from the recharge basin, Berend (1970) derived the following implicit relationship for the optimal t'_u

$$e^{-\delta\phi t'_u} = 1/[\delta\phi(t'_u + t'_r) + 1] \qquad (12)$$

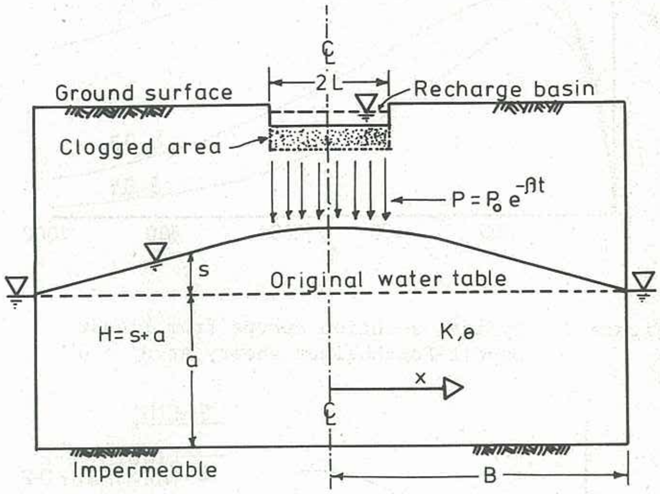

Figure 1 A groundwater mound in a finite domain beneath a strip recharge basin

4 RESULTS AND DISCUSSION

Typical solution curves for various δ's at x' = 0 are plotted in Figure 2. When δ is non-zero the rise of the groundwater mound reaches its peak at different times depending on the magnitude of δ.

The growth and decay of a groundwater mound is depicted in Figures 3 and 4 which typify the rise and fall at x' = 0 and along the aquifer body respectively. From Figure 4 one can see that the interruption of recharge causes the mound to be spread over the whole aquifer width.

Results from the non-linear Dupuit-Forchheimer model have been compared with those from the linear model.

In most cases, they agree quite well when t' is small, when the assumption s/a << 1 is valid and the non-linearity is not apparent.

An examination of Figures 5 and 6 reveals that the discrepancy is greater under the basin than at a distance from the basin, and progressively increases with increasing t'. However, the degree of correspondence improves as δ increases.

As indicated by Amar (1975a) for the non-clogging case, the solution curves are also influenced by p' - the smaller the magnitude of p', the smaller the deviation of non-linear D-F solution curve from the linear D-F solution curve. The results shown in Figures 5 and 7 suggest that this is also true in the clogging case.

Brock and Amar's (1974) experimental curves in the non-clogging case are reproduced in Figures 5 to 7, from which the improvement in the correspondence at low p' is noted.

Comparison is also made in the case of rise and decay occurring due to the interruption of recharge in Figures 3 and 4. The development and decay at the mid-point is shown in Figure 3, and lateral profiles in Figure 4. In general non-linear D-F theory predicts a flatter groundwater mound than that forecast by the linear D-F theory.

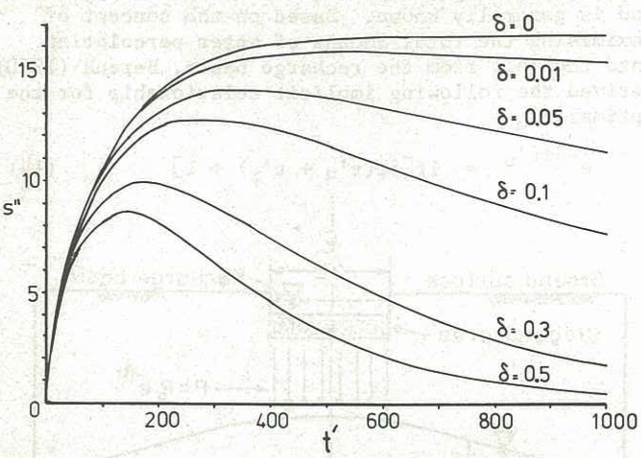

Figure 2 Typical solution curves from linear Dupuit-Forchheimer theory at x' = 0

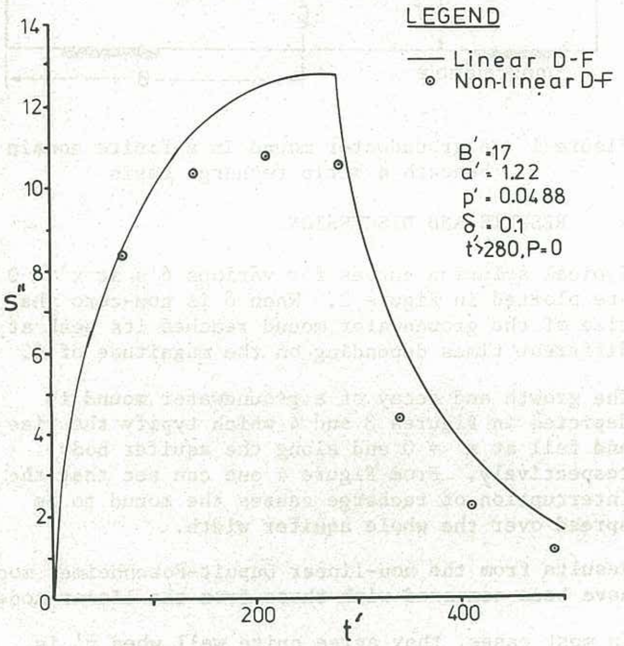

Figure 3 Growth of a groundwater mound due to recharge and decay due to interruption of recharge at x' = 0

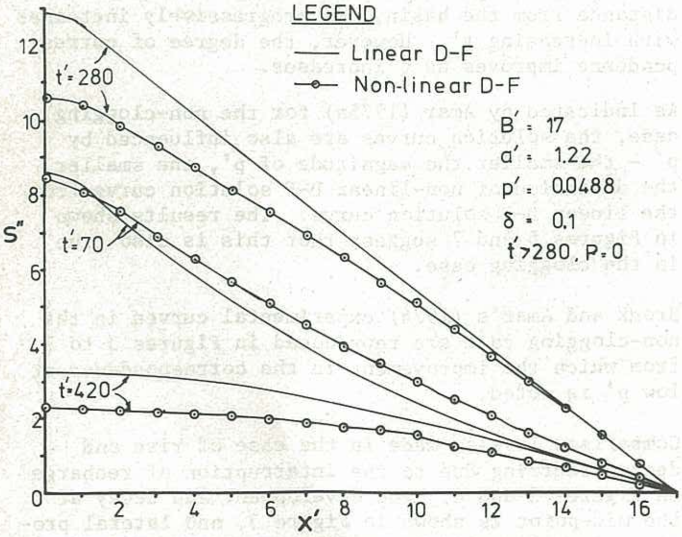

Figure 4 Growth of a groundwater mound due to recharge and decay due to interruption of recharge along the aquifer width

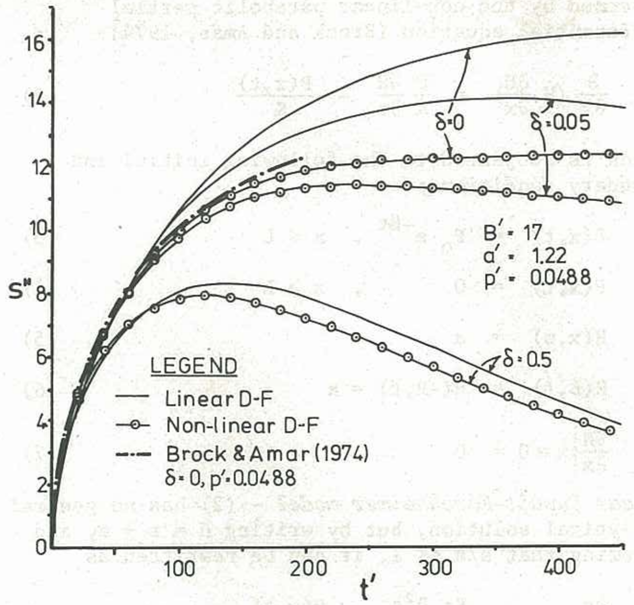

Figure 5 Comparison between linear and non-linear models at x' = 0.1 (p' = 0.0488)

See Fig. 5 for other details.

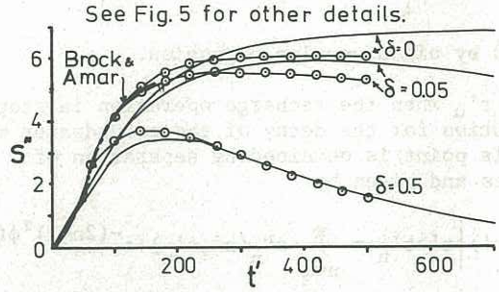

Figure 6 Comparison between linear and non-linear models at x' = 10.2 (p' = 0.0488)

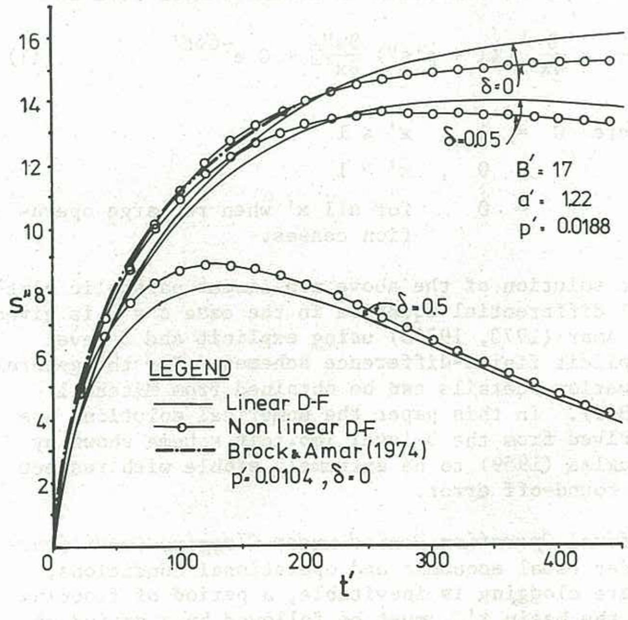

Figure 7 Comparison between linear and non-linear models at x' = 0.1 (p' = 0.0188)

The response of groundwater under the optimal recharge conditions indicated by (12) was investigated using the numerical solution (non-linear D-F theory) which can conveniently simulate periodical recharge operations.

The optimal t'_u is determined by plotting $\hat{s}''$, average of s'' in both time and space during the recharge cycle, against t'_u as shown in Figure 8. It can be seen that in the first cycle optimal t'_u given by (12) is somewhat earlier than the actual t'_u at which maximum $\hat{s}''$ occurs, whereas in the second cycle (12) is a very good approximation. After the second cycle $\hat{s}''$ remain more or less constant because the growth and decay has become periodical. This suggests that (12) can be conveniently used to determine the optimal t'_u.

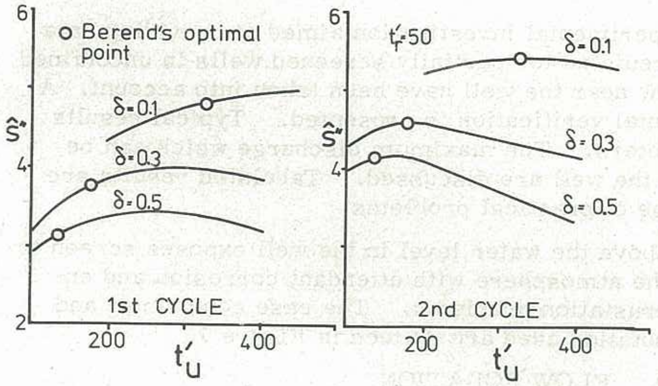

Figure 8 Relationship between $\hat{s}''$ and t'_u at first and second recharge cycles

5 CONCLUSIONS

Solutions are obtained for recharge with clogging from an infinitely long strip basin to a finite aquifer and for the decay of a groundwater mound in such a domain. Both linear and non-linear Dupuit-Forchheimer theories are employed, the former giving an analytical solution and the latter a numerical solution. Comparison between these two solutions indicates that an obvious discrepancy occurs at large t' and p'. However when p' is small and/or δ is large, the linear model does not exhibit any serious deviation from the non-linear model.

An investigation of the optimal period over which the basin should operate under the clogging condition, indicates that when the movement of the groundwater mound in each recharge cycle becomes periodic, the expression of the optimal t'_u employed leads to a maximum average of s''.

6 REFERENCES

AMAR, A.C. (1973). Hydrodynamics of artificial groundwater recharge. Thesis (Ph.D), Univ. of California, Irvine, Calif.

AMAR, A.C. (1975a). Theory of groundwater recharge for a strip basin. Ground Water, Vol.13, No.3, pp.282-292.

AMAR, A.C. (1975b). Groundwater recharge simulation. J.Hyd.Div.,ASCE, Vol.101, No.HY9, pp.1235-1247.

BEREND, J.E. (1970). Clogging processes and optimization of basin recharge. Proc. Artificial Recharge Conf., 21-24 Sept., Water Res. Assoc., Medmenham, England, pp.111-132.

BROCK, R.R. and AMAR, A.C. (1974). Groundwater recharge strip basin experiments. J.Hyd.Div.,ASCE, Vol.100, No.HY4, pp.569-592.

DAGAN, G. (1967). Linearized solutions of free-surface groundwater flow with uniform recharge. J.Geophys.Res., Vol.72, No.4, pp.1183-1193.

DOUGLAS, J. Jr. (1959). Roundoff error in the numerical solution of heat equation. J. Assoc. of Computing Maths., Vol.6, pp.48-58.

GOSS, D.W., SMITH, S.J., STEWART, B.A. and JONES, O.R. (1973). Fate of suspended sediment during basin recharge. Water Resour. Res., Vol.9, No.3., pp.668-675.

HUNT, B.W. (1970). Vertical recharge of unconfined aquifer. J.Hyd.Div.,ASCE, Vol.97, No.HY7, pp.1017-1030.

MITCHELL, A.R. (1969). Computational methods in partial differential equations. John Wiley and Sons.

RIPLEY, D.P. and SALEEM, Z.A. (1973). Clogging of simulated glacial aquifers due to artificial recharge. Water Resour. Res., Vol.9, No.4, pp.1047-1057.

TODSEN, M. (1971). On the solution of transient free surface flow problems in porous media by finite difference methods. J. Hydrol., Vol.12, pp.177-210.