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# Effect of Partial Screening and Non-Darcy Flow on the Performance of Wells in Unconfined Aquifers

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SUMMARY This paper describes a numerical and experimental investigation aimed at providing data to allow discharges and free surface drawdowns to be predicted for partially screened wells in unconfined sand and gravel aquifers. The effects of non-Darcy flow near the well have been taken into account. A brief description of the numerical method and experimental verification is presented. Typical results are given in terms of the relevant dimensionless parameters. The maximum discharge which can be pumped and the maximum lowering of the water table at the well are discussed. Tabulated results are provided to allow these values to be predicted for a range of practical problems.

## 1 INTRODUCTION

Partial screening and non-Darcy flow may cause the discharge from and drawdown near a well in an unconfined aquifer to differ significantly from the values predicted by the Dupuit equation which assumes radial Darcy flow at all points. The complex flow conditions which occur in the vicinity of partially screened wells, particularly when non-Darcy flow exists, have prevented the development of a general analytical solution to the problem. However, numerical techniques applied with the aid of a digital computer may be used to predict flow behaviour for particular well and aquifer geometries and aquifer material characteristics. Computation costs are considerable and for practical purposes the use of a set of dimensionless design charts for commonly met conditions becomes preferable to the direct application of available computer programs to individual problems.

This study was undertaken to investigate the flow behaviour and provide a set of design graphs for the case of partially screened, fully penetrating wells in which the water level is not allowed to fall below the top of the screen. This is considered to be the ideal condition under which wells in unconfined aquifers should be operated since to screen above the water level in the well exposes screen to the atmosphere with attendant corrosion and encrustation problems. The case considered and notation used are defined in Figure 1.

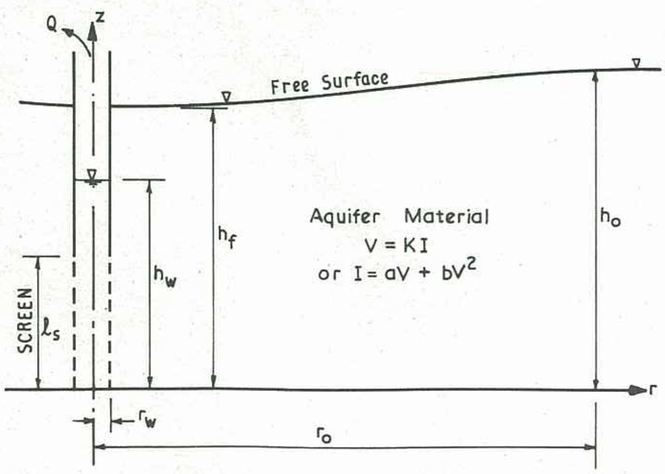


Figure 1 Definition sketch

## 2 FLOW EQUATION

It has long been recognised that there is an upper limit to the validity of the well known and commonly used expression

$$V = KI \tag{1}$$

derived by Darcy in relating flow velocity V, hydraulic gradient I and hydraulic conductivity K. Ideally, the upper limit of validity of Darcy's law and the entire velocity-hydraulic gradient behaviour would be given in the form of a friction factor Reynold's number relationship. No such general relationship which will reliably predict velocities and gradients is available for aquifer materials even though considerable research effort has been expended by a multitude of workers. Thus, the Forchheimer equation

$$I = aV + bV^2 \tag{2}$$

which directly relates V and I was adopted for use in the analysis when non-Darcy flow was considered significant.

The Forchheimer expression has been subjected to theoretical (Stark and Volker, 1967) and experimental (Sunada,1965) validation and found to fit permeameter data with adequate accuracy (Cox, 1976) over the range of velocities encountered in flow towards wells. Use of a single pair of values of the constants a and b was found to give a fit to permeameter data with percentage deviations not exceeding 9 per cent. An alternative to fitting a single pair of constants is to divide the flow region into separate Darcy and non-Darcy zones (Huyakorn, 1973). This procedure gives a somewhat better fit to the data but complicates the preparation of dimensionless graphs by introducing an additional variable, a critical velocity, required to differentiate between Darcy and non-Darcy flow. In view of uncertainties of knowledge of aquifer geometry and homogeneity it is considered that the use of a single pair of constants is appropriate.

## 3 NUMERICAL METHOD

Only an outline of the numerical solution method is presented. Full details of the derivation of the field equation, variational formulation, equation solution procedures, finite element meshes and studies of convergence and accuracy have been given by Cox (1976).

Tensor subscript notation is used to simplify the writing of the equations.

For three dimensional non-Darcy groundwater flow in a homogeneous isotropic aquifer, the Forchheimer equation may be written as

$$\frac{\partial h}{\partial x_i} = -(a + b|V|)\, v_i \tag{3}$$

where $h$ = piezometric head

$v_i$ = the components of velocity in the $x_i$ cartesian co-ordinate system

$|V|$ = magnitude of velocity vector

The continuity equation is

$$-\frac{\partial v_i}{\partial x_i} = S_s \frac{\partial h}{\partial t} \tag{4}$$

where $S_s$ = specific storage of the aquifer

$t$ = time

Combination of equations (3) and (4) yields the field equation

$$\frac{\partial}{\partial x_i}\left[\left(-\frac{a}{2b} + \sqrt{\left(\frac{a}{2b}\right)^2 + \left|\frac{\partial h}{\partial l}\right| / b}\ \right)\left(\frac{\partial h}{\partial x_i}\right) \Big/ \left|\frac{\partial h}{\partial l}\right|\right] = S_s \frac{\partial h}{\partial t} \tag{5}$$

where $\left|\frac{\partial h}{\partial l}\right| = \left(\frac{\partial h}{\partial x_i}\ \frac{\partial h}{\partial x_i}\right)^{\frac{1}{2}}$

For steady flow $S_s \frac{\partial h}{\partial t} = 0$

Equation (5) was solved for steady flow with the appropriate boundary conditions by the method of finite elements. The complexities caused by the non-linearities introduced by non-Darcy flow behaviour and the a priori unknown free water surface were handled by separate over-relaxation iterative procedures. For flows in which at no point does the velocity become high enough for deviations from Darcy's law to become significant, a simplified field equation based on the flow equation

$$\frac{\partial h}{\partial x_i} = -a v_i \tag{6}$$

may be used over the whole flow field.

Solutions were obtained for a range of well geometries and aquifer characteristics considered relevant to water supply and de-watering problems. The range was selected after an examination of records of wells constructed in New South Wales and Queensland.

## 4 EXPERIMENTAL VERIFICATION

The numerical solution procedure was verified by comparing results with those obtained from experiments in an electrolytic analogue tank and a large sand tank model.

For fully Darcy flow the electrolytic tank was used to predict the head distribution, including the position of the free surface. Comparisons were found to be very satisfactory.

Non-Darcy flow solutions were compared with results obtained from a large tank containing a quadrant of a 4.9m radius aquifer, 1.5m thick, made up of coarse sand ($d_{50}$ = 3.7 mm). Water was drawn from the aquifer through a well located at the apex of the quadrant. Figure 2 shows a comparison of the experimental and numerical solutions for a typical case.

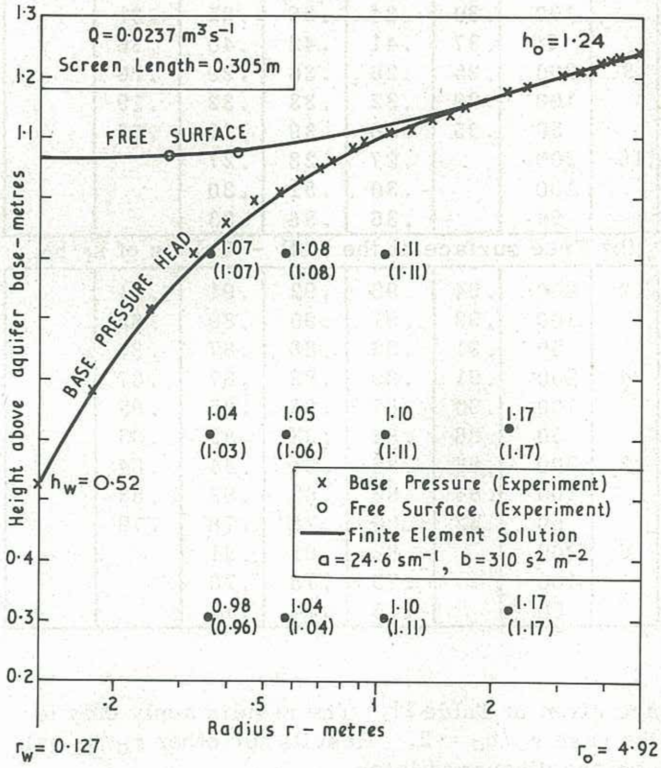


Figure 2 Experimental Verification

## 5 RESULTS

Dimensional analysis of the problem in terms of the variables defined in Figure 1 yields the following relationships:

Well discharge

$$\frac{aQ}{h_o^2} = \phi_1\left(\frac{r_o}{h_o}, \frac{h_o}{r_w}, \frac{l_s}{h_o}, \frac{h_w}{h_o}, \frac{b}{a^2}\right) \tag{7}$$

Water level at the well

$$\frac{h_f}{h_o} = \phi_2\left(\frac{r_o}{h_o}, \frac{h_o}{r_w}, \frac{l_s}{h_o}, \frac{h_w}{h_o}, \frac{b}{a^2}\right) \tag{8}$$

The results of the investigation are set out in terms of the dimensionless ratios of equations (7) and (8).

For cases of wholly Darcy flow Table I gives values of $aQ/h_o^2$ (= $Q/Kh_o^2$) and $h_f/h_o$ for a range of the other relevant ratios. Note that for wholly Darcy flow $b/a^2$ is irrelevant and that for the case treated, water level in the well at the top of the screen, $h_w = l_s$.

Values of $aQ/h_o^2$ and $h_f/h_o$ over a range of $b/a^2$ of practical significance for which the effect of non-Darcy flow near the well may be significant

TABLE I

DARCY FLOW TO A PARTIALLY SCREENED WELL IN AN UNCONFINED AQUIFER.

| $\frac{r_o}{h_o}$ | $\frac{h_o}{r_w}$ | $h_w/h_o = l_s/h_o$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | .2 | .3 | .4 | .5 | .6 | .7 |
| (a) Well performance - Values of $Q/Kh_o^2$ | | | | | | | |
| 2 | 200 | .27 | .32 | .34 | .34 | .31 | .26 |
| | 100 | .31 | .36 | .39 | .38 | .35 | |
| | 50 | .39 | .44 | .46 | .45 | .41 | .34 |
| 4 | 200 | .26 | .30 | .32 | .31 | .28 | |
| | 100 | .30 | .34 | .36 | .35 | .31 | |
| | 50 | .37 | .41 | .42 | .40 | .36 | |
| 8 | 200 | .25 | .28 | .30 | .29 | .26 | |
| | 100 | .29 | .32 | .33 | .32 | .29 | |
| | 50 | .35 | .38 | .39 | .37 | .33 | |
| 16 | 200 | | .27 | .28 | .27 | | |
| | 100 | | .30 | .31 | .30 | | |
| | 50 | | .36 | .36 | .33 | | |
| (b) Free surface at the well - Values of $h_f/h_o$ | | | | | | | |
| 2 | 200 | .94 | .93 | .92 | .91 | .91 | |
| | 100 | .93 | .91 | .90 | .89 | .89 | |
| | 50 | .91 | .89 | .88 | .87 | .87 | |
| 4 | 200 | .91 | .89 | .88 | .87 | .87 | |
| | 100 | .90 | .87 | .86 | .85 | .85 | |
| | 50 | .86 | .84 | .83 | .82 | .83 | |
| 8 | 200 | .88 | .86 | .84 | .84 | .84 | |
| | 100 | .86 | .83 | .82 | .82 | .83 | |
| | 50 | .82 | .80 | .78 | .78 | .79 | |
| 16 | 200 | | .82 | .81 | .81 | | |
| | 100 | | .79 | .78 | .78 | | |
| | 50 | | .75 | .74 | .74 | | |

TABLE II

NON-DARCY FLOW TO A PARTIALLY SCREENED WELL IN AN UNCONFINED AQUIFER $r_o/h_o = 2$

| $\frac{h_o}{r_w}$ | $\frac{b}{a^2}$ | $h_w/h_o = l_s/h_o$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | .3 | .4 | .5 | .6 | .7 | .8 |
| (a) Well performance - Values of $aQ/h_o^2$ | | | | | | | |
| 50 | .01 | .43 | .45 | .44 | .40 | | |
| | .1 | .36 | .39 | .39 | .37 | .32 | |
| | 1 | .20 | .23 | .24 | .24 | .22 | |
| | 10 | | .090 | .099 | .10 | .10 | .090 |
| | 100 | .026 | .031 | .034 | .036 | .036 | .033 |
| 100 | .01 | .35 | .37 | .37 | .34 | | |
| | .1 | .28 | .31 | .32 | .31 | .27 | |
| | 1 | .15 | .17 | .19 | .19 | .18 | |
| | 10 | | .067 | .074 | .078 | .077 | .070 |
| | 100 | .019 | .023 | .025 | .027 | .027 | .025 |
| 200 | .01 | .30 | .32 | .33 | .30 | | |
| | .1 | .24 | .27 | .28 | .27 | .24 | |
| | 1 | .12 | .14 | .15 | .16 | .15 | |
| | 10 | | .054 | .060 | .064 | .064 | .058 |
| | 100 | .015 | .018 | .020 | .022 | .022 | .021 |
| (b) Free surface at the well - Values of $h_f/h_o$ | | | | | | | |
| 50 | .01 | .89 | .88 | .87 | .87 | | |
| | .1 | .91 | .89 | .88 | .88 | .88 | |
| | 1 | .95 | .94 | .93 | .92 | .92 | |
| | 10 | | .98 | .97 | .97 | .96 | .96 |
| | 100 | .99 | .99 | .99 | .98 | .98 | .98 |
| 100 | .01 | .91 | .90 | .89 | .89 | | |
| | .1 | .93 | .92 | .91 | .90 | .90 | |
| | 1 | .96 | .96 | .95 | .94 | .94 | |
| | 10 | | .98 | .98 | .98 | .97 | .97 |
| | 100 | .99 | .99 | .99 | .99 | .99 | .99 |
| 200 | .01 | .93 | .92 | .91 | .91 | | |
| | .1 | .95 | .94 | .93 | .92 | .92 | |
| | 1 | .98 | .97 | .97 | .96 | .96 | |
| | 10 | | 1.00 | .99 | .99 | .99 | .99 |
| | 100 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

are given in Table II. The results apply only to the case $r_o/h_o = 2$. Results for other $r_o/h_o$ values are discussed later.

The general characteristics of the non-Darcy solutions may be observed from Figure 3 on which are plotted the data of Table 2 for $h_o/r_w = 100$. It will be noted that a maximum value of $aQ/h_o^2$ and minimum value of $h_f/h_o$ occur. Since these values are of interest to well designers they were determined for other values of $h_o/r_w$ and plotted in Figure 4. This figure also includes a plot showing the critical screening ratio $(l_s/h_o)_{crit}$ for various $b/a^2$ values. This is the value of $l_s/h_o$ for which maximum discharge and free surface drawdown occur. Within the range $2 \leqslant r_o/h_o \leqslant 16$ and $50 \leqslant h_o/r_w \leqslant 200$ the $(l_s/h_o)_{crit}$ values read from the single curve shown will give the maximum discharge Q and lowest free surface position $h_f$ within 2 per cent.

The following important features of the flow behaviour were observed in all cases:

(i) the flow was radially horizontal below a height $l_s/2$ above the aquifer base;

(ii) partial screening effects were negligible and the flow was radially horizontal beyond $r = 1.5\ h_o$;

(iii) non-Darcy flow effects were negligible beyond $r = 2\ h_o$ even for the extreme value of $b/a^2 = 100$.

Thus for $r > 2\ h_o$ the flow may be described by the Dupuit Equation

$$h_o^2 - h^2 = Q\ln\left(\frac{r_o}{r}\right) / \pi K \qquad (9)$$

## 6 EXTENSION OF NON-DARCY FLOW RESULTS TO VALUES OF $r_o/h_o$ GREATER THAN 2

The computation time required to obtain the non-Darcy solutions for $r_o/h_o = 2$ was so great that for other values of $r_o/h_o$ solutions were obtained for only a few combinations of $b/a^2$, $h_o/r_w$ and $l_s/h_o$. These were then used to check the validity of extending the results for $r_o/h_o = 2$ to greater values of $r_o$ by applying the Dupuit equation beyond $r = 2\ h_o$.

When extending a given solution to a larger value of $r_o$, no control can be exercised over the resultant values of the ratios involving $h_o$. Interpolation is necessary to obtain graphs similar to Figure 3 for other $r_o/h_o$ values. The graphs for the range of values $2 \leqslant r_o/h_o \leqslant 16$ have been prepared and will be made available by the authors in report form.

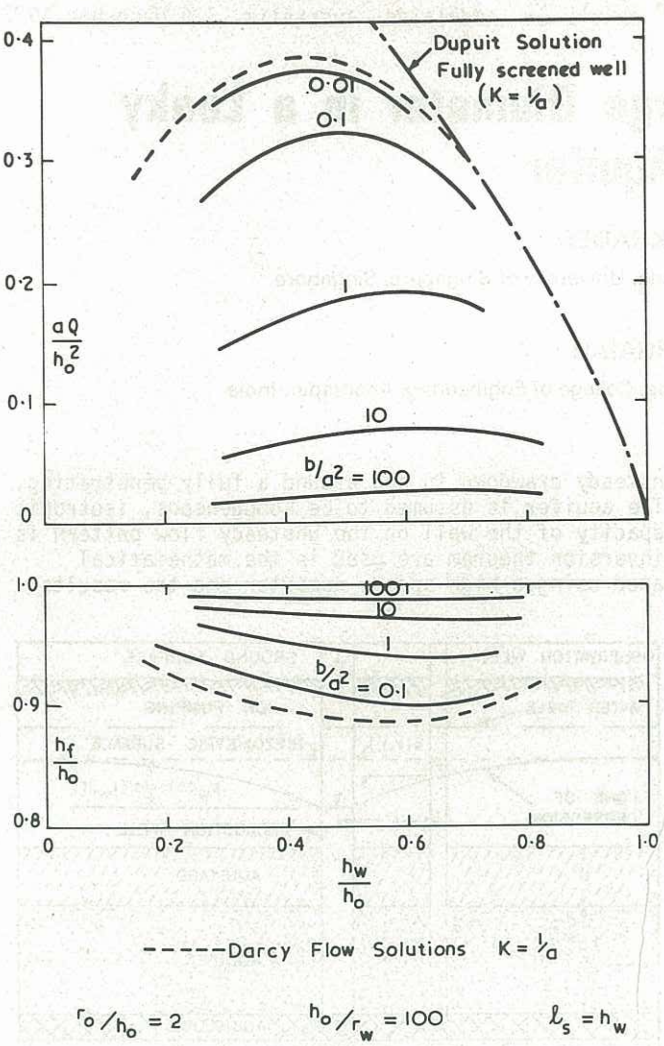


Figure 3 Typical well performance curves

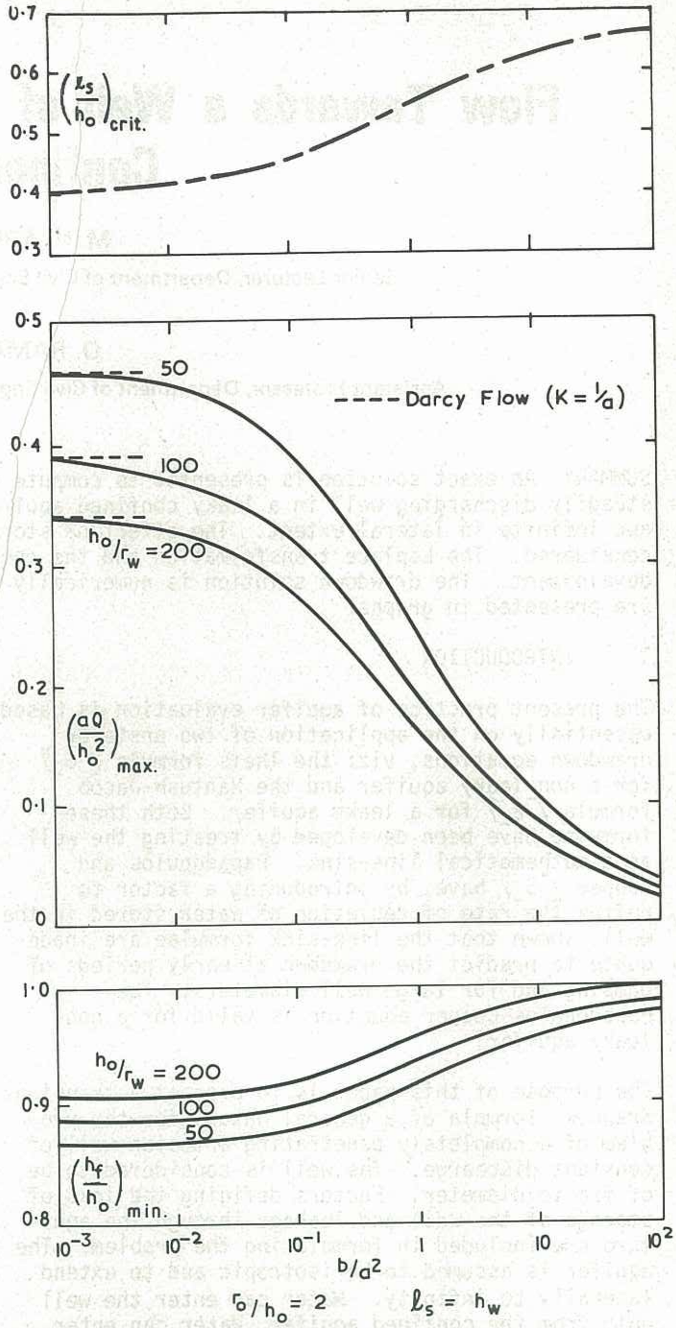


Figure 4 Maximum discharge and drawdown

## 7 CONCLUSION

In the past, lack of data has prevented engineers and hydrogeologists engaged in well design from making comprehensive examinations of the effects of well and aquifer variables on well performance in other than simple cases. It is considered that the results of this investigation will be useful in selecting optimal designs for partially screened wells in unconfined aquifers.

## 8 REFERENCES


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