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Wave Groups: Analysis of Run and Run Length

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SUMMARY The formation of wave groups containing relatively high waves is one of the characteristic features of ocean waves. The statistics of the wave groups, with particular reference to 'Run' and 'Run Length', are analysed for the ocean wave data obtained on the West Coast of India. Studies were also conducted on the phenomenon of wave groups in waves generated in the Laboratory Wind Water Wave flume and the results are discussed. The results are in good agreement with the theoretical model proposed by Goda.

1 INTRODUCTION

The formation of wave groups containing relatively high waves is of common occurrence in seas and swells. A sequence of high waves is called a 'Wave Group' and its length is measured in terms of the number of successive high waves which exceed a certain magnitude. A wave group is also referred to as a 'Run of high waves'. In Statistics, a sequence of elements of the same kind is called a run, and the length of a run is given by the number of elements defining a run. Although wave groups influence many ocean engineering problems, studies on them have been very few and their importance is being realised only now. One of the applications of wave group analysis would be in the study of damage to rubble mound breakwaters, since damage invariably occurs during the run of high waves. The differences in the damages when different wave spectra (all with the same significant wave height) are incident might be possibly due to the different wave group formations in the different spectra. The length of run and more specifically the mean length of run may be a significant parameter in characterising the irregular waves in such tests. Slow drift oscillations of moored objects, ship capsizing tests are other possible applications of the wave group analysis (Goda 1976).

2 RUN LENGTH AND MEAN LENGTH OF RUN

2.1 Theoretical Analysis

With the assumption that successive wave heights are statistically independent, Probability theory can be applied for the analysis of run lengths in terms of the probability of exceedance of wave heights beyond a certain level, H_L. Let the distribution function of wave heights be $P(H)$ and the probability of exceedance be p. Let q be the probability that H is less than or equal to H_L, i.e.,

$$q = P(H_L) \qquad (1)$$

Then, by definition

$$p = 1 - q = 1 - P(H_L) \qquad (2)$$

Consider a sequence of waves where j waves exceed the level H_L, i.e. j is the run length of wave heights that exceed H_L. According to Goda (1976) the probability $P(j)$ of the run with the length j, is equal to

$$P(j) = p^{j-1}\, q \qquad (3)$$

This is because the run with the length of j is the phenomenon that (j-1) consecutive waves exceed the magnitude H_L, after the first one exceeding H_L and the (j+1)th wave fails to exceed H_L. Since successive waves are assumed to be independent, the joint probability is given by the product of the individual probabilities.

Once $P(j)$ is known, the mean and the standard deviation of run lengths can be calculated as:

$$\text{Mean Run Length} = \bar{j} = E(j) = \sum_{j=1}^{\infty} j\, P(j) = 1/q \qquad (4)$$

$$\text{Standard Deviation} = \sigma(j) = [E(j^2) - E(j)^2]^{\frac{1}{2}} = p/q \qquad (5)$$

It is now well established that the probability distribution of the wave heights both in the oceans and the Laboratory Wind Water Wave flumes is adequately described by the Rayleigh distribution (Dattatri, 1973, Dattatri et al. 1976a, Dattatri et al 1976b). The probability density function of the Rayleigh distribution is

$$p(H) = (H/\alpha^2)\exp(-H^2/2\alpha^2) \quad \text{for } H \geq 0 \text{ and } \alpha > 0,$$
$$= 0 \quad \text{for } H < 0, \qquad (6)$$

in which α is the parameter of the distribution. In terms of wave heights, $\alpha = H_{RMS}/\sqrt{2}$, H_{RMS} being the root-mean-square wave height.
It can be shown that $H_{RMS} = \sqrt{8\, m_o}$, m_o being the zeroth moment (or area) of the spectral density function $S(f)$ of $\eta(t)$, the surface elevation. The relationships are

$$m_o = \int_0^{\infty} S(f)\, df = \overline{\eta^2} \qquad (7)$$

in which the bar over any quantity indicates mean values. From the density function the distribution function $P(H)$, which gives the probability that the random variable H, is less than a specific value H_L can be obtained as

$$Pr\ (H \le H_L) = P(H_L)$$

$$= \int_0^{H_L} (H/\alpha^2)\ \exp(-H^2/2\alpha^2)\ dH \qquad (8)$$

$$= 1 - \exp(-H_L^2/2\ \alpha^2) \text{ for } H_L \ge 0$$

$$= 0 \text{ for } H_L < 0 \qquad (9)$$

Since $P(H_L) = q$ and $p = 1-q$, we have

$$p = \exp\ (-H_L^2/2\alpha^2 \qquad (10)$$

$$= \exp\ (-H_L^2/8\ m_o) \qquad (11)$$

If we take the exceedance level H_L to be equal to H_s, the significant wave height, and noting that $H_s = 4\ \sqrt{m_o}$ for the Rayleigh distribution, we have

$$p = \exp\ (-H_s^2/8\ m_o)$$

$$= \exp\ (-16\ m_o/8\ m_o) = 0.1353 \qquad (12)$$

$$q = 1 - p = 0.8647 \qquad (13)$$

If the exceedance level is taken at H_{Mean}, the mean wave height ($H_{Mean} = 2.355\ \sqrt{m_o}$ for the Rayleigh distribution), we get $p = 0.5$ and $q = 0.5$. With these known values of p and q, the probability of the run with length j, P(j) can be evaluated using Eq.3

3 DETAILS OF WAVE DATA USED FOR ANALYSIS

3.1 Ocean wave data

The wave data utilized in the present analysis was obtained off the Mangalore Harbour on the West Coast of India during August 1974. During this period the West Coast will be under the influence of the Indian Monsoon with strong winds and rough seas. The wave measuring device was the Dutch Wave rider accelerometer buoy and the depth of water at the measurement site was 13 metres. About 15 records obtained during the period of measurement were digitized manually according to the zero-crossing method (Draper, 1966) to give the successive wave height data. For the records analysed the range of H_s was 1.57m to 2.14m. The spectral width parameter ε(defined in terms of the spectral moments) varied from 0.51 to 0.61. Many of the records analysed had energy spectra with multiple peaks, the second peak occurring at approximately twice the frequency of the main peak. An earlier analysis of this data (Dattatri et al. 1976b) had shown that the wave height distribution essentially followed the Rayleigh distribution even though the data did not satisfy the narrow band assumption.

3.2 Laboratory Wind Water Wave flume data

The Laboratory wind wave data was obtained using the Wind Water Wave Flume at the Hydraulic Engineering Laboratory, Indian Institute of Technology, Madras, details of which are provided in an earlier reference (Dattatri et al 1976a). Experiments were conducted with different wind velocities and the wave data so obtained was analysed in a manner similar to that used for ocean wave data. The significant wave height varied from 4 cm to 10 cm and the variation in the spectral width parameter was from 0.33 to 0.65. The wave height statistics for this data also is well represented by the Rayleigh distribution (Dattatri et al 1976a).

4 PRESENTATION AND INTERPRETATION OF DATA

The wave heights of successive waves of one of the samples analysed is shown in Fig. 1. The runs for $H > H_s$ are indicated by the closed circles and the corresponding run lengths are also marked in Fig.1. For the sample shown, there are seven runs of $H > H_s$ with the run lengths of 1,3,1,1,4,1 and 1. The results of the analysis of runs and their lengths of all the samples is given in Fig.2(a) for the ocean wave data and in Fig.2(b) for the laboratory wind wave data.

The run lengths for the ocean wave data conforms very well with the theoretical values and the agreement is excellent for the case $H > H_{Mean}$. The mean length of run is 2.23 for $H > H_{Mean}$ and 1.34 for $H > H_s$, and these values compare well with the theoretical values of 2.00 for $H > H_{Mean}$ and 1.16 for $H > H_s$ respectively. In similar studies Goda (1976) for simulated data reports a value of 1.42 (for $H > H_s$) while Rye (1974) obtained 1.35 (for $H > H_s$) for the ocean wave data. Considering the fact that these data were probably under different environments and conditions, the agreement in the values obtained by different investigators can be termed as very good.

Fig. 2 (a) shows that the observed frequencies of long runs are greater than predicted and Goda (1976) who obtained similar trends, attributed it to the dependency of successive wave heights while the theory assumes that they are statistically independent. To study this aspect, the coefficient of correlation between successive wave heights was calculated for the different samples and the values ranged from + 0.055 to + 0.446 with an average value of +0.236 and a standard deviation of 0.104 It is a surprising coincidence that this value of the correlation coefficient is in close agreement with the average value of 0.24 reported by Rye (1974). The correlation coefficient of 0.236 for the present data though low, does indicate some sort of a dependency of successive wave heights, though the dependency can be termed as very weak. Maximum run lengths observed for the Mangalore data were 4 for $H > H_s$ and 14 for $H > H_{Mean}$.

The Laboratory wind wave data as indicated in Fig.2(b) shows trends comparable to the ocean wave data. The observed frequencies of long runs were considerably larger than those predicted indicating higher group activity in laboratory wind waves.

In Fig. 3 is plotted the mean run length for the ocean wave data as a function of the peakedness parameter, Q_p defined by Goda (1976). The expression for Q_p is

$$Q_p = (2/m_o^2) \int_0^{\infty} [S(f)]^2\ f\ df \qquad (15)$$

The peakedness parameter is considered to be a better indication of the spectral band width than the spectral width parameter, ε. One of the reasons for this is the use of higher order spectral moments in the computation of ε and these computations are highly sensitive to the cutoff frequency. In Fig.3 there is a general indication that the mean run length increases with an increase in the peakedness parameter. It should be noted that in the theoretical analysis, narrow band assumption is made and as such the spectral width does not enter into the theoretical equations. According to theory the mean run length would be independent of the peakedness parameter Q_p. These results are consistant with the field observations reported by Goda (1976).

Rye (1974) based on the analysis of ocean wave data has concluded that wave group formations tend to be more pronounced for a growing sea than for a decaying sea. For the ocean wave data analysed, a distinct period of wave growth and wave decay existed. Analysis of the run lengths for the two periods showed slight tendencies towards a more pronounced group formation during wave growth than during wave decay. For $H > H_{Mean}$, there were two runs with run lengths of 12 and 14 during the wave growth period, while during wave decay the maximum run length was 9 (for $H > H_{Mean}$).

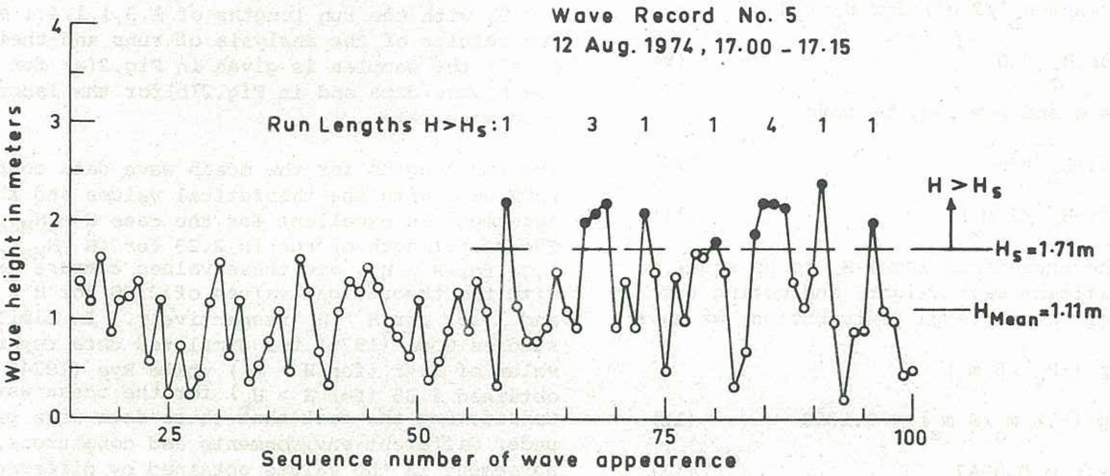

Figure 1 Example of sequence of wave heights, run , and run lengths (ocean wave data)

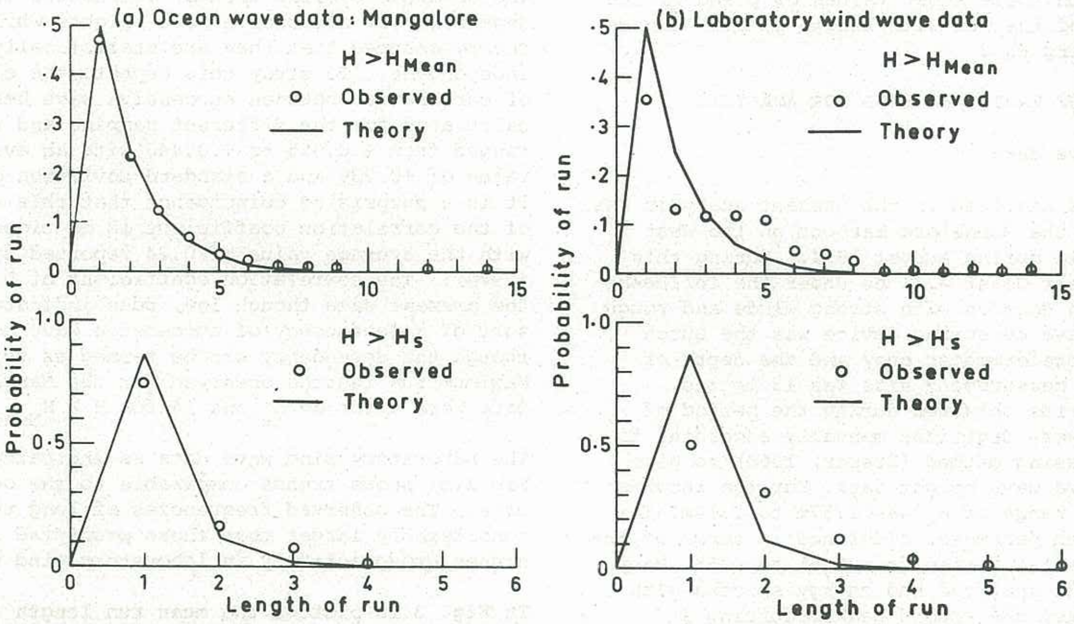

Figure 2 Observed and Theoretical values of the probability of Run Vs the length of run

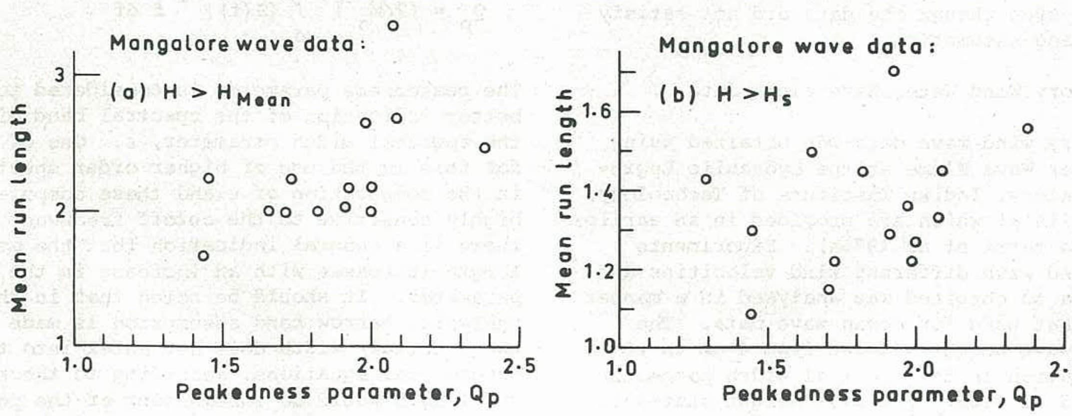

Figure 3 Mean Run Length Vs Peakedness Parameter

5 CONCLUSIONS

Studies concerning wave groups have applications in a variety of Ocean Engineering problems. Analysis of ocean wave data and laboratory wind wave data shows that they conform well with the theoretical model proposed by Goda.

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