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Cell Size and Flow History Effects in the Taylor Vortex Regime

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SUMMARY Measurements of torque with allied visual observations are presented for the Taylor vortex regime using concentric rotating cylinders of radius ratio 0.56, 0.73 and 0.95, with length-clearance ratios L/c up to 100. Cell size, which depends on cylinder length and on flow history, is found to influence Taylor critical speed only very slightly. This result is in line with the results of a rudimentary theory for the effect of annulus length on the critical condition: the prediction is an increase in critical speed as cylinders become shorter although the rise is less than 1% for $L/c > 8$. At speeds above critical, cell size affects measured torque, but only by a few percent, smaller cells producing more torque. Variation of annulus length at a fixed speed in the vortex regime produces larger variations of cell size and torque. A striking effect is that the torque-length relation then becomes non-linear with discontinuities corresponding to changes of cell numbers.

1 INTRODUCTION

Circumferential laminar flow between rotating cylinders becomes unstable as rotor speed is increased and Taylor vortices, in contra-rotating pairs, are set up at a sharply defined critical speed. In turn, the Taylor vortex regime also becomes unstable as speed is further increased, a wave motion developing at a higher critical speed.

The transitions from the laminar regime to the Taylor vortex regime and then to the wavy vortex regime are marked by discontinuous changes of slope of the torque-speed relation at the critical points, the changes being successively upward and downward. It is the nature and magnitude of the friction torque beyond the Taylor critical speed that is of technological interest.

Theoretically, the size of Taylor vortex cells is fixed at a value equal to the radial clearance between the rotating cylinders. As will be discussed later, cell size in practice is dependant on cylinder length and end effects and on flow history effects. By "flow history effect" is meant a dependence of flow pattern not only on operating conditions (geometry, speed, viscosity) but also on the route by which these operating conditions are achieved. D. Coles (1965) first drew attention to flow history effects in connection with the variability of vortex wave number beyond the wavy vortex critical speed. Snyder (1969 a,b) and Cole (1974 a,b) have shown that Taylor vortex cell numbers behave similarly beyond the Taylor vortex critical speed. All these studies are based on visual observations, but the present paper deals with related flow visualisation and torque measurements.

2 EXPERIMENTAL APPARATUS

The experiments have been carried out with three versions of what is effectively a rotating cylinder viscometer. A precise servo-controlled variable speed motor drives a vertical rotor concentrically positioned inside a steel or Perspex stator, with provision for torque measurement. The test oil in the annulus is admitted or removed at the base via an electromagnetically operated valve, and the height of the free liquid surface is indicated by a strain gauge pressure transducer or can be measured with a cathetometer. Taylor vortex cell size is measured also with the cathetometer, the well-known aluminium particle technique being used to delineate the cells. An XY recorder is used to obtain torque-speed or torque-depth plots. Estimates of accuracy and more details of the equipment have been given in previous papers (Cole 1974 a,b).

3 CELL SIZE AND CRITICAL SPEED N_c

Theoretically, individual Taylor vortex cells have an axial length equal to the radial clearance $c = R_2 - R_1$. This arises as the final step in the stability analysis where, as indicated in Figure 1, the critical speed for the onset of Taylor vortices is taken as the minimum point on the calculated stability surve. The minimum is then seen to correspond to a disturbance wavelength of 2c, and this is the size of a contrarotating vortex pair. (Figure 1 shows cell size/c, not wavelength.)

Whereas the theoretical stability analysis assumes an infinite length for the cylinders and annulus, the practical requirement that the length is finite and should accommodate an integral number of cells usually involves individual cell sizes departing from c.

Previously published experimental observations of vortex cell size (Snyder 1969 a,b., Cole 1974 a,b) show that considerable departures from the theoretical size c occur, that end cells are anomalously large, and that length changes may alter cell numbers in steps of one or two at a time.

However, for the purpose of a very rudimentary examination of the influence of cell size, it is simpler to assume that the mean cell size is $\bar{v} = L/n$ when n cells form in an annulus of length L, and that cell numbers alter in increments of

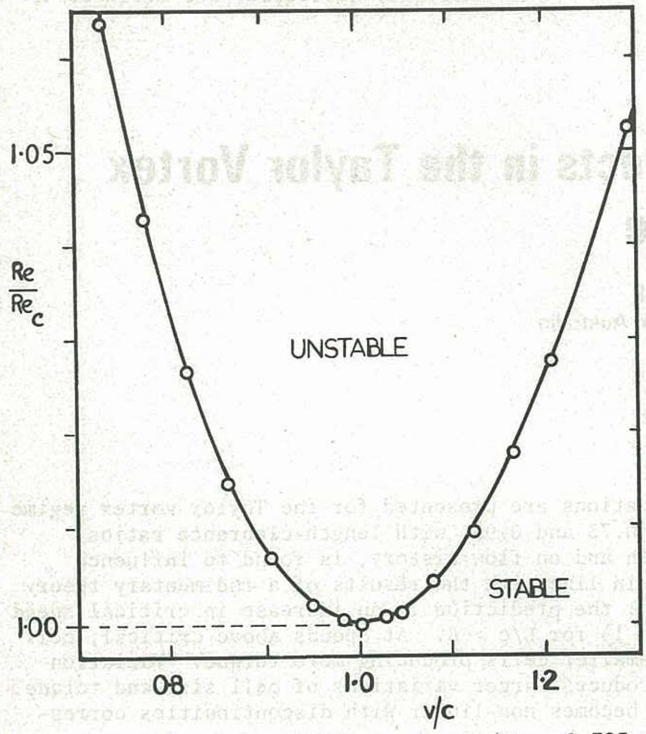

Figure 1 Computed stability curve $R_1/R_2 = 0.723$

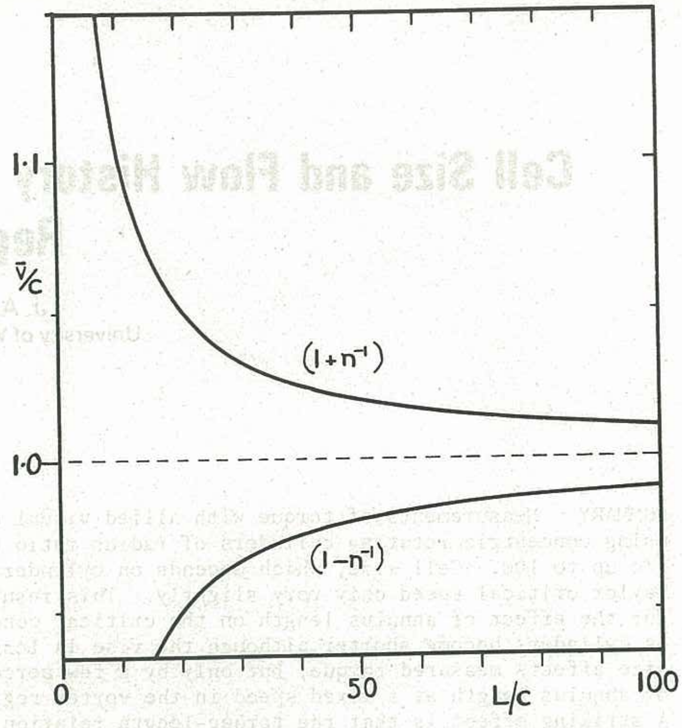

Figure 2 Cell size limits with $\bar{v} = L/n$, $n-1 \rightarrow n+1$

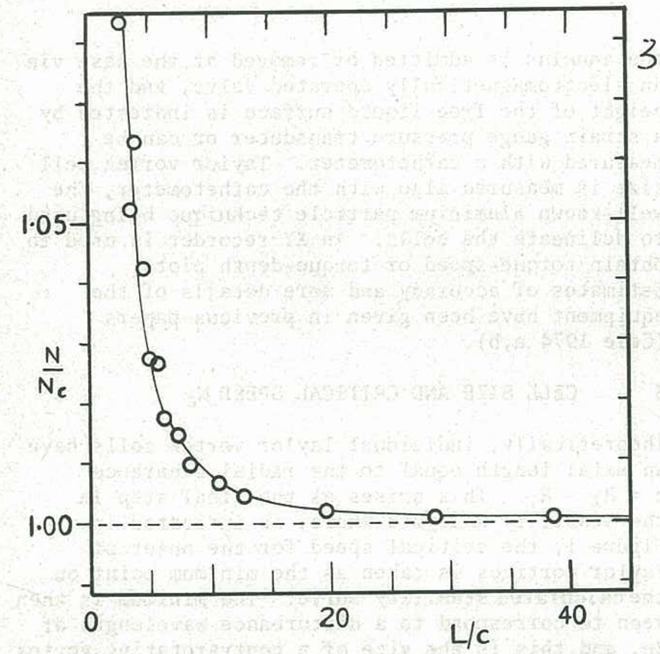

Figure 3 Critical speed & length (computed)

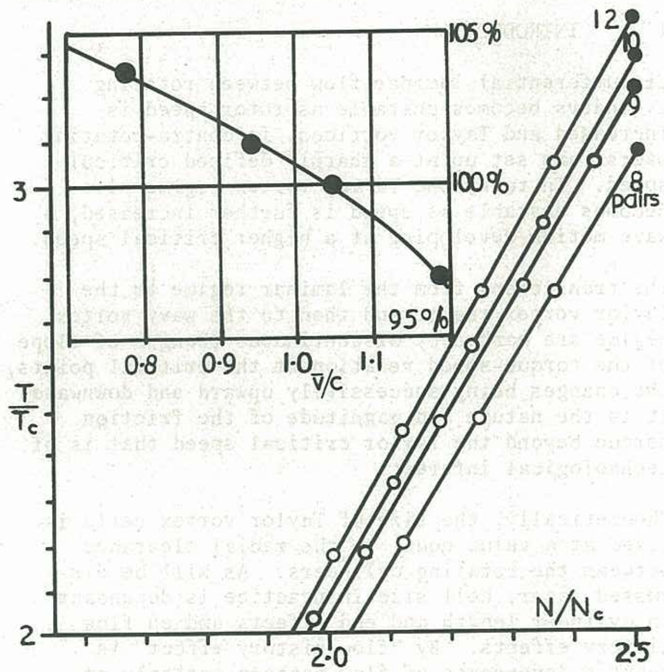

Figure 5 Measured torques, $L/c=19.2$, $R_1/R_2=0.56$
Figure 6 (Inset) Maximum torques & cell size

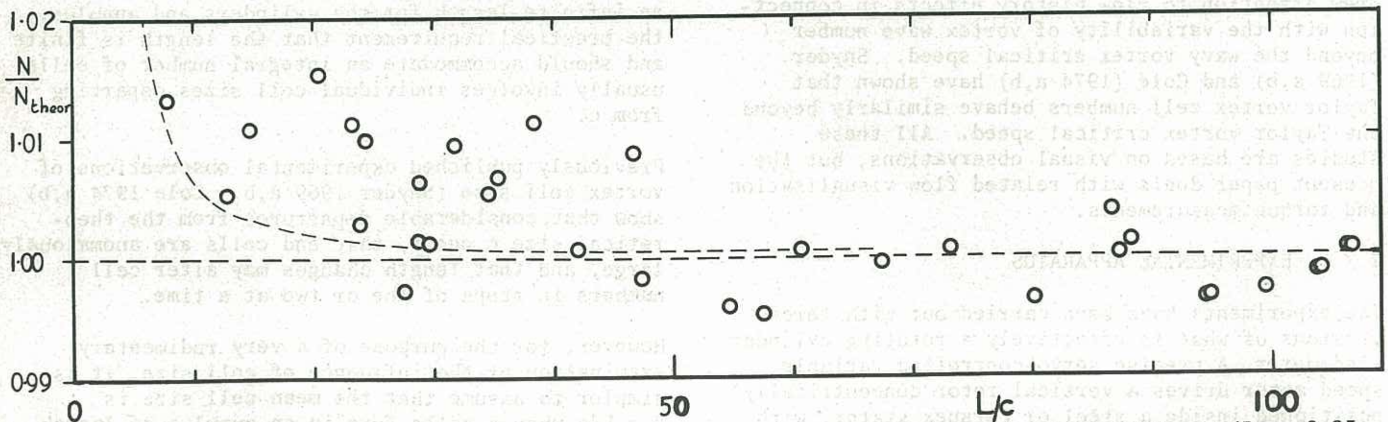

Figure 4 Measured critical speeds N for different lengths L, expressed non-dimensionally. $R_1/R_2 = 0.95$
Dashed lines from Figure 3.

two, $n-1 \rightarrow n+1$, as length is increased. Figure 2 is constructed on this basis, showing variation of cell size with annulus length on the assumption that cell numbers change so as to minimise the departure from the theoretical size. The variations of cell size evidently become large for short annuli, as is confirmed by the author's published observations.

Next, the drastic assumption is made that the standard analysis of stability for infinite cylinders can be applied to the finite cylinder situation by selecting the critical speed on the basis of the cell size imposed by the particular annulus length in use, i.e. by accepting a small departure from the minimum of the stability curve. Thus combining Figures 1 and 2 leads to Figure 3, indicating the variation of Taylor critical speed with annulus length. The calculations apply to a cylinder radius ratio $R_1/R_2 = 0.729$, but variation of the ratio has only a minor effect. The principal results are that critical speeds are higher with short cylinders but that the variation with length is negligibly small, less than 1%, for annuli longer than 8c. Changes of vortex cell numbers in increments of 1 rather than 2, as tend to occur with very short cylinders, could reduce the effect still further.

Detection of so small an effect is obviously not easy. Figure 4 shows the results of experiments in which measurements of outer cylinder torque reaction were used to determine the onset of Taylor vortices from the change in torque-speed gradient at the critical point. For long annuli, $L/c > 60$ say, the measurements are in good agreement with the theoretically predicted critical speed, the maximum deviation being less than 0.4%. However, the scatter increases as annulus length is reduced, for it is found that the gradient change becomes progressively less sharp. At $L/c < 8$, the change is so rounded as to make determination of the critical speed very difficult. Despite these uncertainties the observations confirm the prediction that Taylor critical speed is little affected by length for $L/c > 10$, while the trend of the points in Figure 4 suggests that the increase of critical speed predicted by the foregoing analysis does occur. Attempts are now being made to increase the precision of the measurements.

A contrary result must also be reported. Visual observation of the onset of Taylor vortices using the well-known aluminium particle flow visualisation technique generally gives very good agreement, better than 1% for long cylinders, with the method involving torque measurement. However, re-examination of the former method now suggests a consistent tendency for the vortex pattern to become visually complete at a speed slightly in excess of the value deduced from torque records. On the other hand, for very short annuli, the opposite seems to occur, as in any case the end cells form before the expected critical speed. This also requires further investigation.

4 CELL SIZE AND TORQUE AT CONSTANT ANNULUS LENGTH

At the critical speed, the number n of Taylor vortex cells present in a given annulus length L is single-valued unless the conditions are appropriate for a "change-over length" (Cole 1974a) when two values of n are possible, one giving cells larger than the radial clearance c and the other giving cells smaller than c. At higher speeds, n can be many-valued and the particular number selected depends on the flow history (Snyder 1969 a,b., Cole 1974 a,b). A slow start from the rest gives n corresponding to cell sizes nearest to c. Impulsive starts from rest may give higher values of n, depending on acceleration and final speed. Cell sizes can also be adjusted by expansion or compression, achieved by slow fluid inflow or outflow respectively, when a vortex system has already been set up. Stable cell sizes departing from the theoretical size by as much as 30% can be obtained by the last method (Cole 1974b).

To date, these effects have been investigated by visual observation only, but preliminary results of simultaneous torque and visual measurements can now be presented.

Figure 5 shows torque-speed results for an annulus of length $L/c = 19.2$ with cell numbers n = 16, 18, 20 and 24 obtained by suitably varying the flow history as already outlined. The speed was necessarily varied downwards from the maximum to obtain the plotted points, although in the laminar range, where flow history effects are absent, the direction of speed change is immaterial and a common line is obtained. In the Taylor vortex range, separate torque curves are obtained for each vortex pattern, the greatest torque, for 24 cells, exceeding the lowest, for 16 cells by 7%.

Figure 6 shows the torque at the maximum speed in terms of cell size, bringing out the result that smaller cells produce larger torques but not supporting the view that a minimum energy criterion is involved in cell number selection. Gradual acceleration from rest for this annulus length produces 18 cells, the size being nearest to c, but 16 cells obtained by cell compression, give less torque. Impulsive starts give only smaller cells and higher torques and here at any rate a minimum energy criterion could be considered to apply.

5 CELL SIZE AND TORQUE AT CONSTANT SPEED

Expansion or compression of Taylor vortex cells by very slow oil inflow or out flow while maintaining a constant rotor speed produces large variations of cell size and leads to change-over lengths for cell number changes which differ from those obtained by accelerating the rotor from rest for each of a series of annulus lengths (Cole 1974 b). It has now been found that the corresponding variations of torque with depth can be quite striking. The effects obtained with continuous inflow are illustrated by the X - Y recorder chart shown in Figure 7, The axial flow rates correspond to Reynolds numbers $\bar{u}c/\nu$ of about 0.2, and complete inflow takes approximately 60 minutes.

At the two rotor speeds giving laminar circumferental flow, the expected linear relationship between torque and oil depth is obtained. At speeds above the Taylor critical value, the torque trace is no longer linear and exhibits discontinuous jumps which visual observation shows to coincide with changes in numbers of cells. It is seen that smaller torques correspond to expanded cells, and that the formation of smaller cells as the number of cells increases at change-over lengths leads to a sudden rise in torque. Thus the general behaviour is in line with that reported in Section 3, in that smaller cells involve higher torques.

The circled points in Figure 7, obtained at the end of each inflow run, represent the torques obtained by gradual acceleration from rest for the appropriate depth. These fall on the approximately linear upper envelopes of the inflow plots.

Figure 7 represents a non-steady state, small though the rate of inflow is, and prompts questions concerning response times. Snyder (1969 a) has measured the adjustment times for equalisation of cell sizes following a change of number of cells and has suggested an adjustment time of 0.15 L^2/ν for kinematic viscosity ν. For the results of Figure 7, this adjustment time would be as much as 3 minutes. Measurements of the torque adjustment time (including measuring system response time) in the present experiments show that the torque signal reaches equilibrium in 6 to 12 seconds. Stopping the inflow gives a steady value for the torque, confirming that the expanded cells are in a state of equilibrium.

Figure 8 reproduces another X - Y recorder chart, this time obtained with stepwise changes of oil level. An electromagnetically operated valve admits oil for a few seconds, a period of about one minute is allowed for the vortex pattern to settle down, and then the point plotter is operated. In some cases the change-over point has been caught during the pen contact time, illustrating the rapidity of the final vortex cell number change.

Fluid outflow produces generally similar stepped torque records although some anomalies have been encountered and are being examined.

6 CONCLUSION

A rudimentary theory for the prediction of the influence of annulus length and cell size on Taylor vortex critical speed has shown that the critical speed should rise for short cylinders, but that the effect for $L/c > 8$ is less than 1%.

Experimental measurements seem to confirm this, but a higher degree of precision is needed to settle the matter.

At speeds above the Taylor critical value, the experiments show that cell size influences torque to a small but systematic extent. Variations of torque of a few percent may be expected, with a dependence on flow history. The linear dependence of torque on annulus length which characterises laminar flow, and which is used to provide end corrections in rotating cylinder viscometry, disappears in the Taylor vortex regime. There, measurements involving a preset rotor speed and changes of annulus length can produce departures from non-linearity of as much as 10% at speeds well above the critical. However, measurements involving a gradual acceleration to speed for each successive length give non-linearities of around only 1%. These results have been obtained for cylinders of moderate length, $L/c < 40$.

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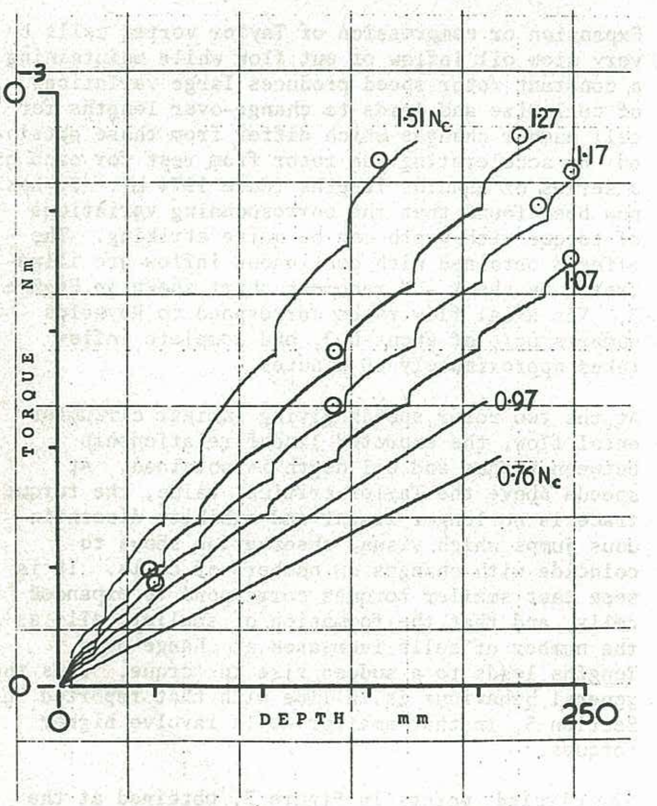

Figure 7 Torque-depth with slow continuous inflow. $n=1,2,3,4,6,\ldots 16$, $R_1/R_2=0.73$, $R_1=72.9$ mm, $\nu=0.50$St

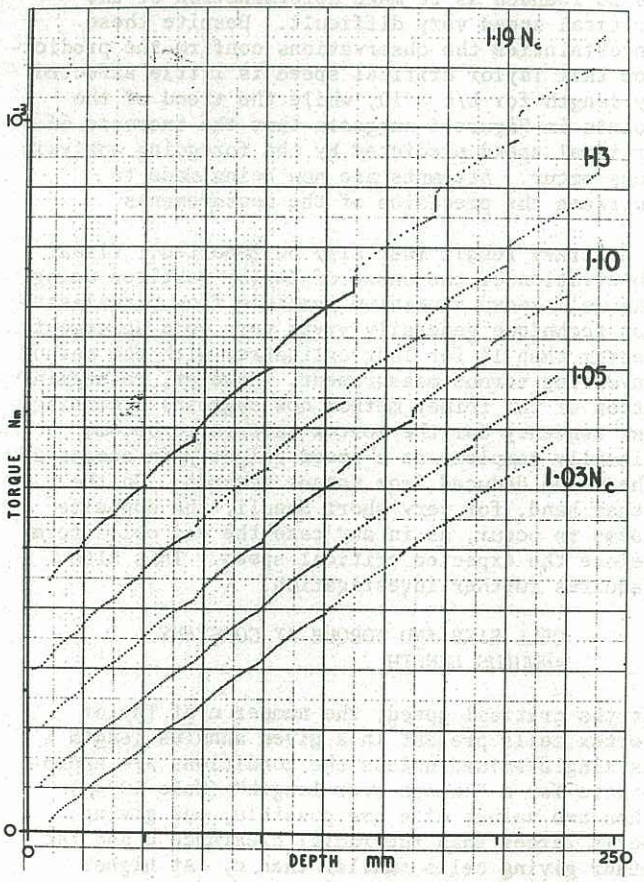

Figure 8 Torque-depth with stepwise inflow. $n=1,2,3,4,6,\ldots 16$, $R_1/R_2=0.73$, $R_1=72.9$ mm, $\nu=0.50$St