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An Analogue of the Wheatstone Bridge Using Critical Flow Nozzles

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SUMMARY A set of critical flow nozzles may be assembled to form the pneumatic equivalent of the electrical Wheatstone bridge; by this means a resolution of a part in 100,000 has been obtained in the relative measurement of the conductance of nozzles with only moderate (1/1000) requirements as to the control of supply pressure and temperature. The differential pressure measurement must be sensitive but need not have very wide span.

This procedure permits the accurate investigation of boundary layer effects in the nozzles, the effect of upstream conditions and of downstream back pressure. It also enables the measurement of the relative conductances of sets of nominally equal nozzles and the comparison of two or more nozzles in parallel with one of larger size, thus stepping up over a wide range. The nozzles can be enlarged by careful polishing so that closely matched sets can be made.

A limited number of high quality absolute measurements should permit the establishment of a standard scale for flow measurement of gases over a wide range of flow rates. This should be transferable to subsonic devices such as the orifice plate or the laminar flow element.

1 INTRODUCTION

Flow rate meters based on inferential devices such as orifice plates, nozzles and venturi tubes have long been in use. In these devices a pressure difference is developed between two points in the system and a measurement of this pressure difference can be used to calculate the flow rate, given a calibration of the device and a knowledge of the properties of the fluid. One rather more recently introduced variant is the Critical Flow Nozzle. It is usable only on gases as it depends for its operation on establishing a sufficiently large pressure drop to cause the velocity in the throat to reach the local velocity of sound in the gas; it cannot be used in circumstances where a high pressure loss is not permissible. However it has characteristics which make it very attractive for setting up calibration systems of high accuracy (Arnberg et al., 1973).

If a sufficiently large pressure ratio is established from the intake to the outlet of a suitable nozzle, the velocity of the gas in the throat reaches the velocity of sound in the gas at that point. If additional pressure is applied at the intake the velocity in the throat does not rise further, the gas density rises, the mass flow rises, but the volume flow remains constant.

Variations in the downstream pressure do not affect the mass flow rate while the velocity in the throat is sonic. The behaviour of the critical flow nozzle has a considerable resemblance to that of water flowing from one pool to another over a waterfall.

For an ideal gas and neglecting the boundary layer effects, the mass flow, M, through a critical flow nozzle is proportional to the upstream absolute total pressure and to the cross-sectional area of the throat. The conductance, C, may be defined by the equation

$$M = P.C. \qquad (1)$$

In practice C is not quite constant as it depends on real gas properties and Reynolds number.

If a means can be found to compare accurately the conductance of two nominally equal critical flow devices and then to compare these with another of double the conductance, it offers an attractive method of calibrating flow measurement standards.

2 WHEATSTONE BRIDGE

The pneumatic equivalent of a Wheatstone bridge can be made using four nozzles of conductance C_1, C_2, C_3 and C_4 as shown in Figure 1a. The power supply is a source gas under pressure, P_0. The equivalent of the galvanometer is a differential pressure detector to measure the out-of-balance pressure, ΔP. The values of the conductances may not be chosen with as much freedom as in the electrical case, as C_1 must be less than C_3 by a sufficient margin to prevent the flow through C_1 from becoming subsonic due to the back pressure induced by C_3. Similarly C_2 must be less than C_4.

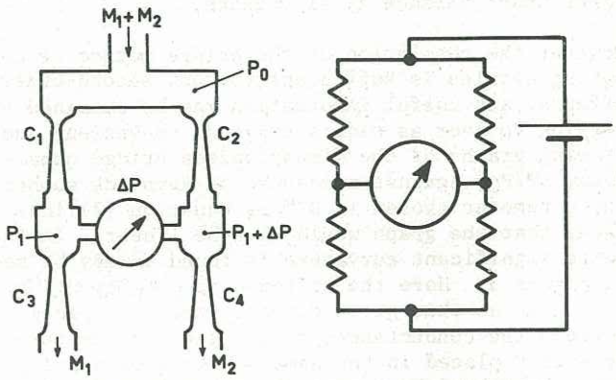


Figure 1a
Pneumatic bridge

Figure 1b
Wheatstone bridge

The left side of the bridge carries a flow M_1, the right side M_2, so for critical flow in all nozzles

$$P_0 = M_1/C_1 = M_2/C_2$$

$$P_1 = M_1/C_3$$

and

$$P_1 + \Delta P = M_2/C_4.$$

Hence

$$\Delta P/P_0 = C_2/C_4 - C_1/C_3. \qquad (2)$$

The nozzles used have the form shown in Figure 2, the radius, r, of the intake plane being twice the throat diameter, d. This design has been generally used as it is a satisfactory compromise between excessive boundary layer thickness in the throat at large values of r/d and an excessive reduction in C as compared with the geometrical area of the throat. Stratford (1964) has made theoretical investigations of such nozzles. His results, simplified for the present purposes, are that the conductance C is a function of Reynolds number, R, of the following form

$$C = C_0(1 - K(r/d)^{\frac{1}{4}} R^{-\frac{1}{2}}) \qquad (3)$$

where C_0 is the flow coefficient for an infinitesimal boundary layer. The equation applies to laminar boundary layers.

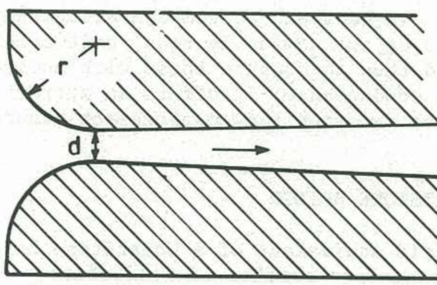
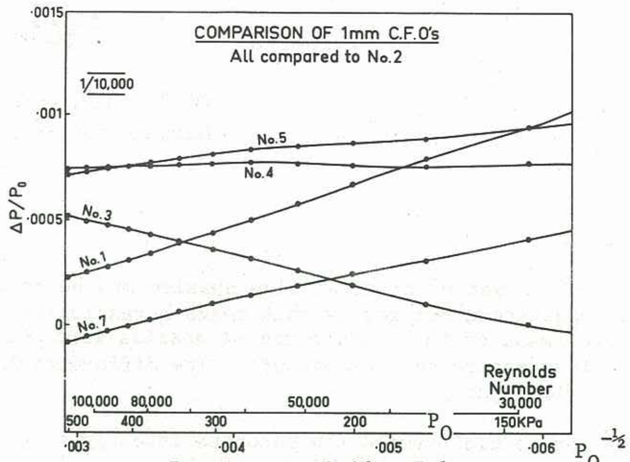


Figure 2 Orifice profile

3 EXPERIMENTAL

Sets of nozzles of nominal diameter 1.0 mm, 1.4 mm, and 2.0 mm have been made and compared under various arrangements. The simplest is that C_1 and C_3 are of 1 mm diam. and C_2 and C_4 are of 1.4 mm. This bridge is symmetrical and hence if it is adjusted (by polishing the bores of the nozzles) to balance at one supply pressure, it should to a first order balance at all others.

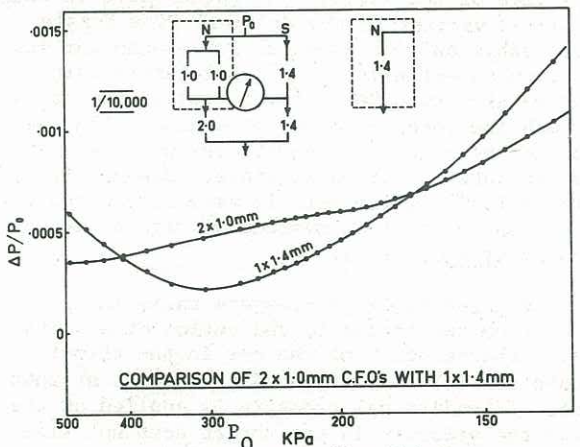
However the resolution of the bridge method of comparing nozzles is sufficient to show second-order effects, and useful information may be obtained by varying P_0 over as wide a range as convenient and drawing graphs of the dimensionless bridge unbalance, $\Delta P/P_0$, against a measure of Reynolds number. The parameter chosen is $R^{-\frac{1}{2}}$ as Equation (3) indicates that the graph would then be linear. In fact quite significant curvature is found as may be seen in Figure 3. Here the differences $(\Delta P_1/P_0 - \Delta P_2/P_0)$ are shown as this gives directly the difference between the conductances of the two nozzles successively placed in the same position of a bridge when the other three nozzles are unchanged. If the ratios r_1/d_1 and r_2/d_2 of a pair of nominally equal nozzles are not equal (due to polishing for fine adjustment of d_1 and d_2) then the lines could be sloped but on first-order boundary layer theory they should not be curved.



Pressure at Bridge Inlet

Figure 3

Nozzles of nominal size 1.4 mm were adjusted and compared with two nozzles of size 1.0 mm connected in parallel. In this case even for nozzles of similar shape a slope may be expected as the Reynolds number in a nozzle of size 1.4 mm is 1.4 times that in a 1 mm nozzle at equal pressures. As before there is also curvature as may be seen in Figure 4.



Pressure at Bridge Inlet

Figure 4

The effect of downstream pressure on the coefficient of a nozzle may be investigated by placing a needle valve downstream of one of the lower nozzles and measuring the pressure upstream of this needle valve. The results of such a measurement are shown in Figure 5, where the out-of-balance is plotted against the dimensionless ratio P_2/P_1 while P_0 is kept constant. It may be seen that over a wide range there is no detectable change in $\Delta P/P_0$ and at a fairly definite value of P_2/P_0 the change is quite large. It should also be noted that the onset of this change occurs at different values of P_2/P_1 depending on the supply pressure P_0.

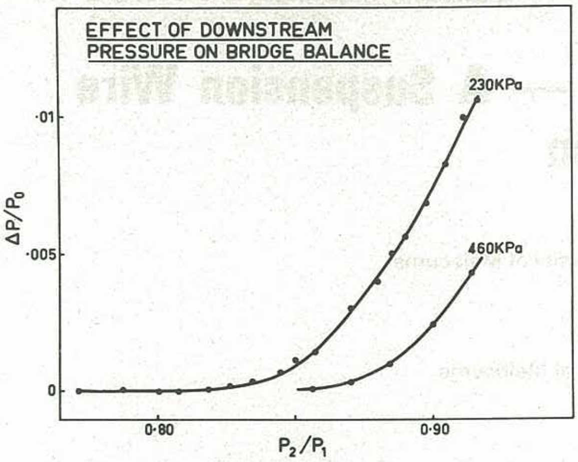


Figure 5 Effect of downstream pressure

4 CONCLUSION

The technique of using various bridges for intercomparing critical flow nozzles should enable the setting up of working standards of flow measurement for gases, it being assumed that (1) adequately accurate pressure measurements can be made, (2) proper corrections for real gas properties are made, and (3) that at one or a few places in the scale absolute flow measurements are established.

It is also possible to make sensitive differential measurements on the small contributions to the orifice coefficients due to boundary layer effects and hence to investigate certain aspects of the nature of such boundary layers.

Using a symmetrical bridge the balance is insensitive to real gas effects and to variations in the temperature of the whole bridge but not to temperature differences between sections of it, so it is neither possible nor necessary to measure real gas properties using such bridges. However if the left and right side of such a bridge were supplied with the same gas at two different controlled temperatures, or with different gases, it should be possible to make sensitive measurements relating to real gas properties; and hence available absolute data on some specified gases may permit easy extension to others not yet well measured.

5 REFERENCES

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