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Unsteady Free Convection Laminar Flow Past an Infinite Plate with Suction

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SUMMARY The two-dimensional incompressible flow of viscous and heat conducting fluid along an infinite flat plate has been considered. Firstly the suction velocity at the wall is taken constant and the velocity and the temperature distribution in the boundary layer are expanded as a series of n terms. Secondly the suction velocity is variable and depends upon certain parameter A. There is a constant magnetic field perpendicular to the wall. The co-efficients of heat conduction, magnetic susceptibility and viscosity are constant. The induced field has bee neglected. The flow phenomenon has been characterized by the magnetic parameter, frequency parameter and variable suction parameter and the role of these parameters on the flow characteristics has been studied.

1 INTRODUCTION

Lighthill [1] considered the response of skin-friction in laminar boundary layer of a fixed cylindrical body to unsteady fluctuations of the free stream velocity. Stuart [2] extended it to the case of an oscillatory flow past an infinite plane wall and obtained an exact solution of the Navier-Stokes equations and also compared some of his results with Lighthill's. Messiha [3] studied Stuart's [2] problem by introducing variable suction at the plate. Puri [4] considered Stuart's [2] problem in a rotating medium and studied the modification on the velocity profile, drag and the lateral stress on the plate.

The phenomenon of natural convection arises in a fluid when the temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This process of heat transfer has important technological applications e.g. in the cooling of nuclear reactors providing heat sinks in turbine blades and high speed re-entry vehicles. When a forced flow in a heat exchanger ceases, a free convention flow takes place when either of the plates on the fluid are at an elevated temperature. In nuclear reactor a significant reduction in heat transfer can be achieved by applying a magnetic field.

The purpose of the present paper is to investigate the influence in the unsteady free convection laminar flow past an infinite plate in the presence of uniform magnetic field. Two cases are considered. In the first case the suction velocity at the wall is constant. In the second case the suction velocity v_s is time-dependant. The Fourier expansion of v_s is assumed and the series expansion solution of the system analogous to the previous case but now with one coefficient dependant on time is sought. Also in this case the solution now approximated with the arbitrary order of accuracy for the small parameter ε is found. The gravitational forces perpendicular to the wall are taken into account. The fluid is electrically conducting and the magnetic field will produce an interaction with the fluid that crosses it. The velocity will not change in the direction parallel to the motion and will be zero in the direction normal to the plane of the magnetic field and plate motion. In the analysis of the present paper the viscous and Joulean dissipation have been neglected. The expressions for temperature distribution in the boundary layer and skin-friction are obtained. The discussion of the results obtained in particular the dependence of the solution on the parameter A is performed making it more accessible to technologists and experimentalists.

2 BASIC EQUATIONS

Let the $\bar{x}$-axis be along the plate, $\bar{y}$-axis perpendicular to the plate. Let $\bar{H}_0$ be the intensity of the magnetic field acting perpendicular to the plate. The equations which describe hydromagnetic free convection flow of a viscous incompressible fluid past an infinite flat plate are:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}_s \frac{\partial \bar{T}}{\partial \bar{y}} = k \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}_s \frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) - \frac{\sigma_o \bar{B}_0^2}{\rho} \bar{u} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \tag{2}$$

where $\bar{t}$ - time; $\bar{u}$ - the velocity component along the plate; $\bar{v}_s$ - a non-zero negative constant suction velocity; $\bar{T}$ - the temperature in the boundary layer; $\bar{B}_0 = \mu_c \bar{H}_0$ - the magnetic induction; σ_o - the electric conductivity of the fluid; ρ - the density; ν - the kinematic viscosity; μ_c - the magnetic permeability of the fluid; β - the coefficient of volume expansion; g - the acceleration due to gravity; k - the thermal diffusivity.

Introducing dimensionless quantities

$$u = \frac{\bar{u}}{|\bar{v}_s|};\ y = \frac{\bar{y}|\bar{v}_s|}{\nu};\ t = \frac{\bar{v}_s^2 \bar{t}}{\nu}$$

$$T = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty};\ Gr = \frac{\beta g \nu(\bar{T}_w - \bar{T}_\infty)}{|\bar{v}_s|^3} \tag{3}$$

$$Pr = \frac{\nu}{k}\ ;\ M = \frac{Pr \bar{B}_0^2 \nu}{\rho \bar{v}_s^2}$$

where $\bar{T}_w$ - the mean wall temperature, Gr - the Grashof number, Pr - the Prandtl number and M - the hydromagnetic parameter.

Thus using equation (3) in (1) and (2), we get

$$\frac{\partial^2 T}{\partial y^2} + Pr\frac{\partial T}{\partial y} - Pr\frac{\partial T}{\partial t} = 0 \tag{4}$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = - Gr + Mu \tag{5}$$

and boundary conditions

$$\left.\begin{aligned} y = 0&: T = 1 + \varepsilon e^{i\omega t} + \text{---} + \varepsilon^n e^{in\omega t},\ u = 0 \\ y \to \infty&: T \to 0,\quad u \to 0 \end{aligned}\right| \tag{6}$$

3 SOLUTION OF EQUATIONS

We consider two cases where in case (i), the suction velocity at the wall is constant and the velocity and temperature distribution in the boundary layer are expanded as a series of n terms and in case (ii) the suction velocity is variable and depends on certain parameter A such that $\varepsilon A < 1$. On putting $A = 0$, we get the solution for case (i).

Case (i): when suction velocity is constant we assume

$$u(y,t) = F_0(y) + F_1(y)\varepsilon e^{i\omega t} + \text{---} + F_n(y)\varepsilon^n e^{in\omega t} \tag{7}$$

$$T(y,t) = T_0(y) + T_1(y)\varepsilon e^{i\omega t} + \text{---} + T_n(y)\varepsilon^n e^{in\omega t} \tag{8}$$

and the boundary conditions are

$$\begin{aligned} y = 0&: \left|\begin{aligned} &T_0 = T_1 = \text{---------} = T_n = 1 \\ &F_0 = F_1 = \text{---------} = F_n = 0 \end{aligned}\right. \\ y = \infty&: \left|\begin{aligned} &T_0 = T_1 = \text{---------} = T_n = 0 \\ &F_0 = F_1 = \text{---------} = F_n = 0 \end{aligned}\right. \end{aligned} \tag{9}$$

Substituting equation (8) into (4) we have on equating the terms independent of $\varepsilon e^{i\omega t}$ to zero and equating the coefficients of harmonic terms

$$\left.\begin{aligned} T_0'' + PrT_0' &= 0 \\ T_0'' + PrT_1' - Pri\omega T_1 &= 0 \\ &\text{-------------} \\ &\text{-------------} \\ T_n'' + PrT_n' - Prin\omega T_n &= 0 \end{aligned}\right| \tag{10}$$

whose solutions by using the boundary conditions (9) are

$$\left.\begin{aligned} T_0(y) &= \exp(- Pry) \\ T_1(y) &= \exp(- h_1 Pry) \\ &\text{-----------} \\ &\text{-----------} \\ T_n(y) &= \exp(- h_n Pry) \end{aligned}\right| \tag{11}$$

where

$$h_n = 1/2[1 + (1 + \frac{4in\omega}{Pr})^{1/2}]\quad n = 1,2,\ldots \tag{12}$$

After the solutions T_0, T_1, - - - are known we may use these to find F_0, F_1 - - - as

$$F_0(y) = \frac{Gr}{Pr(Pr - 1)-M}(e^{-\lambda y} - e^{-Pry}) \tag{13}$$

$$F_1(y) = \frac{Gr}{Pr(Pr - 1)h_1^2 - M}(e^{-\ell_1 y} - e^{-h_1 Pry}) \tag{14}$$

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$$F_n(y) = \frac{Gr}{Pr(Pr - 1)h_n^2 - M}(e^{-\ell_n y} - e^{-h_n Pry}) \tag{15}$$

$$\left.\begin{aligned} \text{where } \lambda &= 1/2\ [1 + (1 + 4M)^{1/2}] \\ \ell_n &= 1/2\ [1 + (1 + 4\ in\omega + 4M)^{1/2}]\quad n=1,2\ldots \end{aligned}\right| \tag{16}$$

After calculating u and T, the non-dimensional form of the rate of heat transfer from the wall to the fluid

$$q = -Gr\ [\frac{\partial u}{\partial y}]_{y=0} = Gr\ Pr\ [1 + \varepsilon h_1 e^{i\omega t} + \ldots + \varepsilon^n h_n e^{in\omega t}] \tag{17}$$

and the expression for skin-friction

$$\tau_w = [\frac{\partial u}{\partial y}]_{y=0} = \frac{Gr}{Pr}\frac{Pr-\lambda}{(Pr-1)-M/Pr} + \frac{h_1\ Pr-\ell_1}{(Pr-1)h_1^2-M/Pr}\varepsilon e^{i\omega t} + \ldots + \frac{h_n\ Pr-\ell_n}{(Pr-1)h_n^2-M/Pr}\varepsilon^n e^{in\omega t} \tag{18}$$

when magnetic field is fixed and ω is large

$$\left.\begin{aligned} q &\sim Gr\ Pr\ (1 + \varepsilon e^{i\omega t}\sqrt{\frac{i\omega}{Pr}} + \ldots + \varepsilon^n e^{in\omega t}\sqrt{\frac{in\omega}{Pr}}) \\ \tau_w &\sim [\frac{Pr-\lambda}{Pr-1-M/Pr} + \frac{Pr\ \varepsilon\ e^{i\omega t}}{(2Pr+1)\ \sqrt{i\omega}} + \ldots + \frac{\varepsilon^n e^{in\omega t}}{(2Pr+1)\ \sqrt{in\omega}}] \\ &\text{where } \lambda = 1/2\ [1 + (1 + 4M)^{1/2}] \end{aligned}\right| \tag{19}$$

Case (ii): when suction velocity is assumed time-dependant $\bar{v}_s$ in (1) is replaced by

$$v_s\ [1 + \varepsilon\ A\ e^{i\omega t} + \ldots + (\varepsilon A)^n\ e^{in\omega t}] \tag{20}$$

from (4), (5) and (16)

$$\frac{\partial^2 T}{\partial y^2} + Pr\ [1 + \varepsilon\ A\ e^{i\omega t} + \ldots +$$

$$+ \ldots . + (\varepsilon A)^n e^{in\omega t}] \frac{\partial T}{\partial y} - Pr \frac{\partial T}{\partial t} = 0 \quad (21)$$

$$\frac{\partial^2 u}{\partial y^2} + [1 + \varepsilon A e^{i\omega t} + \ldots . + (\varepsilon A)^n e^{in\omega t}] \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = GrT + MU \quad (22)$$

Using (8) into (20) and equating the terms independent of time and harmonic terms and solving with the help of boundary conditions (9) we get,

$$To(y) = \exp(-Pr\, y)$$

$$T_1(y) = \exp(-h_1 Pry) + \frac{iPrA}{\omega}[e^{-Pr\, y} - e^{-h_1 Pr\, y}]$$

. . . . and so on.

Similarly,

$$F_0(y) = \frac{Gr}{Pr(Pr-1)-M}[e^{-\lambda y} - e^{-Pr\, y}]$$

$$F_1(y) = c\,(e^{-h_1 Pr\, y} - e^{-\ell_1 y}) + d\,(e^{-Pr\, y} - e^{-\ell_1 y}) + e\,(e^{-\lambda y} - e^{-\ell_1 y})$$

. . . . and so on.

where

$$h_1 = 1/2\,[1 + (1 + \frac{4i\omega}{Pr})^{1/2}] \qquad Re[h_1]>0$$

$$c = Gr\,(\frac{iPrA}{\omega} - 1)/[h_1^2 Pr^2 - h_1 Pr - (i\omega+M)]$$

$$d = -Gr\,Pr\,A\,(\frac{i}{\omega} + \frac{1}{Pr(Pr-1)-M})/[Pr^2 - Pr - (i\omega+M)]$$

After substituting in (7) and (8) and calculating u and T, the non-dimensional form of the rate of heat transfer from the boundary to fluid medium

$$q = -Gr(\frac{\partial T}{\partial y})_{y=0} = Gr\,Pr\,[1 + \varepsilon e^{i\omega t} h_1 (1 - \frac{iPrA}{\omega} + b\,\varepsilon e^{i\omega t}) + \frac{iPrA}{\omega}\varepsilon e^{i\omega t} + h_2 \varepsilon^2 e^{2i\omega t}(1-a-b) + a\,\varepsilon^2 e^{2i\omega t} + \ldots] \quad (23)$$

The expression for skin-friction

$$\tau_w = [\frac{\partial u}{\partial y}]_{y=0} = \frac{Gr(Pr-\lambda)}{Pr(Pr-1)M} + \varepsilon\, e^{i\omega t}[\ell_1(c+d+e) - Pr(ch_1+d) - e\lambda] + \ldots + \ldots + \ldots] \quad (24)$$

where

$$a = \frac{iPrA^2}{2\omega} - \frac{Pr\,A^2}{2\omega^2}\;;\; b = \frac{Pr^2 Ah_1 - iPr^3 A^2 h_1}{h_1^2 Pr^2 - h_1 Pr^2 - 2\,i\omega Pr}\;;$$

$$e = \frac{\lambda GrA}{[Pr(Pr-1)-M][\lambda^2 - \lambda - (i\omega+M)]} \quad (25)$$

When the magnetic field is fixed and ω is large, then we have

$$a \sim \frac{iPrA^2}{2\omega}$$

$$b \sim \frac{-A\sqrt{i\omega Pr} + iA^2 Pr\sqrt{i\omega/Pr}}{\sqrt{i\omega Pr} - i\omega} \quad (26)$$

$$c \sim \frac{Gr}{\sqrt{i\omega Pr} - i\omega(Pr-1)}$$

where d and e keep their original values if ω is not too high and thus

$$T_1 \sim \exp(-y\sqrt{i\omega Pr}) + \frac{iPrA}{\omega}[\exp(-Pr\, y) - \exp(-y\sqrt{i\omega Pr})] \quad (27)$$

$$T_2 \sim \exp(-y\sqrt{2i\omega Pr}) + \frac{iPrA^2}{2\omega}[\exp(-Pr\, y) - \exp(-y\sqrt{2i\omega Pr})] + [\frac{iA^2 Pr\sqrt{\frac{iPr}{\omega}} - A\sqrt{i\omega Pr}}{\sqrt{i\omega Pr} + i\omega}][\exp(-y\sqrt{i\omega Pr}) - \exp(-y\sqrt{2i\omega Pr})] \quad (28)$$

Substituting in (11) we get, the expression for temperature distribution

$$T_n(y) = \exp[-(Pr + \frac{\omega^2 n^2}{Pr})] - \exp[-iy(\omega n - \frac{2n^3\omega^3}{Pr^2})] \quad (29)$$

4 CONCLUSIONS

We see that the skin-friction and the rate of heat-transfer depends upon the fluctuating part of the suction velocity. The rate of heat-transfer from the wall to the fluid decreases as the suction velocity increases. It decreases uniformly for higher values of ω. Both skin-friction and heat-transfer at the wall decrease with the increase in the magnetic field for constant suction velocity. Physically it is also true. This is due to the fact that magnetic field exerts a retarding influence on the motion of the fluid which implies a reduction in the velocity gradient at the wall and consequently the skin-friction is reduced. Also

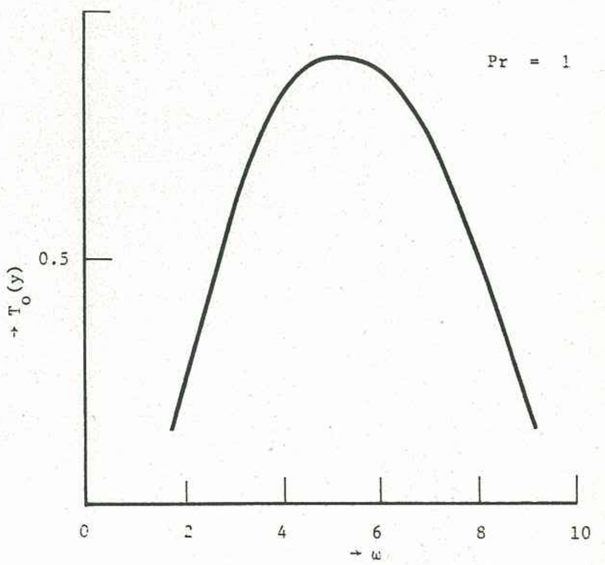

Figure 1 Variation of temperature distribution with ω

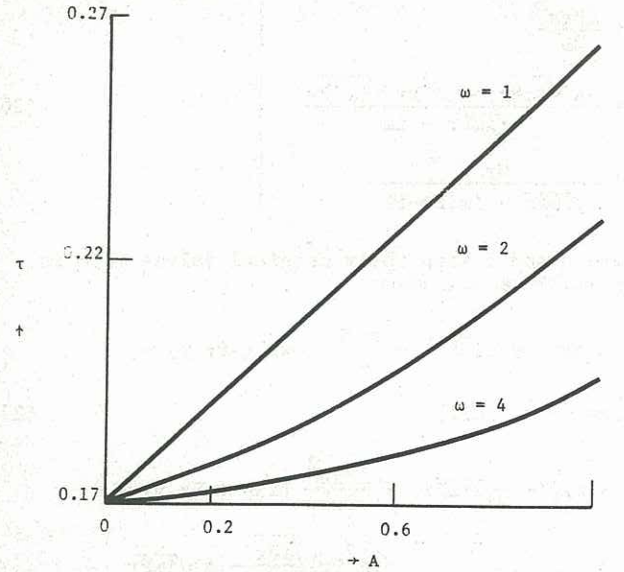

Figure 2 The variation of skin friction with A for large values of ω with fixed M

due to Reynold's analogy reduced skin-friction implies reduced rate of heat-transfer with increase in magnetic field.

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6 REFERENCES

LIGHTHILL, M.J., (1954), "The Response of Laminar Skin-friction and Heat-transfer to Fluctuations in the Stream Velocity", Proc. Roy. Soc. A., Vol. 224, pp. 1.

STUART, J.T., (1955), "A Solution of the Navier-Stokes and Energy Equations Illustrating the Response of Skin-friction and Temperature of an Infinite Plate Thermometer to Fluctuations in the Stream Velocity", Proc. Roy. Soc. A., Vol. 231, pp. 116.

MESSIHA, S.A.S., (1966), "Laminar Boundary Layers in Oscillatory Flow Along an Infinite Plate with Variable Suction", Proc. Camb. Phil. Soc., Vol. 62, p. 329.

PURI, P., (1975), "Fluctuating Flow of a Viscous Fluid on a Porous Plate in a Rotating Medium", Acta Mechanica, Vol. 21, pp. 153.

BRAR, G.S., (1977), "Similarity Solutions for Unsteady Hydro-magnetic Free Convection Flow", Journal de Mécanique (in Press).