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# Similarity of Reynolds Stress, Heat and Moisture Fluxes in Marine Boundary Layer

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SUMMARY Horizontal (u) and normal (w) velocity fluctuations were measured together with temperature ($\theta$) and humidity (q) fluctuations at a nominal height of 6 m above the mean water surface in the marine surface layer under nearly neutral conditions ($z/L = -.045$). The laboratory technique of Lu and Willmarth (1973) has been applied to the atmospheric records of uw, w$\theta$ and wq. In general, "ejections" contribute a larger fraction of the average fluxes than "sweeps", although the "sweep" event occurs more frequently than the "ejection" event. The period between "ejections" or "sweeps" is essentially independent of whichever instantaneous flux signal is processed. The probability that ejections (or sweeps) associated with the different signals uw, w$\theta$ or wq occur at the same time is found to be large. The period between ejections, when scaled with the friction velocity and the Monin-Obukhov length scale L, is in good agreement with the value obtained for a laboratory boundary layer.

## 1 INTRODUCTION

The transfers of momentum, heat and moisture within the Earth's boundary layer play an important role in influencing the atmospheric motions. There are numerous studies which have aimed at improving the basic understanding of transfer processes by examining mean and spectral properties associated with shear, moisture and heat flux variations. Few studies have been made on the similarity between the fluxes by analysing probability density functions of momentum and scalar fluctuations. In this regard, the main contributions have been from Holland (1967), McBean (1974), Antonia (1977a,b).

In the present paper, statistics of the products uw, $\theta$w, u$\theta$ and wq are measured in the atmospheric surface layer at a height of 6 m above a water boundary for a slightly unstable flow $z/L = -0.045$ (L is the Monin-Obukhov length). The technique developed by Lu and Willmarth (1973) to study the structure of Reynolds shear stress near a smooth wall of a laboratory turbulent boundary layer is applied here to instantaneous products uw, $\theta$w, qw, u$\theta$, wq, and $\theta$q.

## 2 EXPERIMENTAL TECHNIQUE

Measurements of streamwise velocity fluctuation u, vertical velocity w, temperature $\theta$ and humidity q were recorded on Kingfish B, the ESSO-BHP oil and gas platform in Bass Strait (Long. 148° 9'E, Lat. 38° 36'S) about 80 km off the Gippsland coast of Victoria. The instruments for recording the above signals were mounted about 6 m above the mean water level (on a vertical pipe) supported at the end of a horizontal boom fastened to one of the western legs of the platform. The boom was of sufficient length (about 15 m) to allow the measurement to be made clear of the platform disturbance. The fluctuation u was obtained with a hot wire (5 μm diameter platinum) operated by a non-linearized DISA 55M01, constant temperature anemometer. The fluctuation w was obtained using a Gill propeller. Temperature $\theta$ was detected with a cold wire (0.6 μm diameter platinum, 0.8 mm length) operated by a constant current anemometer. The value of the current was low enough (0.1 mA) for the wire to be sensitive to temperature fluctuations only. The humidity fluctuation q was obtained using a non-linearized Lyman-alpha humidiometer.

Voltages proportional to u, w, q and $\theta$ fluctuations were recorded on a four-channel Hewlett-Packard 7060 FM tape transport. The recording speed was 24 mm/s (-3 dB point of tape recorder 375 Hz). The tapes were played back and digitized at a sampling frequency of 20 Hz in the Faculty of Engineering Computing Centre of the University of Sydney. Prior to digitization the signals were low-pass filtered with the -3 dB cut-off frequency set at 10 Hz. The digital records were processed on a PDP 11/45 computer. The statistics of two similar records were examined, the experimental details for these runs being summarised in Table I.

A full description of the experimental site, the measurement procedure and background information is provided by Antonia *et al.* (1977).

## 3 RESULTS AND DISCUSSION

Contributions to <xy> from the four quadrants of the xy plane were computed in a manner similar to that outlined in Lu and Willmarth (1973). The contribution to <xy> from the region of the $i^{th}$ quadrant that lies outside the hole (the hole size is set by hyperbola $|xy| = H\sigma_x\sigma_y$) is denoted by $\tilde{xy}_i(H)$, such that

$$\frac{\tilde{xy}_i(H)}{\langle xy\rangle} = \frac{1}{\langle xy\rangle} \lim_{T\to\infty} \frac{1}{T}\int_0^T xy(t)\, S_i(t, H)\, dt$$

where i = 1, 2, 3, 4 and $S_i(t, H)$ is the random square wave function defined as

$$S_i(t, H) = \begin{cases} 1, & \text{when } |xy| > H\sigma_x\sigma_y \text{ and the point (xy) is in the } i^{th} \text{ quadrant} \\ 0, & \text{otherwise} \end{cases}$$

Values of $\tilde{xy}_i(o)/\langle xy\rangle$ are shown in Table I for all the quantities uw, w$\theta$, wq, u$\theta$, uq and q$\theta$ at two different points in time but at effectively the same height above the sea surface and for the same stability condition. The agreement between values of $\tilde{xy}_i(o)/\langle xy\rangle$ for these two points is satisfactory. Also shown in Table I is the occupancy time $\gamma_i$, or proportion of time that the signal spends in the $i^{th}$ quadrant, i.e.,

TABLE I

PERCENTAGE CONTRIBUTIONS TO <xy> AND OCCUPANCY TIME $\gamma$

| Experimental details | x | y | $-R_{xy}$ | Quadrant 1: $\widetilde{xy}_1/\langle xy\rangle$ | $\gamma_1$% | 2 Ejection: $\widetilde{xy}_2/\langle xy\rangle$ | $\gamma_2$% | 3: $\widetilde{xy}_3/\langle xy\rangle$ | $\gamma_3$% | 4 Sweep: $\widetilde{xy}_4/\langle xy\rangle$ | $\gamma_4$% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Date 23/8/76<br>Start Time 17.59 hrs<br>Duration 57.6 min<br>z/L = -0.045<br>$u_* = .31$ m/s<br>$\theta_* = .040^\circ$C<br>$q_* = .080$ g/m$^3$<br>$\sigma_u = .95$ m/s<br>$\sigma_w = .33$ m/s<br>$\sigma_\theta = .25^\circ$C<br>$\sigma_q = .22$ g/m$^3$ | u | w | .32 | -.30 | 23 | .85 | 30 | -.24 | 17 | .69 | 30 |
| | | | | *-.28** | *20** | *.78* | *30* | *-.28* | *20* | *.78* | *30* |
| | -θ | w | .15 | -.74 | 29 | 1.27 | 24 | -.80 | 16 | 1.27 | 31 |
| | | | | *-.82* | *23* | *1.32* | *27* | *-.82* | *23* | *1.32* | *27* |
| | -q | w | .34 | -.23 | 21 | .76 | 31 | -.23 | 18 | .70 | 30 |
| | | | | *-.24* | *19* | *.74* | *31* | *-.24* | *19* | *.74* | *31* |
| | u | θ | .50 | -.09 | 12 | .70 | 27 | -.11 | 19 | .50 | 41 |
| | | | | *-.11* | *17* | *.61* | *33* | *-.11* | *17* | *.61* | *33* |
| | -θ | q | .49 | -.10 | 21 | .64 | 28 | -.11 | 12 | .57 | 39 |
| | | | | *-.11* | *17* | *.61* | *33* | *-.11* | *17* | *.61* | *33* |
| Date 23/8/76<br>Start Time 22.00 hrs<br>Duration 57.6 min<br>z/L = -.045<br>$u_* = .37$ m/s<br>$\theta_* = .067^\circ$C<br>$q_* = .059$ g/m$^3$<br>$\sigma_u = 1.04$ m/s<br>$\sigma_w = .36$ m/s<br>$\sigma_\theta = .33^\circ$C<br>$\sigma_q = .16$ g/m$^3$ | u | w | .35 | -.24 | 20 | .80 | 29 | -.21 | 18 | .65 | 33 |
| | | | | *-.23* | *19* | *.73* | *31* | *-.23* | *19* | *.73* | *31* |
| | -θ | w | .20 | -.55 | 23 | 1.10 | 26 | -.59 | 21 | 1.04 | 31 |
| | | | | *-.55* | *22* | *1.05* | *28* | *-.55* | *22* | *1.05* | *28* |
| | -q | w | .37 | -.19 | 18 | .73 | 31 | -.21 | 19 | .67 | 33 |
| | | | | *-.21* | *19* | *.71* | *31* | *-.21* | *19* | *.71* | *31* |
| | u | θ | .40 | -.21 | 18 | .72 | 28 | -.16 | 18 | .65 | 35 |
| | | | | *-.18* | *18* | *.68* | *32* | *-.18* | *18* | *.68* | *32* |
| | u | q | .64 | -.06 | 15 | .63 | 35 | -.04 | 12 | .47 | 39 |
| | | | | *-.05* | *14* | *.55* | *36* | *-.05* | *14* | *.55* | *36* |
| | -θ | q | .56 | -.09 | 18 | .64 | 31 | -.08 | 14 | .53 | 36 |
| | | | | *-.08* | *16* | *.58* | *34* | *-.08* | *16* | *.58* | *34* |

* Values in italics are contributions from joint Gaussian distribution.

$$\gamma_i = \lim_{T\to\infty} \frac{1}{T}\int_0^T S_i(t, o)\, dt.$$

Values in italics in Table I for $\widetilde{xy}_i(o)/\langle xy\rangle$ and $\gamma$ were obtained from a joint Gaussian probability density of x and y with a correlation coefficient $R_{xy}$ equal to the experimental value.

Although the occupancy time is, in general, slightly larger in the sweep than in the ejection quadrant, the contribution $\widetilde{xy}_2$ is in general greater than $\widetilde{xy}_4$, as a result of the stretching out of probability density contours in the ejection quadrant. In the case of uw, the difference between $\widetilde{xy}_2$ and $\widetilde{xy}_4$ is of the order of 15%. This difference is less marked in the case of wq and, in particular, wθ. In all cases, contributions to <xy> and occupancy times in quadrants 1 and 3 (the interaction quadrants) are nearly equal. In the case of uq and uθ, for which the correlation coefficients are fairly high, contributions to <xy> from interaction quadrants are noticeably small. The reasonable agreement between measured and Gaussian values of $\langle\widetilde{xy}_i\rangle/\langle xy\rangle$ in Table I is consistent with the agreement between measured and Gaussian contours of p(x, y), as reported in Antonia and Chambers (1977).

Values of $\widetilde{xy}_i(H)$, for different values of H, were obtained in the case of uw, wθ and wq. As the behaviour of relative contributions from the different quadrants as H increases, is qualitatively similar for all three products, only contributions $\widetilde{uw}_i(H)$ are shown in Figure 1. The contribution from the ejection quadrant remains larger than that for the sweep quadrant at all values of H, while contributions from the first and third quadrants are approximately equal. The definition of ejections and sweeps used by Lu and Willmarth appears to apply equally well here. At H ≈ 2.5, contributions to <uw> from interaction quadrants become negligible

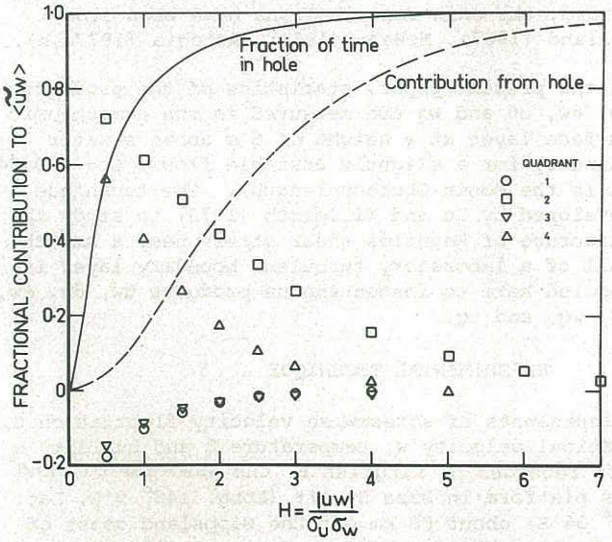


Figure 1 Fractional contribution <uw> from four quadrants in (u, w) plane. Broken curve represent the contribution of hole region to <uw>. Solid curve is the fraction of time that the signal is inside the hole

(less than 1%) while at H ≈ 4.5, the contribution from the sweep quadrant also disappears. We can therefore follow Lu and Willmarth's example and arbitrarily define an ejection event when H ≈ 4.5 and a sweep event when H ≈ 2.5. The mean values of the time between ejection and sweep events is shown, as a function of H, in Figure 2. The only significant difference in the results of $\overline{T}_e$ and $\overline{T}_s$ as derived from the three different signals is that the mean periods in the case of uw are generally slightly

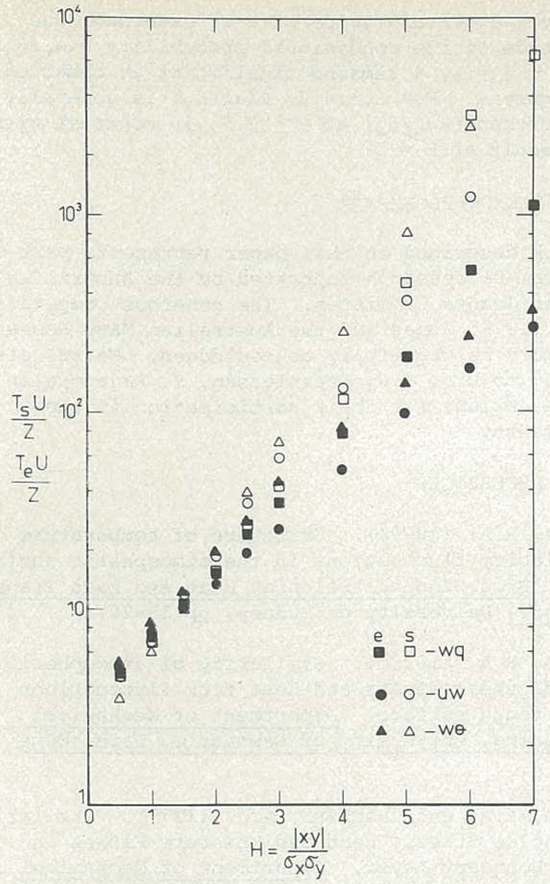


Figure 2 Mean period between ejections and sweeps as a function of hole size

lower than for wθ or wq. For H ≃ 4.5, a reasonable average value of $\overline{T}_eU/z$ appears to be 90 while for H = 2.5, $\overline{T}_sU/z$ ≃ 35. This difference is at variance with the more reasonable observation, by Lu and Willmarth, that $\overline{T}_s \simeq \overline{T}_e$. It may be pointed out that the present values of $\overline{T}_e$ and $\overline{T}_s$ would not be changed if the definition H = constant for an event is replaced by H' = constant where $H' \overset{def}{=} |xy|/\langle xy \rangle = H/R_{xy}$. For uw and wq, $R_{xy}$ is not too different from the value of about 0.4 that was essentially constant across the laboratory boundary layer of Lu and Willmarth. For wθ, $R_{w\theta}$ is in this case only about 0.2 so that lower values of $(\overline{T}_e)_{w\theta}$ and $(\overline{T}_s)_{w\theta}$ would be obtained if the criterion H' = const was used. The situation, in the case of wθ, would be further agravated by the trend which indicates that more appropriate values of H for defining ejections and sweeps may be slightly higher than 4.5 and 2.5 respectively (Antonia and Chambers, 1977). In this connection, it must be noted that Perry and Hoffmann (1977) found that, in a slightly heated laboratory boundary layer the use of wθ led to a value of $\overline{T}_e$ more than twice as large as that obtained from uw.

In spite of the inevitable arbitrariness in defining "ejection" and "sweep" events, it still seems useful to compare the present values of $\overline{T}_e$ (or $\overline{T}_s$) with those obtained by laboratory boundary layer (e.g. Lu and Willmarth, Perry and Hoffmann) with the same hole size technique and identical values of H. This comparison is given in Table II, which also includes estimates of $\overline{T}_b$ the "bursting" period determined by Rao *et al.* (1971) with a procedure that is substantially different from the hole size technique and which does not distinguish between ejections and sweep events. The agreement between the present atmospheric values of $\overline{T}_e$ and the laboratory values is reasonable when the Monin-Obukhov length L is used instead of the boundary layer thickness δ as the normalizing length scale. The replacement of δ by L is not obvious since a reasonable estimate of the neutral boundary layer thickness of the earth is 0.25 $u_*/f$ ($u_*$ is the friction velocity, f is the Coriolis parameter) which, for the latitude of our experimental site and prevailing wind conditions, amounts to a height which is 6-7 times larger than the experimental value of |L|. It is clear that more estimates of $\overline{T}_e$ are required over a wider range of z (preferably extending beyond the region of the constant flux layer) and stability conditions before L can be advanced as the appropriate normalizing length scale for $\overline{T}_e$.

TABLE II

VALUES OF $\overline{T}_e$ OBTAINED BY VARIOUS INVESTIGATORS

| Investigators | Normalized Ejection Period | Reynolds No. (Experimental Technique) |
|---|---|---|
| Lu and Willmarth (1973) | $u_*\overline{T}_e/\delta = 0.151$ | $u_*\delta/\nu = 1.92\times10^3$ (Hole size) |
| Lu and Willmarth (1973) | $u_*\overline{T}_e/\delta = 0.102$ | $u_*\delta/\nu = 1.75\times10^4$ (Hole size) |
| Perry and Hoffmann† (1977) | $u_*\overline{T}_e/\delta = 0.192$ | $u_*\delta/\nu = 4.97\times10^4$ (Hole size) |
| Present | $u_*\overline{T}_e/\|L\| = 0.139$ | $u_*L/\nu = 2.75\times10^6$ (Hole size) |
| Rao *et al.* (1971) | $u_*\overline{T}_b/\delta \simeq 0.18$ | $300 < u_*\delta/\nu < 2500$ (Band-pass filtering technique) |

† Values of $\overline{T}_e$ obtained from uw only is quoted here

It is of interest to note that Monji's (1973) simultaneous temperature traces over a wide vertical separation ($2.9 \times 10^{-4} < z/|L| < .43$), exhibit a marked degree of coherence, especially with respect to the larger scale turbulence structure. This coherence, over a length scale of order |L|, is not dissimilar to that observed by Chen (1975), over a length scale of order δ (boundary layer thickness), in a heated turbulent boundary layer.

Variations of $\overline{\Delta T}_e$ and $\overline{\Delta T}_s$, the average durations of "ejections" and "sweeps" respectively, with hole size H are similar (Figure 3) to those reported by Lu and Willmarth. With characteristic mean durations of ejections and sweeps defined at H = 4.5 and H = 2.5 respectively, we find that $\overline{\Delta T}_e \simeq \overline{\Delta T}_s$, with a magnitude $\overline{\Delta T}_eU/z \simeq 0.4$. This corresponds to a value of $\overline{\Delta T}_eu_*/|L| = 6.2 \times 10^{-4}$ in fair agreement with the value of $\overline{\Delta T}_eu_*/\delta = 7.9 \times 10^{-4}$ derived from Lu and Willmarth's single measurement (z/δ ≃ 0.1)

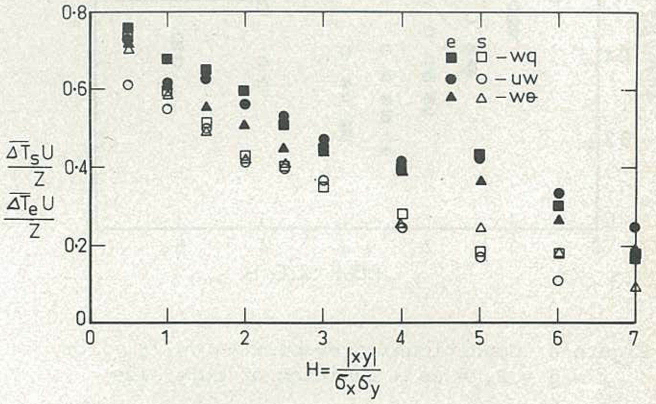


Figure 3 Average duration of ejection and sweep as a function of hole size

at the higher Reynolds number ($u_*\delta/\nu \simeq 17.5 \times 10^4$)+. It is also worth recording that the ratio of the characteristic length scale $U\overline{\Delta T}_e$ of an ejection (or sweep) to the Kolmogorov length scale is about 3000 in the present experiment, compared with about 20 in the case of Lu and Willmarth.

It is useful to compare the joint probability of occurrence of signals such as uw and wθ (or wq) in the various quadrants of the xy plane since this information may shed further light on the similarity between the fluctuations of uw and wθ (or wq). The conditional probability $P(\alpha_i/\beta_j)$ is used here to denote the probability that the event α in the $i^{th}$ quadrant occurs whenever the event β in the $j^{th}$ has already occured (i, j = 1, 2, 3, 4). The conditional probabilities of main interest here are

$$P[(uw)_i/(w\theta)_j] = \begin{pmatrix} .55 & .28 & 0 & 0 \\ .45 & .72 & 0 & 0 \\ 0 & 0 & .47 & .27 \\ 0 & 0 & .53 & .73 \end{pmatrix}$$

$$P[(uw)_i/(wq)_j] = \begin{pmatrix} .69 & .24 & 0 & 0 \\ .31 & .76 & 0 & 0 \\ 0 & 0 & .61 & .20 \\ 0 & 0 & .39 & .80 \end{pmatrix}$$

$$P[(w\theta)_i/(wq)_j] = \begin{pmatrix} .70 & .35 & 0 & 0 \\ .30 & .65 & 0 & 0 \\ 0 & 0 & .59 & .27 \\ 0 & 0 & .41 & .73 \end{pmatrix}$$

with the convention that subscripts i and j denote the positions of the rows and columns respectively (the sum of the elements in any column is equal to unity). It is clear from these probability arrays that the dominant elements are $P(\alpha_2, \beta_2)$ and $P(\alpha_4, \beta_4)$ with the latter being slightly larger than the former, probably as a result of the larger occupancy time of all signals in the sweep quadrant. Previously quoted values of $P(\alpha_i/\beta_j)$ account for all occurrences in a given quadrant, i.e. they correspond to H = 0. If events are considered

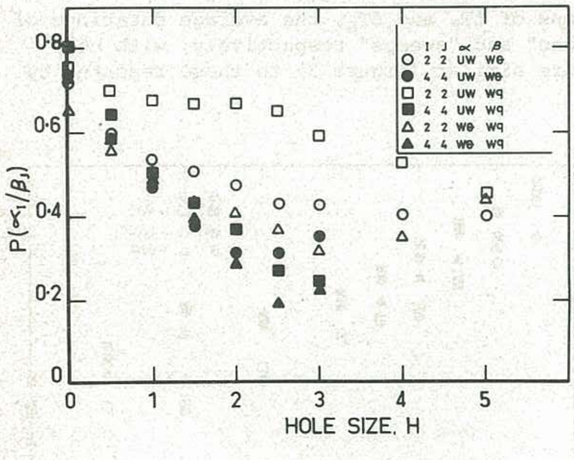


Figure 4 Conditional probability $P(\alpha_i/\beta_j)$ for i = j = 2, 4 as a function of hole size

+ The lower Reynolds number ($u_*\delta/\nu = 1.92 \times 10^3$) measurements suggest a value $\overline{\Delta T}_e u_*/\delta = 2.3 \times 10^{-3}$ almost constant across the layer.

whenever a certain value of H is exceeded, the magnitude of the conditional probability $P(\alpha_i/\beta_j)$ for i = j = 2, 4 remains significant as observed in Figure 4. For H = 4.5, $P(\alpha_2/\beta_2)$ is generally larger than $P(\alpha_4/\beta_4)$ at H ≃ 2.5, in contrast with the result at H = 0.

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