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Filmwise Condensation of Vapour on the Outside of a Rotating Cylinder with Dropwise Removal of Condensate

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SUMMARY When vapour condenses on a cylinder rotating at such speed that centrifugal force greatly exceeds gravity, the condensate will be removed solely by drop-flinging. The present paper is an examination of this process. The mechanics of formation, growth and dislodgment of drops are studied theoretically and numerical solutions of the equations are developed. Confirmatory experimental observations are analysed and presented finally in the form of an equation which enables heat transfer coefficients to be calculated for any given combination of size, speed and liquid.

1 INTRODUCTION

When a vapour condenses on a solid surface, heat transfer is vitally affected by the mechanism of removal of the condensate. On static surfaces it usually flows off by gravity in a film, or as drops if it is non-wetting. On rotating surfaces which are radial, conical, or curved, the flow of condensate may by strongly assisted by centrifugal action and thus heat transfer may be improved. The classical analysis of these situations by Nusselt and other later workers deals with films, waves, drops etc. moving tangentially to the solid surface. Here, however, we intend to consider a rotating cylinder with condensate being removed by drop-throwing in a direction perpendicular to the surface - a mechanism essentially different from the others.

A cylinder does not have to be very big nor rotating very fast to generate a centrifugal action that overshadows gravity. A cylinder of 100 mm diameter rotating at 1000 rev min^{-1} produces a body force of 56 g for instance, while one of 200 mm diameter rotating at 3000 rev min^{-1} generates 1006 g. In this context the formation and removal of drops and the related heat transfer process will now be considered.

1.1 NOTATION

- C Condensation rate expressed as a liquid velocity normal to the surface
- g Acceleration due to gravity
- k Thermal conductivity
- μ Absolute viscosity
- ρ Density of liquid
- σ Surface tension of liquid
- ω Angular velocity

2 THE FORMATION OF DROPS

The basis of this discussion is as follows:

(1) We are dealing with a circular cylinder rotating about its axis, the size and speed being such that gravitational forces may be neglected.

(2) The body force is thus everywhere outwards normal to the surface of the cylinder. It will have the value $\omega^2 \times$(radius of cylinder) and we will call it G.

(3) A rough derivation assuming a spherical shape for a drop as it detaches from the surface gives the radius of the drop as $\sqrt{\frac{3\sigma}{\rho G}}$, whereas some simple experiments suggest it is closer to $\sqrt{\frac{2\sigma}{\rho G}}$. In any event the size of the drop for any known liquid is quite small compared to the radius of the cylinder under the conditions we are considering. Thus the effect of the curvature of the cylinder is neglected and the drops are regarded as detaching themselves from a flat surface under the influence of normal body force G away from the surface.

We may thus quite satisfactorily begin by studying the quiescent formation of hanging drops beneath a wet horizontal surface as in Figure 1. The development of the drop is taken to be slow enough for any internal accelerations to be neglected, so that for the bulk of the drop it is a matter of static equilibrium between surface tension, pressure and body forces. Only at the extreme edges will the inflow of liquid invalidate this assumption.

At some level, which we call the "ambient pressure line" the pressure inside the drop is the same as that outside. Below this line the pressure is greater and maintained so by the surface tension and convexity of the surface. Above the line the pressure is less and maintained so by the surface tension and concavity of the surface, except at the extreme edges. Near the edges there is a pressure gradient up to the ambient value in the surrounding film, which tends to cause flow of liquid into the drop.

Now for the bulk of the drop, using the notation and co-ordinate system shown in Figure 1 we may obtain the equation

$$\frac{\dfrac{d^2z}{dr^2}}{\left\{1+\left(\dfrac{dz}{dr}\right)^2\right\}^{3/2}} + \frac{\dfrac{dz}{dr}}{r\left\{1+\left(\dfrac{dz}{dr}\right)^2\right\}^{1/2}} = \frac{\rho G}{\sigma}(h-z) \qquad (1)$$

which describes the profile of the drop.

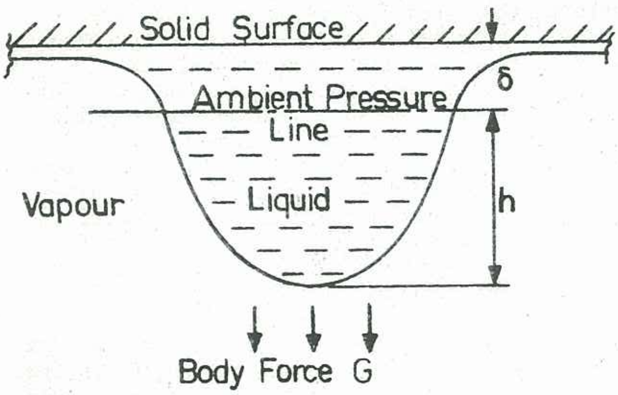
Equation (1) was normalised using the quantity $\sqrt{\frac{2\sigma}{\rho G}}$ as reference length, so that $Z = \frac{z}{\sqrt{\frac{2\sigma}{\rho G}}}$, $R = \frac{r}{\sqrt{\frac{2\sigma}{\rho G}}}$, and $H = \frac{h}{\sqrt{\frac{2\sigma}{\rho G}}}$.

The normalised equation is then

$$\frac{\dfrac{d^2Z}{dR^2}}{\left\{1+\left(\dfrac{dZ}{dR}\right)^2\right\}^{3/2}} + \frac{\dfrac{dZ}{dR}}{R\left\{1+\left(\dfrac{dZ}{dR}\right)^2\right\}^{1/2}} = 2(H-Z) \qquad (2)$$

The quantity H is now a shape factor describing the stage of development of a drop as it is fed with liquid from the surrounding film. Equation (2) was solved numerically for a series of values of H ranging from 0.1 to 2.0. In each case the computation was continued until the slope $\frac{dZ}{dR}$ became 0.01. Thus the profiles obtained represent drops of liquid wetting and clinging to the solid surface with negligible contact angle. The results are shown plotted in Figure 2. The computer programme also included calculations of the surface area, volume and centre of gravity for each profile, so that the stability of the configuration could be examined.

A plot of the volume V as related to H (Figure 3) gives an indication of where the break-away of the drop will occur. Volume V increases with H until



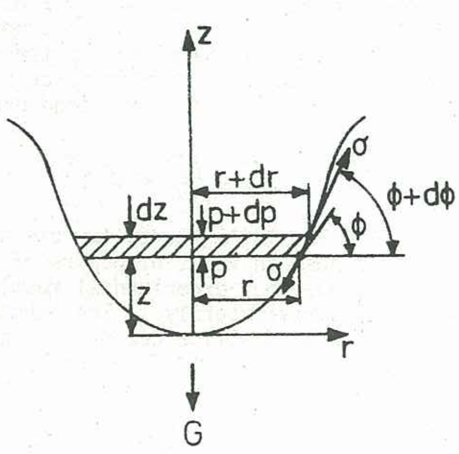


Figure 1

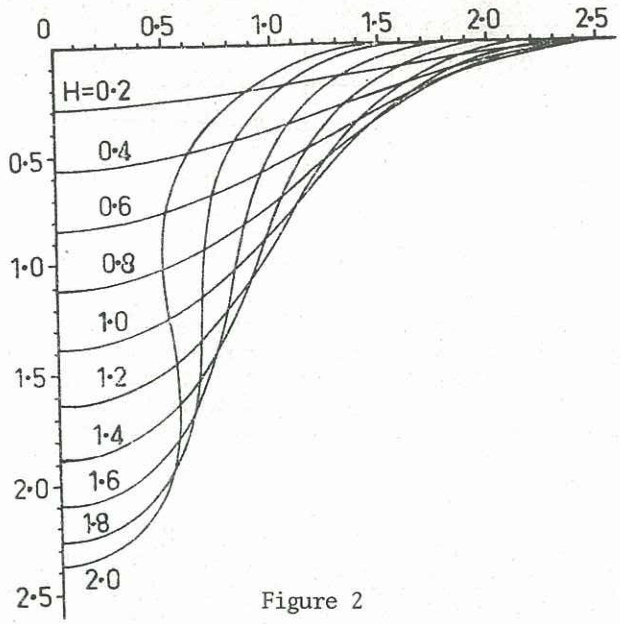


Figure 2

H becomes 1.1 after which it (theoretically) decreases. Since this is physically unrealistic it indicates that breakaway should occur when H reaches 1.1. Energy calculations similarly agree with this indication, which was finally confirmed experimentally by photography and by catching and weighing drops.

The findings of the foregoing work are that

1. a hanging drop profile becomes unstable and a drop breaks away when H reaches 1.1, at which value the volume in the whole profile is $6.7a^3$, where $a \equiv \sqrt{\frac{2\sigma}{\rho G}}$.

2. the volume of the break-away drop is $4.9a^3$ and its effective radius 1.05a.

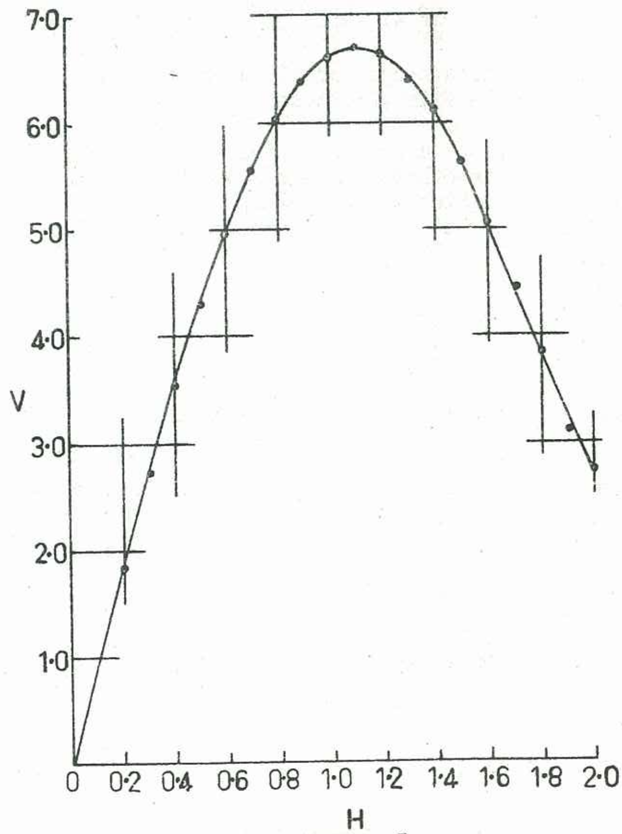


Figure 3

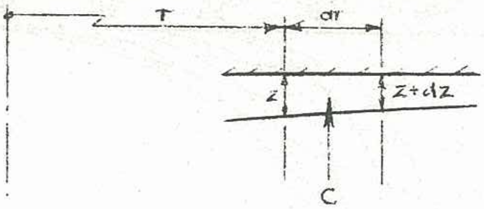


Figure 4

3. after the drop breaks away the remaining liquid re-forms a hanging drop profile corresponding to H = 0.2, with volume $1.8a^3$, and as further liquid is fed to it H will increase and the process will be repeated.

2.1 The Feeding Process

It was initially assumed that the forming drop would be fed by viscous flow of liquid from the surrounding film as impelled by the difference between ambient pressure in the film and the lowered pressure at the drop site. The "head" causing flow would be δ, for which the computed value ranges from 0.08a to 0.42a as H ranges from 0.2 to 1.1.

The Navier-Stokes equation for axi-symmetric flow with cylindrical co-ordinates (with zero terms omitted) is

$$\rho v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + 2\frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial r}\right) + \mu\frac{\partial^2 v}{\partial z^2} + \frac{2\mu}{r}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)$$

Order-of-magnitude examination of the terms above indicates that for the problem in question the term $\mu\frac{\partial^2 v}{\partial z^2}$ is at least two orders of magnitude greater than the other terms, so that

$$\frac{dp}{dr} \simeq \mu\frac{d^2 v}{dz^2}$$

From this it is easy to show that

$$\frac{dp}{dr} \simeq -\frac{3\mu\overline{v}}{z^2} \qquad (3)$$

where $\overline{v}$ is the average radial velocity and z is the film thickness.

We now determine a continuity equation for the flow in the film together with the condensation into the film. The latter is approximated by a constant velocity C normal to the surface at all points, as in Figure 4.

From this we obtain

$$\overline{v} = -\frac{C(r_0^2 - r^2)}{2zr} \qquad (4)$$

where r_0 is the radius at which $\overline{v}$ is zero and z is the film thickness.

At any point along the solid surface the pressure p is $-\rho Gz$ and if we combine this with equations (3) and (4) we get

$$-\frac{dz}{dr} \simeq \frac{3\mu C}{2\rho G z^3} \cdot \frac{r_0^2 - r^2}{r} \qquad (5)$$

Now if (5) is normalised by again using a as reference length we get

$$-\frac{dZ}{dR} \simeq \frac{3}{4}\frac{\mu C}{\sigma} \cdot \frac{R_0^2 - R^2}{Z^3 R} \qquad (6)$$

There is then the problem of interfacing equations (2) and (6) to get some approximate description of the hanging drop together with the surrounding film in which liquid is flowing to the drop. A rough approach was made by calculating the profile using equation (2) up to some arbitrarily selected value of $\frac{dZ}{dR}$, then using equation (6) with an arbitrarily selected value of R_0. It was found that no matter what realistic selections of $\frac{dZ}{dR}$ and R_0 were made, no practically useful value of $\frac{\mu C}{\sigma}$ would be obtainable if R_0 was more than about 2.6. Inspection of Figure 2 will show that this implies that the solid surface would be almost completely covered by drops, and consequently any heat transfer would be very much hindered.

However, everyday observation of, for instance, drops of water forming and falling off under the eave of a building indicates that the whole surface is not covered with incipient drops but rather that these have a sideways mobility and tend to "sweep up" nearby areas. This sweeping up movement appears to be the means of drop growth, rather than flow in the film to a stationary drop. The mobility of forming drops would also be assisted by vapour drag on the rotating cylinder.

If this is so, then what area will a drop sweep under conditions of continuous condensation? We form an idea of this by looking at Figure 2. A hanging drop profile has a radius about 2.5a. Suppose another drop profile exists at centre distance 5a from the first, i.e. almost touching. Then it is easy to believe that any slight disturbance would cause the two to coalesce. This suggests that with continuous condensation going on the centre distance between drops would be about 10a, because any drop that attempted to form between would be quickly "swallowed". The pattern of drops on the surface could thus be expected to average out to something like that in Figure 5, the spacing s being about 10a. This also agrees with visual observation of experiments.

This would mean that about 23% of the surface area would be covered by drops and would thus be ineffective for heat transfer purposes. What then would be the effective average film thickness on the 77% of the surface that is active in heat transfer? A dimensional analysis was made, taking the film thickness t as a function of body force ρG, viscosity μ, surface tension σ and condensation rate C. This gives

$$t = \text{coefficient} \times \sqrt{\frac{\sigma}{\rho G}} \times \left(\frac{\mu C}{\sigma}\right)^x \qquad (7)$$

It may be a little simplistic to express an "average" thickness in terms of powers of the variables as above, but experimental results fit into this form satisfactorily. The unknown exponent x must be positive because an increase in condensation rate C could only thicken the film. Also x must be less than 0.5, otherwise we would have an increase in surface tension causing a decrease in film thickness. So we could expect x to have a small positive value.

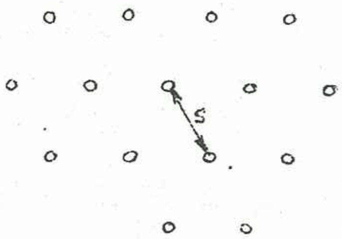


Figure 5

From equation (7) we can determine the film heat transfer coefficient as

$$h = \text{coefficient} \times \frac{k}{\left(\frac{\sigma}{\rho G}\right)^{\frac{1}{2}} \left(\frac{\mu C}{\sigma}\right)^{x}} \qquad (8)$$

It now remains to compare equation (8) with experimental results.

3 EXPERIMENTAL WORK

Some work on this topic has been published by Hoyle & Matthews[1]. The range of size and speed of rotation they have used extends into the area here being studied, i.e. total removal of condensate by centrifugal action, and hence forms a useful background comparison.

In our experiments a carefully machined piece of aluminium 40 mm thick was set horizontally as a dividing plate between two steam chambers (Figure 6).

Steam was supplied continuously to the lower chamber at atmospheric pressure. In the upper chamber a layer of water was kept on top of the aluminium and a vacuum was maintained in the chamber by water-cooled coils. The vacuum and hence the temperature difference between upper and lower chambers was varied by varying the cooling water flow. Steam condensed and fell off as drops beneath the horizontal aluminium surface in the lower chamber, and the amount falling from a known central area was caught and measured. Simultaneously thermocouples embedded in the aluminium at known vertical spacing were read so that heat flux rates could be calculated and cross-checked with those corresponding to the measured rate of condensation. The temperature gradient in the aluminium was substantially linear and hence by extrapolation the metal surface temperature was obtained. Thus with the steam temperature known we obtained the effective temperature difference across the condensate and hence the heat transfer coefficient for the condensate film. The results obtained are shown

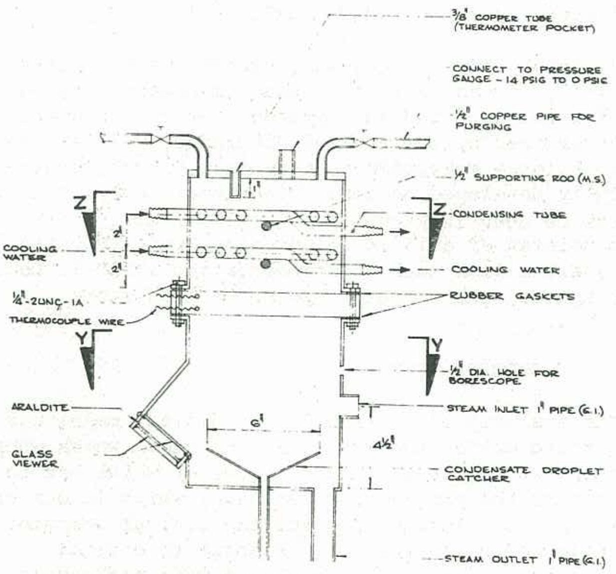


Figure 6

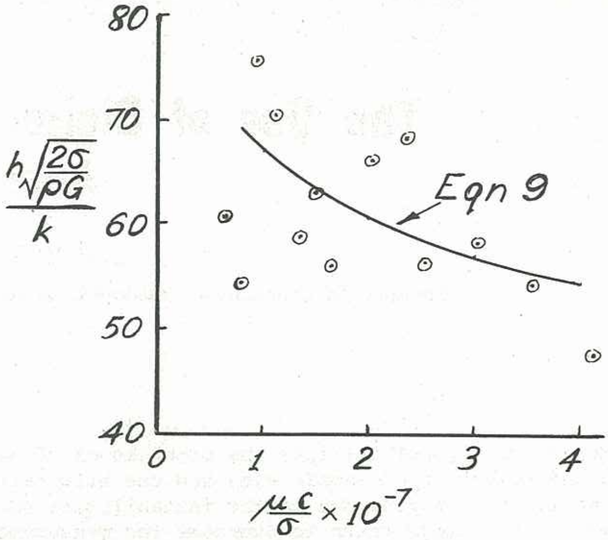


Figure 7

plotted in non-dimensional form in Figure 7. A log-log plot was also made and this indicated that a suitable value for the exponent x is about 0.15, and for the coefficient 4.2. Hence the final equation summarising all the results is

$$h = 4.2\, k \sqrt{\frac{\rho G}{\sigma}} \left(\frac{\mu C}{\sigma}\right)^{-0.15}$$

Now $G = \omega^2 R$, where R is the radius of the cylinder, so for calculation purposes the equation may be stated as

$$h = 4.2\, k\, \omega\, R^{\frac{1}{2}} \rho^{\frac{1}{2}} \sigma^{-0.35} \mu^{-0.15} C^{-0.15} \qquad (9)$$

which is applicable with any consistent set of units.

The results of Hoyle & Matthews[1] were re-calculated into a form comparable to the above as far as this was possible. Their results cover the transition range of values of $\frac{G}{g}$ 0..→ 50 and at the upper end of this range they appear to agree substantially with ours.

4 CONCLUSION

The mechanism of the removal of condensate from the outside of a rotating cylinder has been studied for values of centrifugal acceleration greater than about 50 g. Under these circumstances the heat transfer coefficient of the film is given approximately by the equation

$$h = 4.2\, k\, \omega\, R^{0.5} \rho^{0.5} \sigma^{-0.35} \mu^{-0.15} C^{-0.15}$$

in which any consistent system of units may be used.

5 REFERENCE

[1] R. HOYLE & D.H. MATTHEWS (1964). The effect of diameter size and speed of rotation on the heat transfer from steam to cooled cylinders. Int.J Heat Mass Transfer, Vol 7, pp 1223-1234.