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THE TREATMENT OF FLOW THROUGH HIGH POROSITY COMPRESSIBLE MEDIA, IN TERMS OF A MODIFIED DRAG EQUATION

by

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SUMMARY

The relationship between the liquid permeability and the geometry of fibrous media may be developed on the basis of the drag exerted by individual fibres. A modification to the permeability equation for a three-dimensional arrangement of fibres is presented in this paper.

It is proposed that the pad compression resulting from increasing flow rates is associated with a progressive orientation of fibres from the three dimensional to a two dimensional arrangement. A composite equation is presented which accounts for the change in fibre orientation in terms of pad compression and is shown to improve agreement with experimental results.

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LIST OF SYMBOLS

- A = cross sectional area of pad
- c = Kozeny coefficient
- d = fibre diameter
- f = drag force per unit length of fibre
- k = specific permeability
- k_2 = specific permeability - two dimensional equation
- k_3 = specific permeability - three dimensional equation
- k_v = specific permeability - variable orientation equation
- L = pad thickness
- L_o = initial pad thickness
- L_f = final pad thickness
- n = number of fibres, of unit length, per unit pad volume
- Q = volumetric flow rate
- R = Reynold's number of flow in pores
- S_o = specific surface
- v = linear pore velocity
- V_s = solid volume of pad
- V_b = bulk volume of pad
- ε = porosity of pad
- η = dynamic viscosity
- Δp = pressure drop
- ρ = fluid density

INTRODUCTION

The flow of fluids through porous media may be treated by the application of basic hydrodynamics to a small scale situation, and summing over the whole material. In general, attempts are made to relate the experimentally measured Darcy permeability to the geometrical and physical properties of the porous media. For such work, materials may be classified as either high or low porosity, and their structure may be considered as compressible or incompressible. Low porosity, incompressible materials are commonly treated in terms of the Kozeny theory in one of its modified forms, the most popular being the Kozeny-Carman equation (1). Low porosity, compressible materials are of little practical interest for they imply significant bulk compressibility of the solid phase. High porosity, incompressible materials have been considered in terms of the Kozeny-Carman equation with modifications to the Kozeny coefficient. The limited success of such an approach for fibrous beds reflects the difficulty of applying the physical model of tortuous capillaries, implicit in the hydraulic radius theory, to an open filament structure. It is therefore surprising that for such materials more use has not been made of the drag theory of permeability. The development of this theory by Iberall (2) from the drag equations of Emersleben (3) and Oseen's solutions as given by Lamb (4), have been referred to by Scheidegger (5). The resulting permeability equation has been applied by the author to beds of both synthetic fibres and cellulose fibres in order to understand better the drainage process on a paper machine. Initial attempts along these lines were presented at the First Australian Conference on Hydraulics and Fluid Mechanics in 1962. Further work directly related to beds of paper pulp fibres was presented at the Third Fundamental Research Symposium of the British Paper and Board Makers Association (6). In these papers reasonable correlations were demonstrated between theory and experiment for beds of synthetic fibres of such a length and diameter that bed compressibility was slight. However, as the length to diameter ratio was increased and fibre flexibility allowed the pads to compress under flow conditions the theory became inadequate. In the present paper modifications to the drag equation are considered to account for progressive changes in fibre orientation associated with compression.

EXPERIMENTAL

The apparatus used has been described in reference 6, and the range of experiments performed for this study is indicated by the data in Table 1. In these experiments Δ_p, L, Q and A were measured and the permeability k calculated from the Darcy equation

$$k = \frac{Q\,\eta\,L}{A\,\Delta\,p} \qquad (1)$$

Porosity is defined as the ratio of void volume to bulk volume, and hence may be expressed as

$$\varepsilon = \frac{V_b - V_s}{V_b} = 1 - \frac{V_s}{V_b} \qquad (2)$$

Table 1 also includes, by way of comparison, the value of the Kozeny coefficient, c, required for theoretical agreement with experimental values according to the Kozeny-Carman equation

$$k = \frac{\varepsilon^3}{c\,S_o^{\,2}(1 - \varepsilon)^2} \qquad (3)$$

It may be noted that the values of c vary considerably from the usual value of 5, and although their dependence on porosity (for $\varepsilon > 0.8$) has been proposed in the literature, these results indicate a considerable influence of fibre diameter on c. The table includes permeability values calculated by three different drag equations, the derivations of which will be outlined below.

DRAG EQUATIONS

In Iberall's original development of an equation for permeability based on the drag of individual fibres, it was assumed that the flow resistivity of a random distribution of fibres can be considered as equivalent to that of an equipartition of all fibres in three mutually perpendicular directions, one of which may be taken along the direction of macroscopic flow. From Emersleben (3), the drag force per unit length of single fibre, oriented in the direction of flow is given by

$$f = 4\pi\eta v \qquad (4)$$

Thus if there are n fibres of unit length per unit volume and $n/3$ are arranged parallel to the flow, the drag force per unit volume, due to these fibres, can be equated to the pressure drop per unit length, so that

$$\frac{\Delta p}{L} = \frac{4n\pi\eta v}{3} \qquad (5)$$

From Oseen's solution for fibres transverse to the flow, as given by Lamb (4), the corresponding drag force is given by

$$f = \frac{4\pi\eta v}{2 - \ln R} \qquad (6)$$

where R = Reynold's Number, defined as $dv\rho/\eta$. Thus for two sets of $n/3$ fibres arranged perpendicular to the flow

$$\frac{\Delta p}{L} = \frac{8n\pi\eta v}{3\,(2 - \ln R)} \qquad (7)$$

The total pressure gradient is then obtained by addition of equations 4 and 6, so that

$$\frac{\Delta p}{L} = \frac{4\pi n\eta v}{3}\;\frac{4 - \ln R}{2 - \ln R} \qquad (8)$$

Since the number of fibres of unit length, n, equals the total fibre length per unit pad volume, the solid volume of the pad, V_s, may be expressed as

$$V_s = \frac{\pi d^2 n}{4}\,V_b \qquad (9)$$

It follows from the definition of porosity in equation 2 that $1 - \varepsilon = V_s/V_b$, so that

$$n = \frac{4\,(1 - \varepsilon)}{\pi d^2} \qquad (10)$$

Also the pore velocity, v, is related to the volumetric flow rate Q, and the macroscopic cross sectional area of the pad A by the equation

$$v = \frac{Q}{\varepsilon A} \qquad (11)$$

Substituting for n and v from equations 10 and 11, in equation 7 gives

$$\frac{\Delta p}{L} = \frac{16\eta Q\,(1 - \varepsilon)}{3d^2 A\varepsilon}\;\frac{4 - \ln R}{2 - \ln R} \qquad (12)$$

from which further substitution for k from equation 1 gives

$$k_3 = \frac{3d^2\,\varepsilon}{16\,(1 - \varepsilon)}\;\frac{2 - \ln R}{4 - \ln R} \qquad (13)$$

This equation, which is given in Scheidegger's book (5), will be referred to as the three dimensional drag equation. As mentioned earlier, this equation did not fulfil all expectations when applied to compressible pads. Close observation of such pads, formed in a transparent tube, indicated that fibres appeared to become progressively oriented transverse to the flow as the pad thickness decreased. This suggested (6), as an initial approach, the development of a permeability equation for all fibres transverse to the flow. Such a treatment commences with equation 5, from which the total pressure gradient, with all fibres perpendicular to the flow, is given by

$$\frac{\Delta p}{L} = \frac{4\pi n\eta v}{2 - \ln R} \qquad (14)$$

Substituting for n and v from equations 10 and 11, equation 13 then gives

$$\frac{\Delta p}{L} = \frac{16\eta Q\,(1 - \varepsilon)}{d^2 A\varepsilon\,(2 - \ln R)} \qquad (15)$$

Further substitution for k from equation 1 gives

$$k_2 = \frac{d^2\varepsilon\ (2 - \ln R)}{16\ (1 - \varepsilon)} \quad \text{-----------------------------------} \quad (16)$$

corresponding to equation 4 of reference (6). This will be referred to as the two dimensional drag equation.

VARIABLE FIBRE ORIENTATION

When experimental results for a range of fibre diameters are compared with the theoretical predictions of the two equations, the line of exact agreement generally lies between the two theories. This is shown in Figure 1. An explanation of this situation is proposed in terms of an initial three dimensional pad formation, with the reduction in pad thickness, as flow is increased, being accompanied by a progressive rearrangement of those fibres initially oriented in the general flow direction, towards positions transverse to the flow. This requires an assessment of the proportion of fibres along the flow and across the flow in terms of pad thickness, and the application of these numbers to the two and three dimensional drag expressions respectively. Two assumptions are made at this stage. After pad formation from a suspension, the flow is stopped and the pad is allowed to expand. The resultant initial thickness (L_o) is considered to represent a three dimensional situation. A final thickness which would be attained at very high flow rates is extrapolated from an empirical relationship between measured thickness and flow rate, and, at this value (L_f) the pad is assumed to be two dimensional. The contribution to the pad thickness by all fibres transverse to the flow by those fibres initially oriented along the flow (e.g. vertical fibres) is assumed negligible. Considering the number of vertical fibres in the pad to be a linear function of pad length, L, with boundary conditions that when $L = L_o$, $^n/_3$ fibres are vertical, and when $L = L_f$ no fibres are vertical, then for pad length, L, the number of vertical fibres is given by

$$\frac{n}{3}\left(\frac{L - L_f}{L_o - L_f}\right)$$

and the number of horizontal fibres by

$$n - \frac{n}{3}\left(\frac{L - L_f}{L_o - L_f}\right)$$

Returning to the derivation of the drag equations using this partition of fibres, and summing as for equation 7, we get

$$\frac{\Delta p}{L} = \frac{4\pi\eta v n}{3}\left(\frac{L - L_f}{L_o - L_f}\right) + \frac{4\pi\eta v}{(2 - \ln R)}\left[n - \frac{n}{3}\left(\frac{L - L_f}{L_o - L_f}\right)\right]$$

$$= \frac{4\pi n\eta v}{3\ (2 - \ln R)}\left[4 - \left(\frac{L_o - L}{L_o - L_f}\right) + \left(\frac{L_f - L}{L_o - L_f}\right)\ln R\right] \quad \text{---------} \quad (17)$$

Again substituting for n and v from equations 10 and 11 and for k from equation 1 gives

$$k_v = \frac{3d^2\varepsilon}{16\ (1 - \varepsilon)} \ \frac{2 - \ln R}{\left[4 - \left(\frac{L_o - L}{L_o - L_f}\right) + \left(\frac{L_f - L}{L_o - L_f}\right)\ \ln R\right]} \quad \text{-------} \quad (18)$$

Comparisons of calculated permeabilities, based on this equation, with experimental values of k from equation 1 are shown in Figure 2.

CONCLUSIONS

The agreement of the modified drag equation with experimental data is clearly an improvement on either the two or three-dimensional equations, although there are still some anomalies, such as the high experimental values at higher flow rates and the lower values at the larger fibre diameter (50μ). This may be due to a departure from the ideal geometry of the structure; also the proximity of the fibres to each other (approximately 20 to 30 fibre diameters) might invalidate the assumption that the flow around any fibre is undisturbed by adjacent fibres. Furthermore the laminar flow regime around drag producing elements may not persist at Reynolds numbers greater than 1 or 2. It is to be noted that the basic theory, and its modifications, are based on the same model, and, unlike most treatments, a feature of the resulting equation is that it does not require an empirical constant to produce experimental agreement. This should encourage further use of this type of approach to flow through such media as may have similar physical characteristics.

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TABLE 1 Experimental data and derived hydrodynamic quantities for pads of synthetic fibres

Area of pads = 570 mm^2 Fibre length = 5 mm	Q m^3/sec x 10^{-5}	Δp kPa	L mm	c	ε	R	k m^2 x 10^{-4}	k_2 m^2 x 10^{-4}	k_3 m^2 x 10^{-4}	k_v m^2 x 10^{-4}
Fibre dia. - 50 µm	0.408	0.05	40.31	15.5	0.961	0.372	5.89	11.52	6.92	7.99
Nylon	0.783	0.10	37.20	15.0	0.958	0.716	5.21	8.32	5.77	6.71
L_o = 48.3 mm	1.650	0.24	32.33	13.0	0.951	1.520	3.97	4.80	4.01	4.50
	2.475	0.39	29.65	14.4	0.947	2.290	3.28	3.27	3.09	3.23
L_F = 24.0 mm	3.467	0.61	27.07	14.4	0.942	3.224	2.71	2.11	2.23	2.12
	4.017	0.75	26.04	14.1	0.939	3.748	2.46	1.64	1.83	1.65
Fibre dia. = 39 µm	0.317	0.04	42.68	11.9	0.963	0.228	6.04	8.84	4.84	5.17
Nylon	0.833	0.13	36.84	10.9	0.957	0.602	4.22	5.44	3.62	3.97
L_o = 50.0 mm	1.650	0.31	31.88	11.4	0.950	1.202	2.94	3.37	2.63	2.87
	2.500	0.53	28.37	11.5	0.945	1.830	2.36	2.34	2.07	2.18
L_F = 22.0 mm	3.350	0.77	25.91	11.1	0.939	2.470	1.96	1.65	1.60	1.62
	3.958	0.98	25.08	11.4	0.937	2.925	1.77	1.34	1.22	1.36
Fibre dia. = 20 µm	0.358	0.14	42.88	13.4	0.971	0.123	1.51	3.10	1.53	1.71
Terylene	0.833	0.32	42.76	13.5	0.971	0.286	1.60	2.46	1.41	1.55
L_o = 45.8 mm	1.625	0.76	40.45	16.2	0.969	0.559	1.37	1.82	1.19	1.38
	2.450	1.25	37.78	17.0	0.967	0.845	1.26	1.43	1.03	1.23
L_F = 32.0 mm	3.300	1.77	35.25	17.0	0.965	1.140	1.12	1.16	0.90	1.09
	4.000	2.26	33.49	16.6	0.963	1.385	1.04	0.98	0.80	0.96
Fibre dia. = 12 µm	0.333	0.44	45.24	8.7	0.972	0.072	0.60	1.45	0.65	0.66
Terylene	0.825	0.73	40.53	9.7	0.969	0.179	0.81	1.05	0.55	0.62
L_o = 45.8 mm	1.633	1.76	33.39	10.3	0.962	0.357	0.54	0.69	0.41	0.53
	2.567	3.03	28.68	9.7	0.956	0.564	0.42	0.50	0.33	0.44
L_F = 23.0 mm	3.350	4.18	26.26	9.1	0.952	0.740	0.37	0.41	0.29	0.39
	4.000	5.10	24.98	8.7	0.949	0.886	0.34	0.36	0.26	0.35

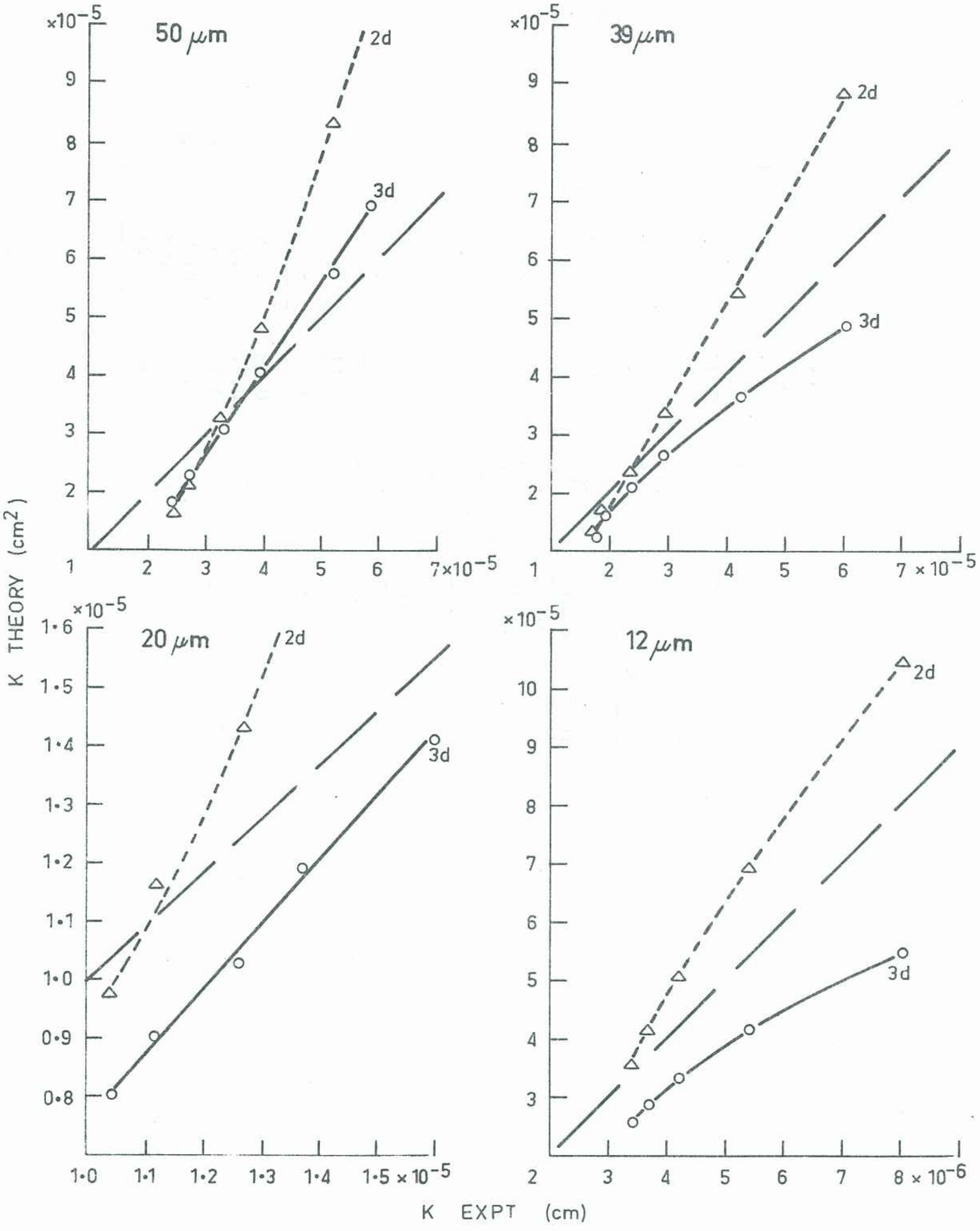

Figure 1 - Specific permeability calculated from equations (13) and (16), compared with observed specific permeability.

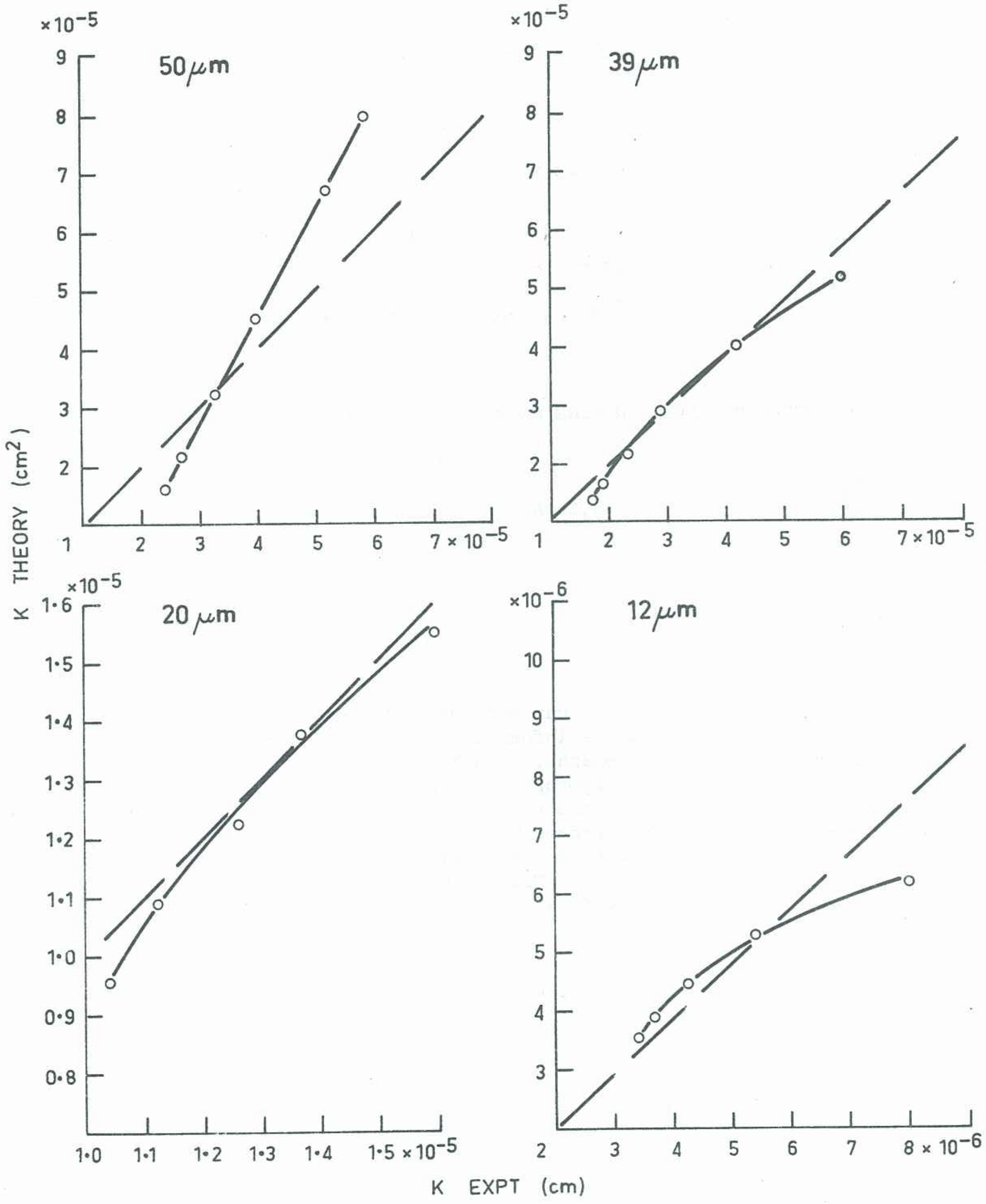

Figure 2 - Specific permeability calculated from equation (18), compared with observed specific permeability.