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NUMERICAL DETERMINATION OF SEA WATER INTRUSION IN AQUIFERS

by

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SUMMARY

Various methods have been developed for calculating the mean position of the salt water interface in aquifers discharging to the sea, but most of these methods are restricted to idealized flow situations with simple boundary conditions. The solution procedure employed in this paper utilizes the finite element technique which overcomes most of the restrictive assumptions.

The mean long term position of the interface in an aquifer is determined by solving in two-dimensional vertical sections taken through the aquifer discharge face at the sea.

Conditions include unconfined flow with pumping and recharge at various locations in the aquifer. The effects of anisotropy and pumping on the interface and free surface positions and shapes are outlined.

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INTRODUCTION

The problems of salt water intrusion caused by over-development of aquifers discharging to the sea have been recognized for over a century. The salt water front forms a transition zone which varies in thickness according to the conditions under which it is established and maintained. In some situations where water is being extracted from a position above, or close to, the transition zone it is important to know the thickness of the zone. For other cases where the major pumping stresses occur further inland the chief difficulty in management of the aquifer lies in assessing the liklihood of occurrence of sea water intrusion as a long term hazard. In such cases it will normally be sufficient to know where the mean position of the interface will be under the average steady conditions of development and in these circumstances the assumption of an abrupt interface may be justified provided its position can be determined.

A summary of salt water intrusion problems and available solutions has been given by Todd (1) and Bear (2,3). One of the most commonly used methods for estimating the interface position for steady flow is the Ghyben - Herzberg approximation (see Reference 1). Although it has been applied to both confined and unconfined aquifers significant errors occur especially toward the seaward end of the interface and the method cannot account for inhomogeneity or anisotropy of the aquifer. Santing (4) extended the scope of the method by adding the Dupuit assumption of horizontal flow and applied it to a variety of practical problems.

As noted earlier, in practice hydrodynamic dispersion (5) results in a transition zone between the salt water body and the fresh water discharging from the aquifer. Reddell and Sunada (6) who treat the problem as one of dispersion derive two equations, a flow equation and a convective-dispersion equation. These two equations are then solved alternately by numerical methods, the convective-dispersion equation using a method of characteristics approach and the flow equation by an implicit finite difference technique. Pinder and Cooper (7) use a similar approach to solve the dispersion problem.

In this paper an abrupt interface between fresh and salt water is assumed and the solution procedure employed is based on the numerical finite element technique. This technique overcomes most of the limitations on methods previously available and enables a solution to be obtained for inhomogeneous and anisotropic aquifers with irregular boundaries, for confined or unconfined flow and with pumping or recharge at any desired location in the aquifer. The method has special advantages in dealing with boundaries whose positions are initially unknown and have to be calculated in the solution process. Examples of such boundaries are the interface between fresh and saline water and the free surface for an unconfined aquifer.

Previous solutions to the problem have usually necessitated the adoption of the Dupuit assumptions of horizontal flow with consequent errors in the vicinity of the interface where there are significant vertical velocity components. The finite element method allows the Laplace equation to be solved without the Dupuit approximation so that more accurate solutions both for the interface position and for seepage quantities can be obtained.

METHOD OF SOLUTION

Typical flow situations at the seaward end of coastal aquifers when an abrupt interface is assumed are shown in Figure 1. Note that the diagrams in this Figure are not to scale and are highly distorted horizontally to vertically. Details of the solution procedure for confined aquifers have been given elsewhere [Cantatore and Volker (8)]. The procedure for unconfined aquifers is outlined below.

For steady flow conditions in Figure 1(b) the interface CD will be in a fixed position and there will be no flow in the salt water region. However the location of the interface CD is initially unknown as is that of the free surface EB and these locations must be obtained during the solution process. For Darcy flow in the aquifer the equation applicable in the flow field ACDEB is the Laplace equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \ldots (1)$$

where h is hydraulic head, and x and y are othogonal co-ordinate directions.

This equation is then solved numerically using the finite element approach.

Boundary Conditions

With reference to Figure 1(b) the following boundary conditions apply:

AB is a constant head boundary, h is constant;

AC is an impervious boundary along which .. (n being normal to the boundary)

$$\frac{\partial h}{\partial n} = 0 \quad \ldots (2)$$

DF is a boundary along which the head is given by

$$h = \frac{\gamma_s(h_s - y)}{\gamma_f} + y \quad \ldots (3)$$

where γ_f and γ_s are specific weights of fresh and salt water respectively and h_s is the height of the salt water surface above the datum;

EF is a seepage face along which the head is equal to the height vertically above a horizontal datum

$$h = y \quad \ldots (4)$$

EB is the top flow line for which equation (2) and equation (4) both apply.

CD is the fresh water-salt water interface along which, for steady conditions, pressures are equal in the fresh and salt water and therefore

$$y = \frac{\gamma_s\, h_s - \gamma_f\, h_f}{\gamma_s - \gamma_f} \quad \ldots (5)$$

where y is the height of a point on the interface, and h_s and h_f are heads in salt water and fresh water respectively at that point.

For a semiconfined aquifer portion of the length EB will be a top flow line and the remainder an impervious boundary.

For a completely confined aquifer the boundary EB will be impervious for which equation (2) applies.

Solution Procedure

Positions of the interface CD and, for the unconfined aquifer, of the top flow line EB are assumed. Equation (1) is solved in the region ACDEB by the finite element method. The appropriate boundary conditions are applied along CD and EB and are used to obtain new estimates of their locations.

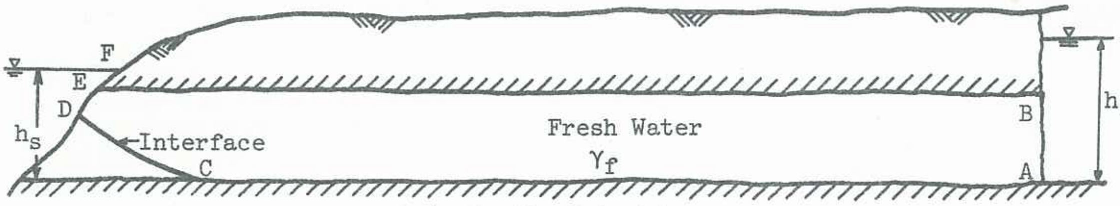
The adjustment procedure for the free surface EB has been described by Neuman and Witherspoon (9) while details of the iterative process used to obtain successively better approximations to the true interface position are given in Reference (8). This routine is continued until successive locations calculated for the free surface and also for the interface agree to within some predetermined accuracy. For both boundary types the adjustment is terminated when successive positions agree to within 1 per cent of the average aquifer depth at all points on the boundary being adjusted.

A semi-automatic mesh generation program is used to reduce the work of data preparation for the finite element routine and this program produces nodes at the end points of the interface.

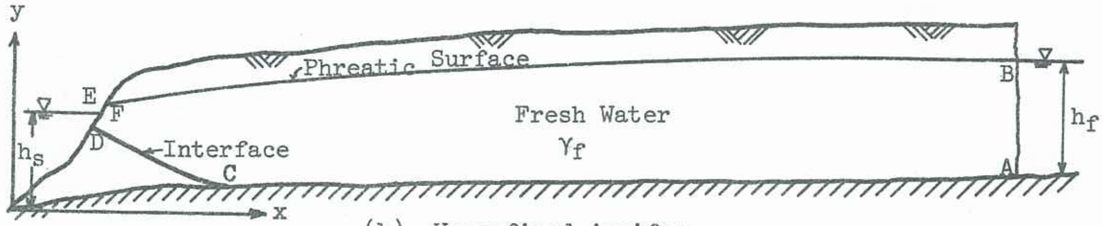
A well discharging or recharging at a point within the aquifer is accounted for by a constant head or constant discharge imposed on the solution as a boundary condition at the appropriate point. The mesh generation program is designed to give nodes at required locations of wells.

Inhomogeneity and anisotropy of the aquifer material are readily accounted for by specifying magnitudes and directions of principal hydraulic conductivities for each element of the mesh.

For problems involving flow in homogeneous isotropic aquifers which can be readily simulated with a Hele-Shaw analogue the method has been shown to yield good agreement with the results of the Hele-Shaw model (8).



(a) Confined Aquifer



(b) Unconfined Aquifer

FIGURE 1. EXAMPLES OF SEAWATER INTRUSION IN COASTAL AQUIFERS

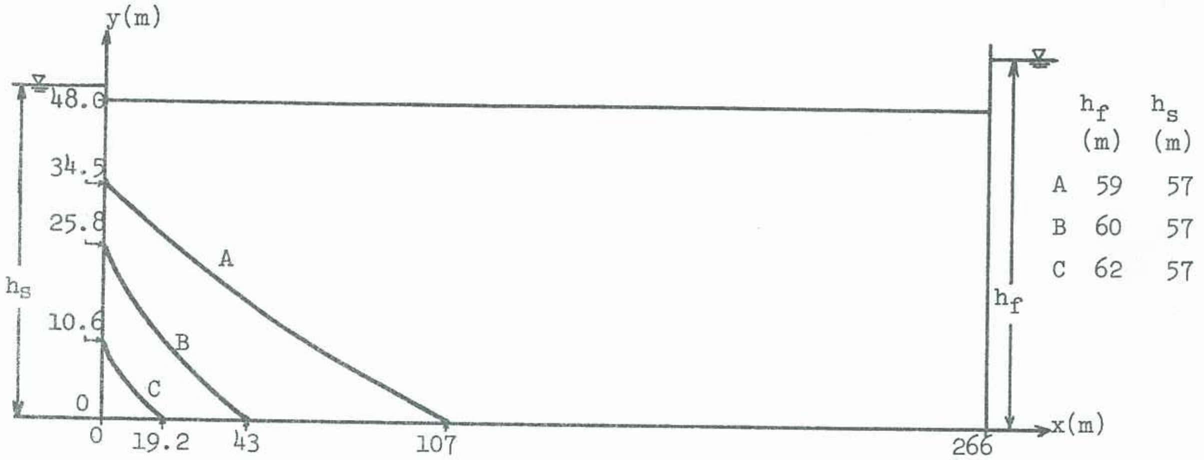


FIGURE 2. EFFECT OF DIFFERENT UPSTREAM HEADS IN A CONFINED AQUIFER

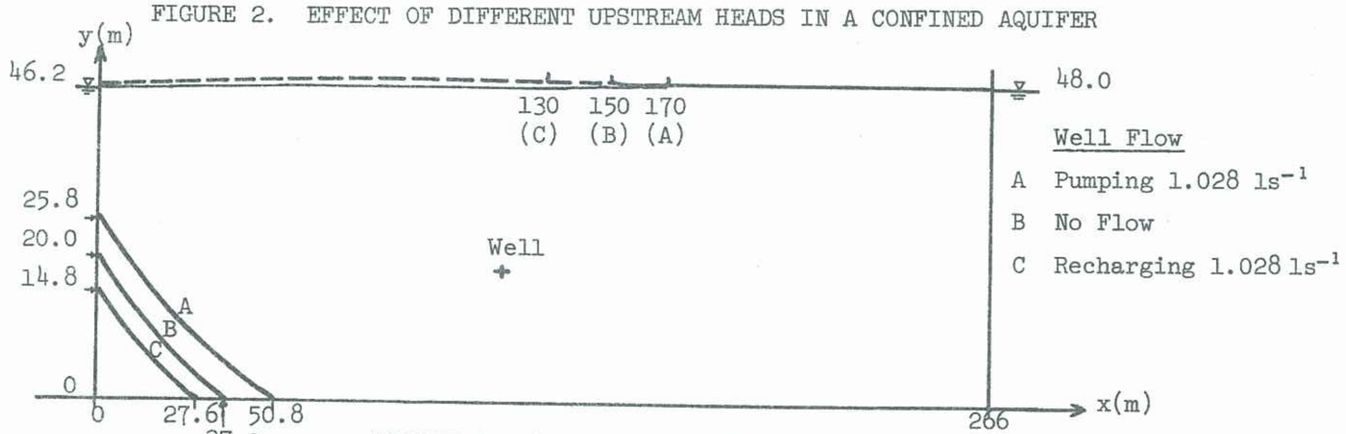


FIGURE 3. SEMI-CONFINED AQUIFER

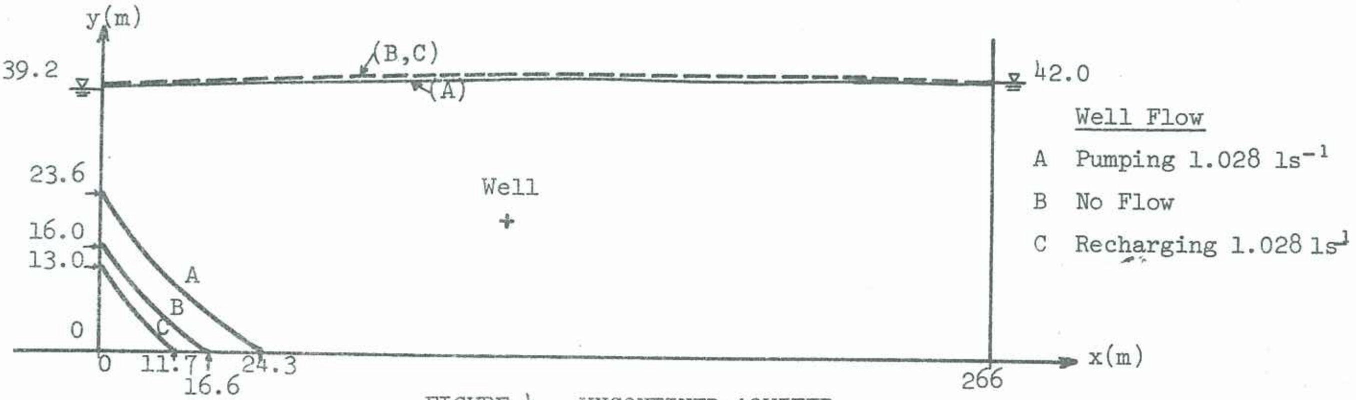


FIGURE 4. UNCONFINED AQUIFER

SOLUTIONS OBTAINED

Confined Aquifer

The interface position in a section of confined aquifer 265 m in length with a saturated depth of 48 m was calculated for a constant downstream sea level and three different upstream heads. The upstream head distribution was assumed to be hydrostatic in each case. The results are shown in Figure 2.

As discussed previously the Dupuit-Ghyben - Herzberg equation is often used to obtain a first approximation for the interface position. However in this equation either a value for the freshwater discharge to the sea is required or a knowledge of the piezometric head distribution. Thus when a given upstream head is maintained special observations in the field may be necessary to obtain flow rate or piezometric head values before applying the Ghyben - Herzberg equation. An approximate value for discharge can be calculated assuming horizontal flow throughout the aquifer although some error is involved even in the confined aquifer because of the presence of the interface. For the three cases given in Figure 2 this assumption was used to calculate the discharge which was substituted in the Ghyben - Herzberg equation to give the intrusion length to the toe of the salt water wedge along the horizontal lower impervious boundary. The results are compared with those from the finite element solution in Table I.

TABLE I - Lengths to Toe of Salt Water Wedge

Case	h_s* (m)	h_f* (m)	Length to Toe (m)	
			Finite Element	Ghyben - Herzberg
A	57	59	107	76.7
B	57	60	43	51.1
C	57	62	19.2	30.7

* see Figure 2.

Semi-confined Aquifer

The computer programs developed by Cantatore (10) have been extended by Arndt (11) to cater for semi-confined and unconfined aquifers. Figure 3 shows the results for an aquifer which is confined at the upstream end but which becomes unconfined close to the sea. A well is included in the field at the location shown and free surface and interface positions are calculated for three different cases: no flow from the well; discharge from the well; and recharge by the well. As expected pumping from the well increases the amount of sea water intrusion and the length for which the aquifer is unconfined.

Unconfined Aquifer

Similar results are observed for the fully unconfined aquifer as depicted in Figure 4. It is worth noting that the solution procedure provides an answer for the flow rate through the aquifer and for the positions of the interface and the phreatic surface (top flow line). Other methods such as the Dupuit-Ghyben - Herzberg approximation or that suggested by Rumer and Harleman (12) require a prior knowledge of either the phreatic surface or the flow rate before the interface position can be calculated. These cannot usually be simply calculated when upstream and downstream head conditions are specified.

Effect of Several Pumping Wells

Figure 5 shows the result of introducing several discharging wells at the locations indicated in a confined aquifer.

Effect of Anisotropy

The degree and orientation of anisotropy influence the position and shape of the intruding interface. Figure 6 gives a result for an isotropic medium (interface A) together with results for two different anisotropic media, the aquifers being confined in each case. For vertical and horizontal principal hydraulic conductivity directions, the effect of increasing the ratio of horizontal to vertical hydraulic conductivity is a smaller degree of penetration of the salt water as shown by interface B. When the ratio remains the same but the major hydraulic conductivity direction is at 45^0 to the horizontal as shown in Figure 6, the interface changes orientation to position C. This result is as expected since the equipotential lines will tend towards an alignment parallel to the principal hydraulic conductivity direction thus allowing

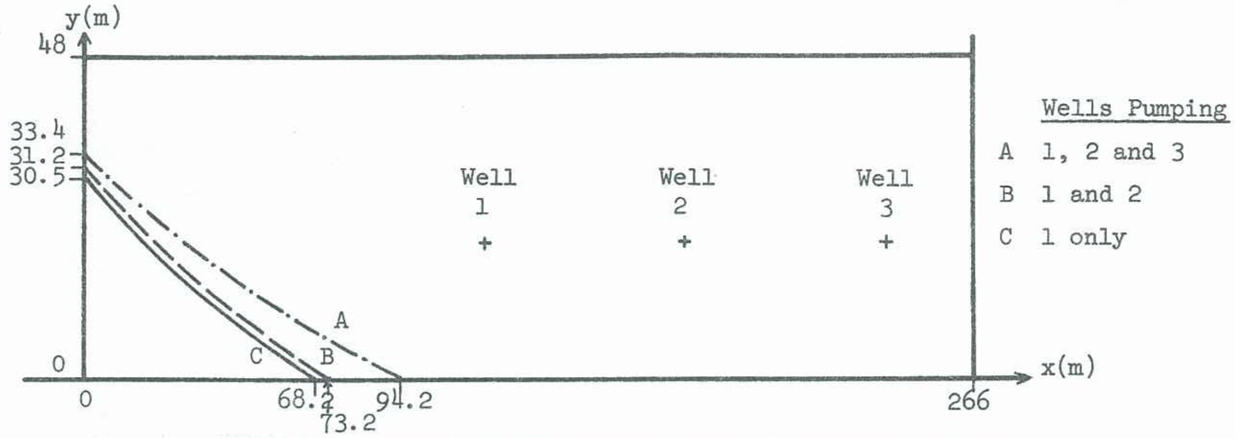


FIGURE 5. SEVERAL WELLS PUMPING FROM A CONFINED AQUIFER

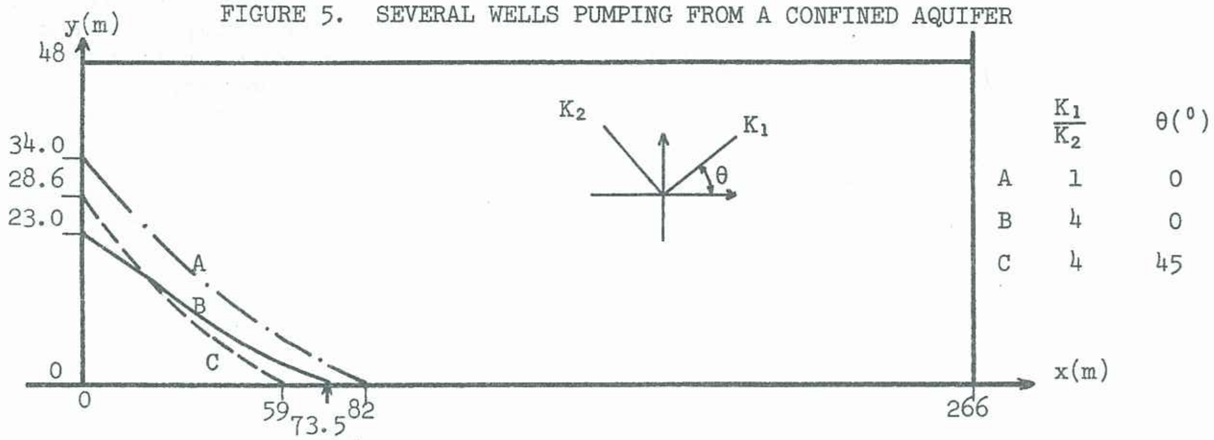


FIGURE 6. EFFECT OF ANISOTROPY IN A CONFINED AQUIFER

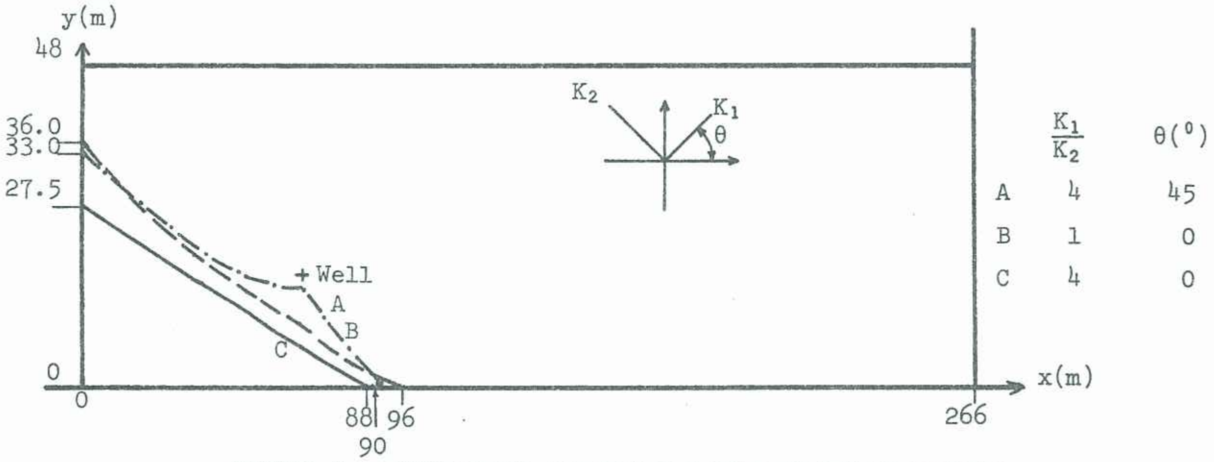


FIGURE 7. ANISOTROPY AND PUMPING IN A CONFINED AQUIFER

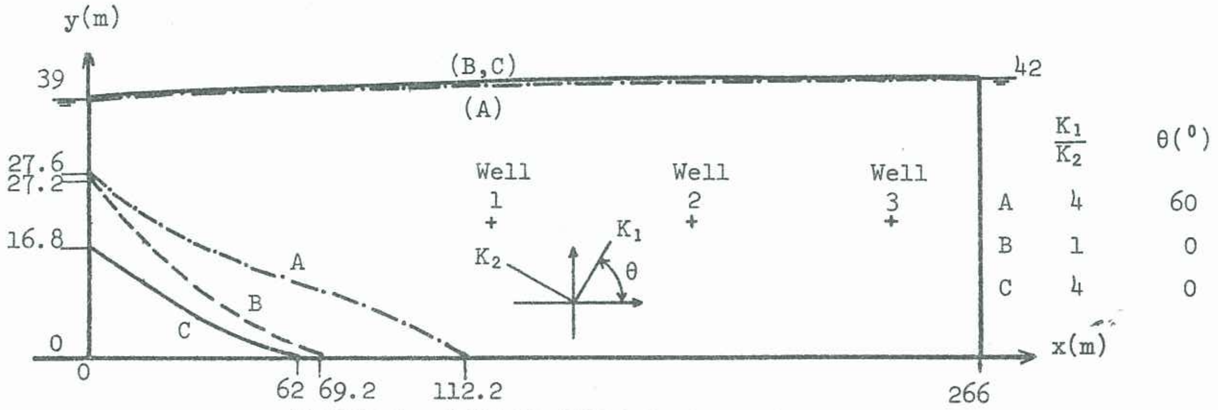


FIGURE 8. SEVERAL WELLS IN AN UNCONFINED AQUIFER

the rotation of the interface from B to C.

Figure 7 shows the effect of anisotropy on the interface when pumping occurs from a well in the field. Interface C exhibits an upconing effect toward the well situated above the interface because the hydraulic conductivity is larger in the direction approximately at right angles to the interface. Relatively few solutions to the problem of upconing of interfaces have been reported although the degree of upconing is of course critical when a well extracts fresh water from above the interface. Muskat (13) obtained an approximate solution to the problem of upconing of an initially horizontal interface toward a sink vertically above it. He assumed relatively small interface rises to obtain a solution. Bear and Dagan (14) used the method of small perturbations to solve a similar problem.

Figure 7 confirms that the finite element method can adequately account for upconing of an interface that is initially not horizontal even when the medium is anisotropic.

The influence of anisotropy on the interface in an unconfined aquifer with three pumping wells is shown in Figure 8. The three wells are discharging in each case. The intrusion is largest when the principal hydraulic conductivity direction is rotated from the horizontal and for this case some upconing again occurs.

CONCLUSIONS

It has been shown that a method based on the finite element technique has wide application in solving problems of sea water intrusion in coastal aquifers for steady flow situations provided an abrupt interface assumption is valid.

Results have been obtained for different flow rates in confined, semi-confined and unconfined aquifers. The effects of pumping and recharging wells on the location of the interface have been successfully modelled as have the effects of anisotropies of varying degree and orientation. The analyses have demonstrated that the solution procedure adequately predicts interface upconing due to a well pumping fresh water from above the interface for anisotropic confined and unconfined aquifers.

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