FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

HIGHER-ORDER STATISTICAL PROPERTIES OF TURBULENCE IN THE MARINE BOUNDARY LAYER AND ATMOSPHERE

by

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SUMMARY

Measurements of higher-order structure functions in high Reynolds number turbulence exhibit a behavior consistent with that predicted by simple dimensional theories, but the measured higher-order spectra have been found to exhibit an apparent paradoxical invariance of form that is contrary to the predictions of corresponding dimensional theories. In the present work, theoretical calculations based on a Gaussian model for the velocity fluctuations furnish predictions for spectra of arbitrary order that are in very good agreement, both in functional form and in absolute value, with measurements obtained in the atmospheric marine boundary layer. The present experiments and analysis suggest that the inertial subrange dimensional arguments of Kolmogorov and Obukhov apparently may be applied only to the first order (energy) spectrum, and that trends observed in earlier measurements of higher-order spectra in the atmospheric boundary layer and stratosphere correctly anticipated the invariance of the spectral form in the inertial subrange. It remains to explain why the dimensional analysis arguments of the Kolmogorov theory (that lead to the $k^{-5/3}$ behavior of the inertial subrange energy spectrum) are so decidedly unsuccessful when applied to higher-order spectra.

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I. Introduction

Interest in higher-order statistical properties of turbulent flows is generally stimulated by both fundamental interest in the closure problems of statistical formulations and in the subsequent application of closure schemes to numerical modelling of turbulent flows of practical interest in engineering, atmospheric sciences, etc. Here, we discuss some measurements related to the more fundamental and simpler aspects of high Reynolds number turbulence, in particular the behavior of statistical properties of the velocity field in the range of scales known as the "inertial subrange." For this idealized situation the theory and interpretation of results is relatively simple, and both can be carried out with some success to fairly high order.

Following the arguments of Kolmogorov and Obukhov (1), the inertial range of length scales r, defined by $\eta \ll r \ll L$, where $\eta = (\nu^3/\langle \varepsilon \rangle)^{1/4}$ is the viscous scale and L the external scale, all correlations and structure functions may depend only upon r and $\langle \varepsilon \rangle$, the mean rate of dissipation of kinetic energy. The usual assumption is that all spectral properties may depend only upon the wavenumber k and $\langle \varepsilon \rangle$. Extending (2) Kolmogorov's argument to structure functions of arbitrarily high order, and working only with a single (longitudinal) component of velocity and separation r for clarity, we have by dimensional analysis

$$\langle (u(x+r) - u(x))^n \rangle = C_n (\langle \varepsilon \rangle r)^{n/3} \qquad (1)$$

A corresponding result has been given by Dutton and Deaven (3) for the mth order spectrum ϕ_m, defined by

$$\int_0^\infty \phi_m(k)\, dk = \langle (u^m - \langle u^m \rangle)^2 \rangle = \langle u^{2m} \rangle - \langle u^m \rangle^2 \qquad (2)$$

so that $\phi_m dk$ is the spectral contribution to $\langle u^{2m} \rangle - \langle u^m \rangle^2$ from the wavenumber interval k to $k + dk$.

Again, by dimensional analysis

$$\phi_m(k) = \alpha_m \langle \varepsilon \rangle^{2m/3} k^{-(2m+3)/3} \qquad (3)$$

For $m = 1$, equation (3) reduces to the classical result that the energy or power spectrum of the velocity fluctuations is proportional to $k^{-5/3}$ in the inertial interval.

Stimulated by the comments of Landau and Lifschitz (4), there has been considerable speculation as to whether results like equations (1) and (2) should be modified because of a possible dependence on the form of the probability density of ε, and not just on its mean value $\langle \varepsilon \rangle$. Applying the modifications formulated by Kolmogorov (5), Obukhov (6), and Yaglom (7), which incorporate a lognormal distribution of ε (averaged over a region of characteristic dimension r) with variance σ given by $\sigma^2_{\ln \varepsilon_r} = A + \mu \ln L/r$, leads to the following modified form of equations (1) and (3)

$$\langle (u - u')^n \rangle = \tilde{C}_n (\langle \varepsilon \rangle r)^{n/3} (L/r)^{\mu n(n-3)/18} \qquad (4)$$

$$\phi_m = \tilde{\alpha}_m \langle \varepsilon \rangle^{2m/3} k^{-(2m+3)/3 + (2m-3)\mu m/9} \qquad (5)$$

These conflicting predictions for the original and modified theories are compared in figure 1 for $\mu = 1/2$ (a nominal average of available data, e.g., see (2)). We see that while the dependence of the moments or spectra on separation distance or wavenumber, respectively, is practically unaffected for commonly measured second or third-order quantities, the differences increase rapidly for higher-orders, and the predictions diverge rapidly for larger values of n or m. Because of this sharp divergence, one immediately questions the validity of the modified theories, whose behavior is somewhat strange, producing in both cases a relatively complex dependence on n or m, in which the power of r or k actually changes sign for sufficiently

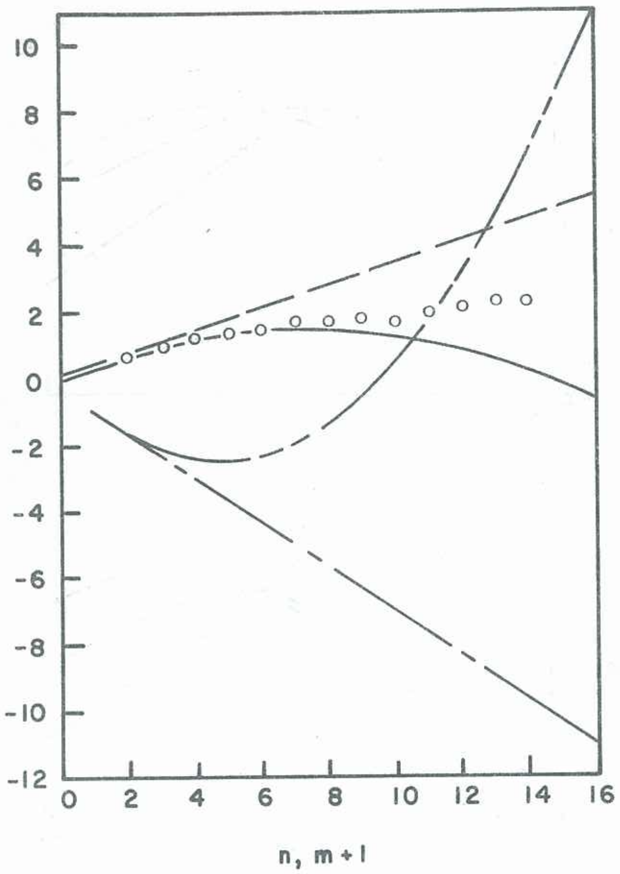

Figure 1. Power of r or k in inertial subrange variation of nth order structure function or mth order spectrum, respectively. Structure functions: ———— , Kolmogorov (1962); — — — , Kolmogorov (1941). Spectra: — - —, Dutton and Deaven extension of Kolmogorov (1941); —— - - —— , Modification of Dutton and Deaven for lognormal $p(\varepsilon)$. o; Van Atta and Park (8).

large n or m. That is, for sufficiently high order, the modified Kolmogorov theory predicts that structure functions should decrease with increasing r, and spectra increase with increasing k, a behavior outside our experimental experience up to the present time. For structure functions, the data of Van Atta and Chen (2) and Van Atta and Park (8) summarized in figure 1 support the modified theory up to about eighth order, but are inconclusive for higher order. Measurements of higher-order structure functions are difficult because of the requirements of large dynamic range and very long samples of data required to adequately resolve the tails of the probability densities that become crucial for the higher-order moments of the velocity differences. Higher order spectra, as defined here, do not involve measurement difficulties of this type, and good statistical convergence can be obtained in the frequency range of interest, without special efforts to achieve a large dynamic range of recording or very long samples of data. However, except for second-order structure functions and first-order spectra ($n = m + 1 = 2$), there is no apparent direct mathematical correspondence between the higher order correlations and spectra of the turbulent field. The closure problem for the Navier-Stokes equations can be written in a form involving higher-order structure functions (1), but it probably cannot be expressed in terms of the higher-order spectra as defined by equation (5). Hence, in testing either eqs. (4) or (5) experimentally, we are looking at two very different aspects of the problem. To avoid confusion, it should be mentioned that the Fourier transformed Navier-Stokes equations can be expressed in terms of another variety of higher-order spectra involving multiple wavenumbers. This analytical approach has been discussed for third-order quantities (bispectra) by Yeh and Van Atta (9).

II. Measurements of Higher-order Spectra

Recently, Dutton and Deaven (2) computed the spectra defined in eq. (2) up to fourth order for all three velocity components from samples of atmospheric turbulence obtained at four different altitudes with instrumented aircraft. As illustrated in figure 2, the behavior described by eqs. (3) or (5) was not observed in any of their data (actually, they had only equation (3) available for comparison). Rather than increasing in slope with increasing order, in the inertial subrange the log-log spectral plots either retained an approximately -5/3 power law slope or decreased in slope. Dutton and Deaven offered no physical or analytical explanation for the observed behavior, but speculated that "it is clear that the -5/3 power law generally observed in atmospheric turbulence in the range 100 to 1000 m does not indicate an inertial range in which spectral properties depend only on $\langle \varepsilon \rangle$ and k." It is not at all obvious that such a strong sweeping conclusion as this is justified on the strength of the available data and present ideas on turbulence. There is strong experimental evidence (1, 2) for the validity of Kolmogorov's -5/3 (spectral), +2/3 (second-order structure function), and +1 (third-order structure function) "laws," and detailed theoretical calculations that support the dimensional results for spectra and second-order structure functions are available (10). One objective of the present work is to attempt to develop alternate analytical interpretations of the experimental results which do not require the abandonment of the original energy cascade idea of Richardson (11) and the consequences put forward originally by Kolmogorov and Obukhov in 1941 and in modified form in 1962.

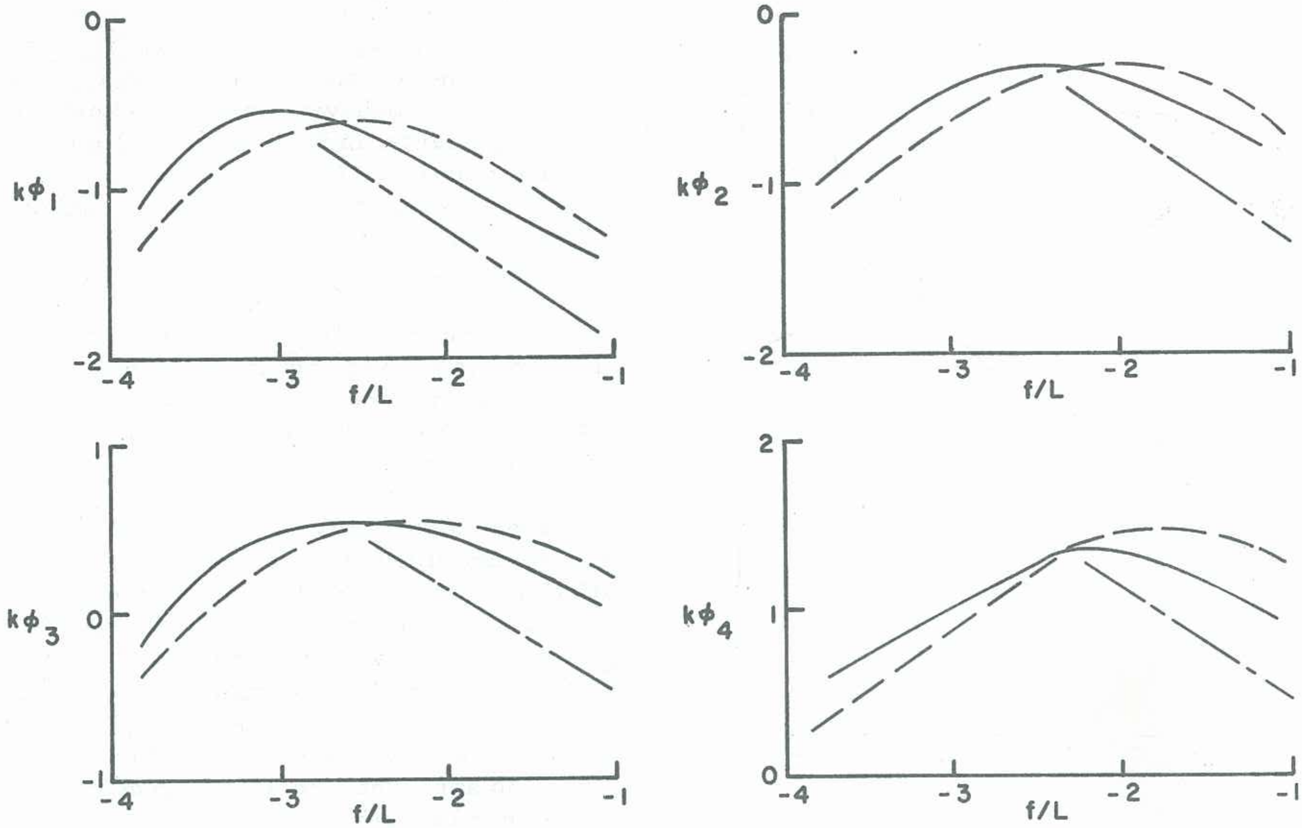

Figure 2. The spectra of the first, second, third and fourth powers of the Dutton and Deaven LO-LOCAT 750 ft data. Note that the function $k\phi(k)$ is scaled logarithmically on the vertical axis and that f (or k) is scaled logarithmically on the horizontal axis, with the integers denoting powers of 10. The dashed line gives the -2/3 slope that illustrates a -5/3 power law on a plot with these coordinates. ————————, u; — — — —, v.

The Dutton and Deaven spectra, which cover a large frequency range including quite low frequencies, do not resolve the inertial subrange sufficiently well to provide convincing evidence for the present comparison. To obtain suitable data, the present experimental results were calculated using data obtained in the atmospheric boundary layer over the ocean. These hot-wire data (2, 8) were known to exhibit an extensive inertial subrange behavior in the energy spectra and second and third-order structure functions, and the data used here are exactly the same data used to calculate the structure functions reported in (2) and (8). The power spectra were first recomputed emphasizing frequencies in the inertial subrange, and then the higher order spectra were computed with the same resolution. These results showed that all the computed higher-order spectra preserve the high-frequency end of the -5/3 behavior usually associated with an inertial subrange, and that the measured spectra deviate from this simple behavior at a lower frequency bound which increases as m increases. This data confirmed the suggestion from the Dutton and Deaven data that the form of the spectra in this range might be invariant, and motivated a search for a theoretical explanation.

III. Gaussian Theory for Higher-order Spectra

In general, there is no close correspondence between the probability density and spectrum of a random variable. Exact mathematical relations are apparently available only for a Gaussian process.

If u(t) is assumed to be Gaussianly distributed, then the higher order spectra defined in eq. (2) can all be calculated in theory from a knowledge of the first-order spectrum ϕ_1 alone. The probability density of the velocity fluctuations in atmospheric turbulence is only approximately Gaussian, as illustrated in figure 3 for the present data obtained in the atmospheric

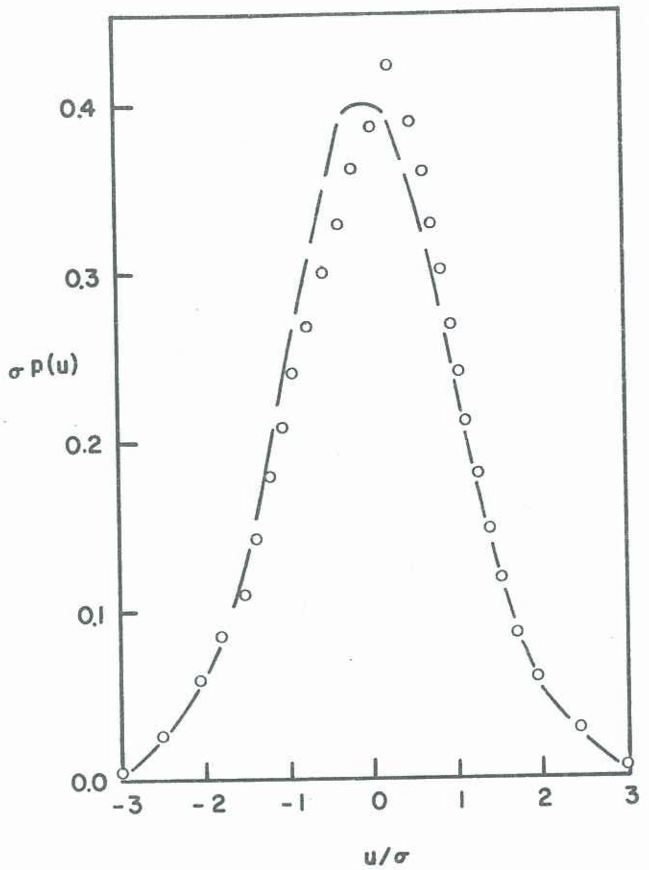

Figure 3. Probability density of u(t) measured in atmospheric boundary layer over open ocean. Height = 3 m, U = 8.4 m/sec. Dashed curve is Gaussian distribution.

boundary layer over the open ocean. However, although the measured probability densities are skewed and deviate from the Gaussian for small values of u, for intermediate and larger values of u the densities are well approximated by a Gaussian distribution. The question thus naturally arises as to whether or not the Gaussian assumption for the probability density of u(t) will produce realistic predictions for higher-order spectra of u(t), and how these would compare with the apparent paradoxical disagreement between the measured spectra and those predicted by the original and extended Kolmogorov arguments.

For a Gaussian random process, the second order spectrum is related to the first order (energy or power) spectrum by the relation given by Rice (12)

$$\phi_2(f) = \int_{-\infty}^{\infty} \phi_1(s)\, \phi_1(f-s)\, ds \qquad (6)$$

i.e., the second order spectrum is equal to the convolution of the first order spectrum with itself. In a general case, the convolution may be performed directly on the measured values of ϕ_1, but for spectra with extensive inertial subrange behavior the simple form of ϕ_1 allows one to use simple analytical expressions as approximating functions. For atmospheric boundary layer turbulence, a suitable form is given by

$$\phi_1(f) = b\,\lambda^{5/3}/[1 + (f\lambda)^2]^{5/6} \qquad (7)$$

The values of b and λ are easily determined by fitting two of the measured spectra in the inertial subrange. For the present atmospheric data obtained in the atmospheric boundary layer over the ocean, the values of b and λ determined for the raw (uncalibrated) spectra shown in figure 4 were b = 6498 and λ = 3.32. For purposes of interpretation, we assume that k and f are related according to $k = 2\pi f/U$ by Taylor's "frozen turbulence" hypothesis, where k is radian wavenumber, f is frequency in Hertz, and U is the mean velocity. Using eq. (7) the convolution integral of eq. (6) was performed numerically. The result fits the experimental data in figure 4 very closely. ϕ_2 exhibits an extensive region where $\phi_2 \sim f^{-5/3}$ corresponding to that for $\phi_1 \sim f^{-5/3}$, and no $\phi \sim f^{-7/3}$ region (the behavior predicted by eq. (3)) is observed. Note that the numerical values of the spectra increase rapidly with increasing m, and the absolute magnitude of the second order spectrum is 2.6×10^5 larger than that of the second order spectrum.

All the higher-order spectra could be numerically computed from a knowledge of ϕ_1 alone, but the task becomes increasingly more involved as the order increases. This result has been of considerable use in the study of noise in communication systems, and discussions can be found in Rice (12) and in Davenport and Root (13). In the present case, it is not necessary to numerically perform the m-fold convolutions on the data, as an analytical asymptotic expression for ϕ_m for large frequency in the range of interest can be found by employing (7) and the formalism for computing higher-order spectra developed by Bedrosian and Rice (14) and Rice (15) for the more general problem of the output properties of Volterra systems driven by Gaussian inputs.

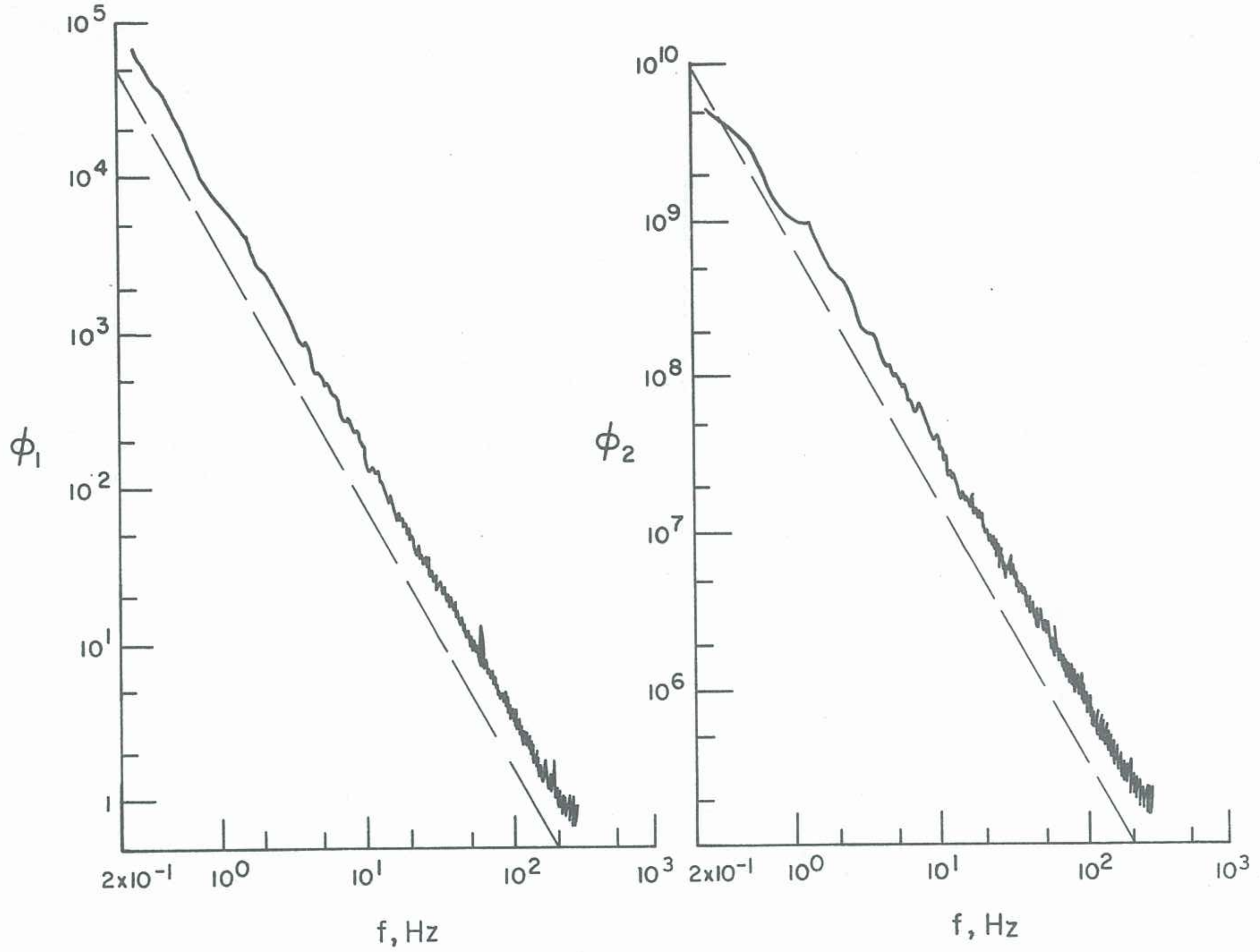

Figure 4. ϕ_1 and ϕ_2 measured in atmospheric boundary layer over open ocean. Dashed line has -5/3 slope. ϕ_m data for larger m are similar.

Our problem corresponds to the simplest possible memoryless case. The asymptotic result, which requires a considerable amount of algebra, is:

$$\phi_m(f) \sim (\text{D. C. spike if } m \text{ is even}) + b\,F^{-5/3}(4.2\,b\,\lambda^{2/3})^{m-1}\, m^2[1\cdot 3\cdot 5\cdots(2m-3)] \quad (8)$$

where the bracket [ ] is replaced by 1 if $m=1$, and $F = \lambda f$, where f is the frequency in Hertz. The leading term in the asymptotic expansion for large F is thus always proportional to $f^{-5/3}$, independent of the order m. This agrees with the experimentally observed behavior. For $b = 6498$ and $\lambda = 3.32$, the magnitudes of the spectra predicted from eq. (8) are in excellent agreement for all the measured spectra up to the highest order so far computed (9th). The predictions of eq. (8) also appear to be consistent with the data of Dutton and Deaven; however, at the time of this writing a full comparison has yet to be completed. It appears that the Gaussian model for the spectrum in the inertial subrange is capable of predicting (both qualitatively and quantitatively) all the observed features of the higher-order spectra in the inertial subrange.

IV. Discussion and Conclusions

The present experimental results and theoretical analysis demonstrate that the invariance of the form of the higher order spectra of velocity fluctuations in high Reynolds number turbulence can be explained on the basis of a Gaussian approximation for the velocity fluctuations. The close agreement of the present analysis and the experimental results is strong evidence in favor of the validity of the Gaussian approximation for representing spectral properties. The measured and predicted behavior is in sharp conflict with the predictions for higher-order spectra of the dimensional similarity theories, but in basic agreement with the trend of earlier experimental data obtained by Dutton and Deaven. These results raise a fundamental question: Why are the dimensional analysis arguments of the Kolmogorov theory (that lead to the $k^{-5/3}$ behavior of the inertial subrange energy spectrum) so spectacularly unsuccessful when applied to higher-order spectra? Hopefully, a way can be found to reconcile the present Gaussian theory, which represents the data well, and some alternate extension for general ϕ_m of the concept of an energy cascade in the inertial subrange, as successfully applied to the case of ϕ_1 by earlier workers.

Acknowledgements: The work was supported by NSF Grant GK-33799 and the Advanced Research Projects Agency of the Department of Defense and was monitored by the U.S. Army Research Office under contract No. DAHC04-72-C-0037. I especially wish to thank Dr. S. O. Rice for the analysis leading to equation (8).

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