FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand.

1974 December 9 to December 13

PRESSURE DROP PREDICTION FOR FLOW OF SOLID-LIQUID MIXTURES IN HORIZONTAL PIPES

by

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SUMMARY

The flow of solid-liquid mixtures or slurries in horizontal pipes is discussed under two main headings, settling and non-settling slurries. The available methods of predicting the pressure drop for these two cases are reviewed and compared. In the flow of settling slurries there are a number of possible mechanisms of particle suspension. These are discussed in connection with the prediction methods.

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INTRODUCTION

Solid-liquid mixtures can be classified under two main headings; settling and non-settling. A settling mixture is one in which the solid particles will quickly settle out if the mixture is left standing. A non-settling mixture is one in which the solid particles do not settle out or settle only very slowly. An often used rough criterion is that particles of diameter less than 50 microns can be classed as forming a non-settling mixture in water. A non-settling mixture will generally behave as a single liquid with viscosity and density different from that of the carrier fluid, and will flow in a pipe in a homogeneous manner. The manner in which a settling mixture flows in a pipe will depend on the velocity. At sufficiently high velocities all particles will be kept suspended although there will be a concentration gradient vertically across the pipe. This type of flow is termed heterogeneous flow and is a true two-phase flow in that the solid and liquid phases retain their identity. If the velocity is progressively reduced the concentration gradient will increase until a stage is reached when the motion of the liquid will no longer be sufficient to suspend all of the particles. At this velocity, termed the deposit velocity, some particles will separate out and form a stationary or moving bed of solids at the bottom of the pipe. It should be noted that if there is a wide particle size distribution the fine particles may be transported homogeneously whilst the coarser particles could be transported heterogeneously.

In this paper various methods of predicting the pressure drop for the two types of mixtures will be reviewed and discussed in the light of the possible mechanisms by which the particles are suspended in a flowing liquid.

PRESSURE DROP PREDICTION FOR FLOW OF NON-SETTLING MIXTURES

Non-settling mixtures of solid granular particles in water often exhibit non-Newtonian behaviour such as a yield stress and a non-linear stress-shear rate relationship, especially at high solids concentration. The problem of pressure drop prediction of a homogeneous mixture is therefore one of pressure drop prediction for a non-Newtonian fluid. There are two main approaches to the prediction of pressure drop in a commercial size pipeline. The first is to use viscometric results obtained under laminar flow conditions to establish a rheological model of the mixture. Using this model the laminar pressure drop may then be calculated theoretically and the turbulent pressure drop by semi-empirical methods. The second method is to use a scale-up procedure from small pipe data. In the laminar regime a chart of T_w against 8 V/D will usually correlate data for different pipe diameters and enable scale-up. For turbulent pressure drop prediction the small scale laminar results are discarded and small scale turbulent data are used to scale-up in the turbulent regime. These two basic methods will now be discussed and compared.

Pressure Drops from Viscometric Data

From viscometric tests a rheological model of the mixture can be evaluated. Cheng (1) has found that a wide range of commercial slurries can be characterised by the generalised Bingham model in which the shear stress is given by $T = T_o + K\,\gamma^n$ (1)

where T_o is the yield stress, K is the consistency index, γ is the rate of shear strain, and n is the behavioural index.

Once T_o, K and n are determined the laminar pressure drop can be predicted theoretically. For turbulent flow it is necessary to resort to semi-empirical correlations. Most of these are based on either the Blasius type equation for the friction factor

$$f = A\,(Re)^b \qquad (2)$$

or the Nikuradse type equation

$$\frac{1}{\sqrt{f}} = A \log (Re\ \ f^b) + c \qquad (3)$$

In the evaluation of the Reynolds number, Re, various values for the viscosity have been used. One of the earliest correlations used the viscosity of the suspending medium. Others have used the plastic viscosity and some the limiting viscosity at infinite shear rate. Dodge and Metzner (2) replaced the conventional Reynolds number in equation (3) by a generalised Reynolds number

$$Re' = D^n\,V^{2-n}\,\rho\,/K \qquad (4)$$

after applying a dimensional analysis approach to a power law fluid. Cheng (1) has modified their method to accommodate a generalised Bingham fluid and has applied it to a large number of commercial pipelines. In a recent paper Kemblowski and Kolodziejski (3) have reviewed most previous correlations and have proposed a new one based on the Blasius equation. Their correlation allows for the decrease of non-Newtonian behaviour at high levels of turbulence in which region the pressure drops approach the Blasius value for a Newtonian fluid. Hanks and Dadia (4) employed Prandtl's mixing length approach and applied it to a Bingham fluid. Their theory has been modified by Kenchington (5) to accommodate a generalised Bingham fluid. Murthy and Zandi (6) present an analysis in which is included a "viscous interaction co-efficient" to modify the laminar rheological parameters under turbulent conditions. Kenchington (5) has compared the predictions of all the above methods with experimental pressure drops for clay and kiln feed slurries for pipe diameters ranging from 16 mm to 329 mm, and found that all the above methods generally predicted turbulent pressure drops with comparable accuracy with no single method being significantly superior to the others. In the case of laminar flow he found that Cheng's (1) method gave reliable results except for the case of the clay slurry in the largest (329 mm) diameter pipe where it overpredicted by about four times the measured pressure drop. He attributed this gross overprediction to the occurrence of "wall slip" in the large pipe which was not present in the smaller pipe. "Slip" can also occur in the viscometer and give erroneous readings. This "slip" effect is caused by the formation of a particle free layer near the wall giving reduced values of the local shear stress. This particle free layer has been observed in laminar tube flow (Segre and Silberberg (7)) in turbulent pipe flow (Roberts et al (8)), and in Couette cylindrical viscometers (Harris (9)). Harris presents a method of identifying slip in a co-axial cylinder viscometer. Olroyd's method (10 and 11) allows slip to be detected in laminar and turbulent pipe flows. Kenchington (5 and 12) found that slip did not significantly affect turbulent pipe flow prediction but could cause large errors in laminar pressure drop prediction.

Pressure Drop Prediction by Scale-Up

Bowen (13) has presented a complete design procedure for scaling up small pipe results to large pipes in both the laminar and turbulent regimes. In the laminar regime he uses the method of Metzner and Reed (14). This method is applicable to any purely viscous liquid ie., one which is time independent and inelastic. For such a liquid it can easily be shown that a log-log plot of wall shear stress τ_w against 8 V/D will correlate the data for different pipes providing there is no slip effect. For a power law fluid this plot will be a straight line. If slip occurs plots of data from different pipes will be displaced and this effect will need to be allowed for in any scale up procedure. This is discussed by Kenchington (12).

For turbulent pressure drop prediction Bowen's method uses small scale turbulent data for scaling up. It has as its basis the assumption that for a non-Newtonian fluid the Blasius equation for the turbulent friction factor can be generalised.

The Blasius friction factor $f = 0.079\,(D V \rho/\mu)^{-.25}$ for a Newtonian fluid. (5)

Combining this with the Fanning equation $\Delta P/L = 2 f \rho V^2/D$ (6)

one obtains $\Delta p/L = 2 A V^{2-b} D^{-1-b} \rho^{1-b} \mu^b$ (7)

where b = .25. For a particular Newtonian fluid (or Bingham plastic) ρ & μ are constant Equation (7) becomes:

$$\Delta P/L = B_1 V^{2-b} D^{-1-b} \qquad (8)$$

For a pseudo plastic fluid Bowen substituted the generalised Reynolds number of Dodge and Metzner (2), Re', into the Blasius equation ie.,

$$Re' = D^n V^{2-n} \rho/K \quad \text{where K and n are power law constants} \qquad (9)$$

Once again combining with the Fanning equation and removing ρ and K which are constant for a particular pseudo plastic fluid.

$$\Delta P/L = B_2 V^c D^d \qquad (10)$$

This equation is analagous to equation (8). In the case of a Newtonian fluid or a Bingham plastic only two parameters, b and B_1 need to be determined from experimental data. In the case of a pseudo plastic fluid three parameters c, d and B_2 must be determined experimentally. Once these parameters have been determined by tests on a pilot plant scale the pressure loss in a full scale pipeline can be obtained. For pipes of diameters D_1 and D_2 equation (10) can be written

$$J_1/J_2 = (V_1/V_2)^c (D_1/D_2)^d \quad \text{where J is the pressure gradient} \qquad (11)$$

In the scale-up process any uncertainty in the small scale data will be magnified. Kenchington (15) has extended Bowen's method to allow calculation of confidence limits for the full scale results. He found that in any scale-up over a large diameter range (say 8 to 1) it is imperative to calculate the confidence limits as calculations based on the best fit value could give results seriously in error.

Comparison of the Two Methods

Criticism can be levelled at both methods. Harris (16) has argued that methods using laminar viscometric data to predict turbulent behaviour, in particular the Dodge-Metzner method, are theoretically unsound. He advocates the Bowen scale-up procedure. However, as pointed out by Kenchington (12), inherent in the Bowen method is the assumption that the fluid obeys a particular law throughout the scale-up range. This may not be the case since the "turbulent viscosity" of a power law fluid tends to become constant at very high shear rates.

Thus experiments carried out in small pipes where shear rates are high may give erroneous results for a large pipe where lower shear rates prevail.

Kenchington (12) has compared the two methods and came to the conclusion that they both predicted pressure drops in the turbulent regime with accuracies of the order of 30%. Of course one big advantage of using viscometric tests is that they are a lot less costly than pipe loop tests. However Kenchington points out that the presence of slip, especially under laminar flow conditions, can cause large errors which could be overlooked when using a viscometric method. A pilot test would enable this slip to be detected. Kenchington recommends the use of both methods so that any discrepancy between the two can be investigated.

PRESSURE DROPS FOR FLOW OF SETTLING SUSPENSIONS

As mentioned in the introduction the character of the flow of settling suspensions depends on the velocity. Large heavy particles may never be suspended and may be transported by rolling or sliding along the bottom of the pipe. Smaller particles will be maintained in suspension provided the mean pipe velocity is high enough. Only the turbulent regime is of interest since no suspension of particles will occur in the laminar regime. At present no purely theoretical methods are available for pressure drop prediction with settling suspensions. Numerous empirical equations have been proposed which correlate data in specific instances but no equation has yet been proposed which will allow accurate prediction of pressure drop from the properties of the liquid and solids phases. Because of this it is necessary to resort to pilot scale tests. However if the pipe diameter of the pilot plant is significantly smaller than the full scale pipeline there is not even a reliable scaling up procedure which can be used with confidence. Some of the correlations which have been proposed will now be briefly discussed.

1. Empirical Correlations of the form $\Phi = K\,\Psi^{m}$ (12)

where $$\Phi = \frac{J - Jw}{\bar{C}\ \ Jw} \qquad \Psi = \frac{V^2 \sqrt{C_d}}{g\ D\ (S-1)} \qquad (13) \text{ and } (14)$$

K is a constant, J and Jw are the head loss per unit length of pipe for the mixture and for water alone respectively, $\bar{C}$ is the delivered volumetric concentration, V is the average mixture velocity in the pipe, Cd is the steady state drag co-efficient of a particle, D is the pipe diameter, and S is the ratio of the density of the solids to that of water.

Durand (17) was the first to propose a correlation of this form. From experiments with a number of materials of grain size from 2.5 mm to 80 mm in pipes of diameters from 40 mm to 700 mm he gave the value of the index m as -1.5. Zandi and Govatos (18) gave two values of m = - .35 and m = -1.93. The choice of which one to use depended on the mixture velocity and concentration. Hayden and Stelson (19) gave a value of m = -1.3. Inherent in all correlations of this form is the assumption that the increase in the head loss over that for water is proportional to the delivered concentration. Babcock (20) questioned this assumption and performed accurate experiments which showed that whilst it was true in some cases it was invalid in others. He also found that in some cases a better correlation could be obtained by not including the drag co-efficient. Furthermore he found that the Froude Number $\frac{V^2}{gD}$ did not adequately correlate data from pipes of widely differing diameters. Despite these shortcomings there is no doubt that a rough correlation does exist between Φ and Ψ and this type of correlation remains one of the most used.

2. Correlations Derived from a Dimensional Analysis Approach

Recognising the limitations of the above correlations Rose and Duckworth (21) attempted an entirely experimental approach employing only the techniques of dimensional analysis. They assumed that the mixture friction factor f_m was given by the sum of the fluid friction f_w and a friction factor due to the presence of solids.

$$\text{ie., } f_m = f_w + f_s \qquad (15)$$

They then included all variables which they thought could possibly affect f_s amd obtained the following non-dimensional expression.

$$f_s = F\left(\frac{\rho_w VD}{\mu_w}, \frac{V^2}{gD}, \frac{M_s}{M_w}, S, \frac{d}{D}, \frac{k}{D}, Z, \beta\right) \qquad (16)$$

where M_s and M_w are the mass flow rates of solids and fluid respectively d and D are the diameters of the particles and the pipe respectively, k is a roughness factor, Z is a shape factor, and β is a parameter defining the spread of the sizes of the particles. Experiments were then performed on a wide range of materials in air and water in different pipe sizes up to 75 mm diameter but only for spherical particles of closely graded size so that the influence of Z and β was not determined. The influence of pipe roughness was also not determined but because they obtained good correlation of data from pipes of widely varying roughness they deduced that the effect of $\frac{k}{D}$ on f_s was relatively small. Their final results are given in graphical form. The large number of variables affecting the pressure drop of slurry flow and the wide range of values that these variables can have makes any purely experimental approach immensely difficult. Because of this many of the graphs presented are drawn through only a few points and no great accuracy is possible using their method. Its main value lies in the link which it provides between the fields of hydraulic and pneumatic solids transport.

Turian et al (22) also approached the problem using dimensional analysis but reduced the number of non-dimensional groups by assuming that some of the groups could be combined. By considering only spherical particles of closely graded size flowing in smooth pipes their initial analysis indicated the following groups:

$$f = F\left(\frac{D\rho_w V}{\mu_w}, \frac{d\rho_w V}{\mu_w}, S, \frac{V^2}{gD}, \bar{C}\right) \qquad (17)$$

By postulating that the free settling velocity of particles in still water was an important variable and after consideration of the forces involved they reduced the number of groups thus

$$f = F\left(\frac{D\rho_w V}{\mu_w}, C_d, g\frac{(S-1)D}{V^2}, \bar{C}\right) \qquad (18)$$

They then assumed the functional relationship

$$f - fw = K\bar{C}^{a}\, fw^{b}\, C_d^{\,c}\left[g(S-1)\, D/V^2\right]^{e} \qquad (19)$$

and obtained the values of K, a,b,c and e by non-linear least squares analysis of 1511 data points. The final form of their correlation was

$$f = f_w + 0.32\,\xi \qquad \text{for } \xi \geqq 1 \qquad (20)$$

$$f = f_w + 0.32\,\xi^{1.36} \qquad \text{for } \xi < 1 \qquad (21)$$

$$\text{where } \xi = \bar{C}^{0.49}\, f_w^{0.56}\, C_d^{-0.11}\left[gD(S-1)/V^2\right]^{0.90} \qquad (22)$$

The data used in obtaining this correlation was obtained for a reasonable range of particle sizes (from 31 microns to 4.38 mm) and particle densities (from 2.3 to 11.3 g/cc). However only a limited range of pipe sizes was covered (12 mm, 25 mm and 50 mm diameter) and its use outside this range is of course highly suspect. The correlations of the type $\Phi = K\,\Psi^m$ of the previous section can be compared with this by writing them in friction factor form

$$\text{or} \qquad f = f_w + K\, f_w\, \bar{C}\, C_d^{\,m/2}\left[gD\,(S-1)/V^2\right]^{-m} \qquad (23)$$

3. Semi-Theoretical Correlations

A number of correlations have been developed by applying some theoretical reasoning about the flow situation. Some of these will now be discussed. Newitt et al (23) reasoned that the work done in maintaining particles of settling velocity V_o in suspension would be proportional to their effective weight and their settling velocity. Equating the work done on and by the particles they obtained the equation

$$\Phi = K_1\,(S-1)\,(V_o/V)\,(gD/V^2) \qquad (24)$$

From their data they gave K_1 to be a constant equal to 1100. Wasp et al (24 and 25) have developed a systematic method for pressure drop prediction in the heterogeneous flow regime which they have successfully applied to commercial size coal-water pipelines. Their method splits the pressure drop up into two fractions - that due to the solids which are transported homogeneously and that due to solids transported heterogeneously. They then use single phase Newtonian methods for estimating the pressure drop of the homogeneous portion and employ the Durand correlation (equation 12) for estimating the heterogeneous pressure drop. The homogeneous portion increases the effective density and viscosity of the "carrier fluid" and the consequent reduction in the settling velocity of the coarse particles reduces the heterogeneous pressure drop compared to what it would have been if the carrier fluid were pure water. This is an attempt to allow for the observed fact that as the amount of fine material in a turbulent stream is increased the carrying capacity for coarse material is increased. To apportion the fraction of solids in the homogeneous and heterogeneous regimes they employ the theory of Ismail (26) to predict the in-situ concentration q at a height y = 0.8D above the bottom of the pipe. This theory employs the equation

$$\varepsilon_s\frac{dq}{dy} + V_h\, q = 0 \qquad (25)$$

which equates the rate of upward transfer of particles resulting from turbulent exchange with the rate of settling. Here ε_s is the mass transfer co-efficient and V_h is the hindered settling velocity. Equation (25) is integrated by assuming that ε_s is equal to the momentum transfer co-efficient. The concentration at 0.8D is then assumed to be the portion of the solids which is homogeneously distributed. If the solids have a wide size distribution they are broken up into 5 to 10 size fractions and the above calculations are performed for each size fraction and the pressure drops added.

The method of Wasp et al, employing as it does the Durand correlation, is subject to the same shortcomings as the Durand correlation. In addition the choice of the concentration at 0.8D as representing the homogeneous portion of the slurry is rather arbitrary.

In spite of its partly theoretical background the method is largely empirical and as Wasp et al (25) themselves admit its success stems in part from "fitting" of arbitrary constants to experimental data. None the less it is of importance because of its successful use in the design of commercial pipelines.

Shook and Daniel (27) have developed a theory for the flow of suspensions of fine particles where turbulence is the dominant form of suspension by assuming that the suspension behaves essentially as a variable density single phase fluid. As the basis of their analysis they used the method of Julian and Dukler (28) in which it is hypothesized that the solids make their influence felt primarily by modifying the local turbulence in the fluid. In other words the solids contribute to the increased pressure drop by increasing the eddy viscosity of the pseudo fluid. Unlike Julian and Dukler however they allowed the solids concentration to vary across the pipe in the vertical direction rather than use the average value. Their final equation for the friction factor f is

$$(1/\sqrt{f_n}) - (1/\sqrt{f}) = (1 - \rho_c/\bar{\rho})(1/\sqrt{f_n} + 2.65) \qquad (26)$$

where f_n is the friction factor for a Newtonian fluid of the same density and viscosity as the slurry flowing in the pipe at the same pressure gradient as the slurry. The difference between f and f_n is then a measure of the degree to which the slurry departs from the behaviour predicted by its bulk properties because of the variation of concentration across the pipe. The value of ρ_c needs to be determined independently and Shook and Daniel suggest using the method of Ismail mentioned previously. Using experimentally determined values of ρ_c the method was shown to be significantly superior to the Durand equation. However when the error in predicting ρ_c is allowed for it is not likely to be any better than the Durand method.

Shook and Daniel (29) also investigated flows at high concentrations where particle-particle interactions became important. They suggested that in such a situation particles may be supported by Bagnold stresses. Bagnold (30) performed experiments on coarse particles at high concentrations above 15% by volume and found that when such a suspension was sheared there was a normal or dispersive stress,P set up, the magnitude of which was proportional to the shearing stress. ie.,

$$T_B = K\ P \qquad (27)$$

Under conditions of coarse particles at high concentrations Bagnold stresses may be responsible for particle support. In such a situation P will be equal to the integral of the in-situ concentration q across the pipe-cross-section and T_B, the shear stress due to Bagnold stresses will be calculable from equation (27). In general the shear stress due to the fluid friction alone will be impossible to calculate. To overcome this Shook and Daniel confined their investigation to flow in a channel with a stationary bed present and a steep concentration gradient above the bed such that there was a moving layer of high concentration immediately above the bed and essentially clear liquid towards the top of the channel. In such a situation they found it possible to estimate the fluid shear stress T_T at the top of the channel and obtained

$$J = f\ V^2/2gh + (S - 1)\bar{q}'\ K \quad \text{where h is the channel height , and} \qquad (28)$$

where $\bar{q}'$ is the average concentration above the stationary bed. A series of experiments was performed for sand, lead and nickel particles of various sizes. It was found that in all cases where turbulence was unlikely to be responsible for particle support (ie., for coarse, heavy particles) the values of K were within the range 0.4 to 0.75 which compare well with Bagnold's values. Newitt et al (23) using an entirely different line of reasoning proposed an equation similar to equation (28) for flow with a sliding bed which has been found to apply equally well to situations where there is a stationary bed present and to where there is no bed at all. Shook and Daniel suggest that this could indicate that their derivation based on a sliding bed is questionable and that this lends support to the Bagnold dispersive stress hypothesis.

Vocadlo and Charles (31) present an equation of the form

$$J = \frac{a_m}{a_w}\left(\frac{\mu_m}{\mu_w}\right)^{2-t}\left(\frac{\rho_m}{\rho_w}\right)^{t-1} J_w + k_s\ \bar{C}\ (S - 1)\ \frac{V_o}{V} \qquad (29)$$

where subscripts m and w refer to the mixture and the liquid respectively. The first term arises from a dimensional analysis and reflects the increased friction due to the increase in the density and viscosity. a_w is related to the co-efficient in the f-Re relation for liquid flowing alone. Any change in the turbulence structure due to the presence of the solids will cause a_m to differ from a_w. The second term results from a consideration of the work done to maintain the particles in suspension and is similar to that obtained by Newitt et al (equation 24). Suspensions of coarse particles often exhibit Newtonian type behaviour in laminar shear flow and in such a case the term μ_m/μ_w is dependent only on the particle concentration. Vocadlo and Charles present a means of calculating this ratio or alternatively it could be measured in a viscometer. To compare the method with experimental results and with other prediction methods they put $a_m = a_w$, $t = 1.80$ and $k_s = 10$ and found that the method consistently predicted the pressure gradient with more accuracy than the other methods. However all comparisons were for a 25 mm diameter pipe only. From equation (29) for two pipes of diameters D_1 and D_2

$$\frac{J_1 - k_{S1}\,\bar{C}_1(S-1)\,(V_o/V_1)}{J_2 - k_{S2}\,\bar{C}_2(S-1)\,(V_o/V_2)} = \frac{a_{m1}}{a_{m2}}\left(\frac{V_1}{V_2}\right)^{t}\left(\frac{D_1}{D_2}\right)^{t-3}\left(\frac{\mu_{m1}}{\mu_{m2}}\right)^{2-t}\left(\frac{\rho_{m1}}{\rho_{m2}}\right)^{t-1} \qquad (30)$$

This equation offers possibilities as a scale-up procedure in a manner analagous to that of Bowen's method described earlier. For a non-settling slurry the terms $k_s\,\bar{C}(S-1)V_o/V$ vanish. If in addition it is assumed that μ_m and ρ_m depend only on the bulk delivered concentration $\bar{\bar{C}}$ and that $a_{m1} = a_{m2}$ equation (30) becomes identical to equation (11) for slurries of equal concentration.

Discussion

The pressure drop experienced by a turbulent suspension flowing through a pipe will be due to one or more of the following interactions. (a) Fluid-wall interaction, (b) Fluid-particle interaction, (c) Particle-particle interaction, (d) Particle-wall interaction. The relative importance of each of these depends on firstly the concentration of the particles and secondly on the size of the particle with respect to the scale of turbulence in the fluid.

1. Flow at low concentration. At low concentrations type (a) and (b) interactions will predominate. If the particles are large compared with the scale of turbulence the particles will only follow the larger eddies and the main effect of the turbulence on each particle will be to alter its flow resistance. Uhlher and Sinclair (32) and Clift and Gauvin (33) have shown that in most cases the drag coefficient of a sphere is increased in a turbulent field although in some cases a decrease can occur. Many of the correlations presented for flow of settling suspensions involve a particle settling velocity and invariably no account is taken of the effect of turbulence on this settling velocity. Furthermore with non-spherical particles the settling velocity in still water will indicate the drag for one particular orientation only whereas the orientation between a particle and the fluid in a turbulent field will be changing continuously.

If the particles are small compared with the smallest scale of turbulence they will tend to follow all turbulent fluctuations. At very low concentrations (less than 5%) pressure drops lower than that of water alone have been observed (See Zandi (34)) however in general the pressure drop is greater than that of water. The variable density model of Shook and Daniel described previously is applicable for fine particles at low concentrations. This model assumes that the particle-fluid interactions increase the turbulent fluctuations and thus the friction drop. As pointed out by Wiles et al (35) this method does not take into account the phenomenon of particle migration away from the wall and this effect may be significant.

2. Flow at high concentrations. At high concentrations the effects due to particle-particle and particle-wall interactions become increasingly important. With fine particles the suspension may exhibit non-Newtonian effects. With large particles collision between particles and the rolling of particles along the bottom of the pipe may help to form large intensive eddies which in turn help to support the particles. Kazanskij (36 and 37) has proposed this phenomenon following experiments in which he compared the intensity of the large scale turbulent fluctuations in water with and without coarse sand present. For concentrations of sand above 15% he found a large increase in intensity over that for pure water. No such increase was observed with fine sand. At very high concentrations particle-particle and particle-wall effects would be expected to become dominant, and the Bagnold dispersive stress discussed earlier may become increasingly important. Shook and Daniel (29) employ this reasoning in the correlation for flow with a stationary deposit which was discussed previously.

In a later paper Shook et al (38) again investigated Bagnold stresses. This time they performed experiments on fully suspended flow in a channel with fine and coarse sand and nickel. They measured the concentration profiles and compared them with concentration profiles obtained using the theory of Ismail (26) assuming turbulent diffusion was responsible for particle support. For the fine particles they obtained good agreement but with the coarser particles at higher concentrations the agreement became progressively worse. This discrepancy could be qualitatively explained by the presence of Bagnold stresses.

CONCLUSIONS

For a non-settling slurry there are two basic approaches available both of which give acceptable results. One of these methods requires only bench scale tests although to confidently predict pressure drops in large scale pipes it would seem necessary to perform pilot scale tests.

The situation with settling slurries is still very unsatisfactory and there is no method which can be used to confidently predict pressure drops. Until the fundamental processes involved can be identified and understood the situation is not likely to improve. In the interim it may be possible to modify the method of Vocadlo and Charles (31) to allow scale-up from pilot scale tests.

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