FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

TWO ASPECTS OF THE PHYSICS OF WARM WATER DISCHARGES

by

R J Sobey*

SUMMARY

The physical significance of both the equation of state and lateral water surface gradients for warm water discharges are discussed. It is suggested that the equation of state should be retained in its elementary, essentially tabulated, form rather than adopt the conventional Taylor series approximation. It is subsequently indicated that the relative source buoyancy should be retained as an independent parameter, in addition to its inclusion in a densimetric Froude number. For surface discharges it is shown that lateral density gradients are negligible in comparison with lateral water surface gradients, although the density deficit is responsible for the water surface superelevation.

* Lecturer, Department of Engineering, James Cook University, Townsville, Australia 4811.

1. INTRODUCTION

The disposal of the huge amounts of waste heat from thermal power plants (approximately two-thirds of the energy production) to the environment is most commonly achieved by once-through cooling; cooling water from a nearby river or large body of water is passed through the power station condensors and subsequently discharged back to the same water body at approximately 10°C warmer. The hydraulic design of the outfall near-field region utilises the entrainment properties of momentum jets and plumes to rapidly dilute the temperature excess to tolerable ecological levels. Other aspects can include the prevention of thermal recirculation to the same or nearby power stations and the prevention of possible navigational difficulties in adjacent waters; such constraints, however significant, do not alter the essential buoyant jet character of the outfall, except perhaps in the limiting case of a cooling pond design.

Consideration is given below to two particular and somewhat neglected aspects of the dynamics of warm water discharges, namely the equation of state and lateral water surface gradients. Attention is concentrated on their physical rather than their quantitative significance, especially in the near-field region.

2. CONSERVATION EQUATIONS AND THE EQUATION OF STATE

The conservation equations for the steady flow of a turbulent viscous fluid in a gravity field are respectively

Mass: $$\nabla \cdot \rho \underline{u} = 0 \qquad 1.$$

Momentum: $$\rho \underline{u} \cdot \nabla \underline{u} = -\nabla p - \rho \underline{g} + \mu \nabla^2 \underline{u} \qquad 2.$$

Thermal Energy: $$\rho C_p \underline{u} \cdot \nabla \theta = k \nabla^2 \theta, \qquad 3.$$

where ρ is the mass density, u the instantaneous velocity vector, p the pressure, g the gravitational acceleration, μ the dynamic viscosity, θ the temperature excess above ambient temperature T_a, k the thermal conductivity and C_p the specific heat at constant pressure. For conciseness of presentation the mean flow and turbulent transport terms have not been separated from the left-hand sides of the above equations; turbulent flow is intended and it is the mean flow that is assumed steady.

These equations must be supplemented by an equation of state, relating the mass density to the thermodynamic state variables pressure p and temperature T, ie

$$\rho = \rho(p,T).$$

This is normally established (1) in the present circumstances by a double Taylor series expansion about the atmospheric pressure p_a and the ambient temperature T_a, giving

$$\rho = \rho_a(1 - K_a \Delta p - \beta_a \theta), \qquad 4.$$

where ρ_a is the mass density at pressure p_a and temperature T_a, K_a is the isothermal compressibility and β_a the isobaric compressibility:

$$K_a = -\frac{1}{\rho}\frac{\partial \rho}{\partial p}\bigg|_{p_a,T_a} \qquad \text{and} \qquad \beta_a = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_{p_a,T_a}.$$

K_a can be considered constant for pressures and temperatures encountered in hydraulic engineering and the consequent density difference is so small (0.05 kg/cbm for 1 atm. pressure increase above atmospheric at 20°C) that water is conventionally assumed incompressible under practical pressure changes.

However the temperature excesses of cooling water discharges result in density differences that are substantial in terms of their influence on the flow pattern; and β_a varies significantly with temperature (Fig. 1 and Ref. 6) in the range normally encountered in practice. A few figures will illustrate this variation. A cooling water discharge that is 10°C warmer than an ambient water temperature of 15°C has an equivalent source density deficit $\Delta\rho_o$ of 2.1 kg/cbm; where the excess temperature has been diluted to 20% (2°C) of the outfall excess, the density deficit has been diluted to 15% (0.31 kg/cbm) of the outfall deficit. A constant isobaric compressibility is often established from the outfall characteristics, in which case there is a discrepancy of 33%. The variation with ambient temperature is also significant; had the ambient temperature been 20°C, the outfall density deficit would be 2.6 kg/cbm, 23% more buoyant than the same outfall at an ambient of 15°C.

An ambient temperature variation of 5°C can not be considered extreme; Partheniades et al (7), for example, have recorded ambient temperatures ranging from 18.7°C to 31.5°C in the same laboratory study and field temperatures can range from near 0°C to over 20°C. An outfall temperature excess of 10°C is also fairly typical of current practice. Source momentum flux and source buoyancy flux (hence the outfall density deficit rather than the temperature excess) are the forcing mechanisms (9) of buoyant jet flow in the near-field region; some caution then needs to be exercised with the equation of state when thermally-induced density differences are a significant influence on the flow field.

In such circumstances it would seem more appropriate to retain the equation of state in its elementary form

$$\rho = \rho_a - \Delta\rho(T_a, \theta), \qquad 5.$$

with the density deficit $\Delta\rho$ computed from tabulated (6) values of β as

$$\Delta\rho = -\int_{T_a}^{T_a+\theta} \beta(T)\, dT \doteqdot \beta_a \theta,$$

where β_a is evaluated at temperature $T_a + \frac{1}{2}\theta$; it is not necessary to know the ambient density ρ_a to the same precision. In situations where density differences can also result from dissolved solids (eg salt), Eq. 5 would still be suitable but there would be an additional dependence of $\Delta\rho$ on the ambient concentration of the dissolved solids. For most mathematical models the function $\beta(T)$ can easily be incorporated into the computer program; a quadratic function is a very good fit to tabulated data in the temperature range of interest. The use of Eq. 5, essentially a tabulated equation of state, in place of Eq. 4 is physically more realistic and should not result in any real inconvenience in application.

Using the assumption that the density deficit is much smaller than the ambient density in the mass conservation equation and non-dimensionalising with the outfall velocity V_o as the velocity scale, outfall diameter D_o as length scale, outfall temperature excess θ_o as temperature scale, ρ_a as density scale, outfall density deficit $\Delta\rho_o$ as density deficit scale, and $\rho_a V_o^2$ as pressure scale gives the conservation equations as

$$\nabla \cdot \underline{u}^* = 0 \qquad 6.$$

$$\underline{u}^* \cdot \nabla \underline{u}^* = -\frac{1}{\rho^*}\nabla p^* - \frac{1}{Fr^2}\underline{k} + \frac{1}{Re}\nabla^2 \underline{u}^* \qquad 7.$$

$$\underline{u}^* \cdot \nabla \theta^* = \frac{1}{RePr}\nabla^2 \theta^* \qquad 8.$$

and the equation of state as

$$\rho^* = 1 - B\Delta\rho^*. \qquad 9.$$

The star superscripts represent non-dimensional dependent variables, $\underline{k}$ is the unit vector directed vertically upwards, and

$$Fr = \frac{V_o}{(gD_o)^{\frac{1}{2}}}, \quad Re = \frac{\rho_a V_o D_o}{\mu}, \quad Pr = \frac{\mu C_p}{k} \quad \text{and } B = \frac{\Delta\rho_o}{\rho_a}.$$

The temperature dependence of the dynamic viscosity, the thermal conductivity, the specific heat and the mass density has been ignored in the molecular transport terms, which are very small in comparison with their turbulent counterparts. The role of buoyancy in the above equations of motion is thus restricted to the pressure term of the momentum equation, an assumption consistent with the classical Boussinesq approximation (9). The conventional statement of the Boussinesq approximation includes the subtraction of a reference state of hydrostatic equilibrium in the ambient fluid from the momentum equation. This reference state has not been introduced as it looses its usefulness in the presence of free surface deformations; most warm water outfalls are into relatively shallow water and free surface deformations are almost inevitable.

3. THE BUOYANCY PARAMETER

Note now the interactions among this system of equations. The fluid dynamics of warm water outfalls is described by the continuity and momentum equations (Eqs. 6 and 7) and the non-dimensional flow is dependent upon the Froude and Reynolds numbers. The thermodynamics (the source of the buoyancy) is described by the thermal energy equation (Eq. 8) and is non-dimensionally dependent on the product of the Reynolds and Prandtl numbers. The fluid dynamics and thermodynamics are coupled by the non-dimensional velocity and by the equation of state, Eq. 9, which depends on the buoyancy parameter B. In general it may be said that the non-dimensional near-field flow is dependent, at least, on the four above-mentioned parameters - Fr, Re, Pr, B - although Pr is essentially constant for water. Certain other parameters, mainly geometric but also including surface heat exchange, enter into particular situations via the boundary conditions; their exclusion here to simplify the discussion does not imply their unimportance. It has proved useful (see Section 4) to combine Fr and B to define a densimetric Froude number

$$Fd = Fr/B^{\frac{1}{2}}$$

in place of Fr. Hence the non-dimensional flow field can be represented as

$$flow = f(Fd, Re, B).$$

It has become common (3,5,7) to use Fd in place of both Fr and B, which is equivalent to the additional assumption that β_a is a constant, independent of both T_a and θ. The time scale for significant (in the sense of Fig. 1) changes in the ambient temperature is of the order of a few hours so that the implicit neglect of the ambient temperature has some justification. Moreover in situations where buoyancy results solely from concentrations of soluable matter the omission of the equivalent buoyancy parameter is an acceptable assumption as the relationship between concentration differences and density differences is very nearly linear in the practical range. However the discussion in Section 2 indicates that this practice can be questioned where the buoyancy is thermally induced and that both Fd and B (or Fr and B or Fr and Fd - see Section 4) should be retained as significant non-dimensional parameters.

The omission of B appears to warrant careful examination where experimental results (5,7) are presented as excess temperature fields, non-dimensionalised as θ/θ_o. This has become a fairly general practice, largely because thermal pollution regulations are framed in terms of temperatures, and it is ultimately essential for the engineer to present his thermal pollution predictions for official approval in this manner. Many existing experimental studies of the near-field region however would seem more useful to the engineer if they had been presented in terms of density deficit fields, $\Delta\rho/(\rho_o-\rho_a)$; for a particular T_a and θ_o such predictions could readily be converted to an excess temperature field as required. Unfortunately it is generally not possible to revaluate existing results as they have mostly been conducted on the implicit basis of source heat flux rather than source buoyancy flux, the nett result being that B cannot be calculated from the information supplied and/or it varies over a very considerable range. Some careful experiments at low densimetric Froude numbers might clarify this point.

A buoyancy parameter is generally implicitly included in most existing theoretical studies of warm water outfalls, although β_a normally (4,8) has been assumed constant. The above discussion indicates that B should still be listed among the field parameters when a thermal pollution prediction is computed. It is also indicated that the assumption of a constant β_a is physically inaccurate and may be mathematically unnecessary; the error involved however is probably smaller than errors due to other assumptions (eg entrainment) involved in the model formulation.

4. WATER SURFACE GRADIENTS

In the case of surface discharges of warm water from single outfalls, the assumption of hydrostatic pressure can be written (Fig. 2 and Eq. 7) as

$$-\frac{1}{\rho^*}\frac{\partial p^*}{\partial z^*} - \frac{1}{Fr^2} = 0.$$

Integrating this equation in the vertical from non-dimensional elevation z* to the free surface η^*, and using the equation of state (Eq. 5) and the assumption that the density deficit is much smaller than the ambient density gives the pressure field as

$$p^* = \frac{1}{Fr^2}\int_{z^*}^{\eta^*} (1-B.\Delta\rho^*)dz^*. \qquad 10.$$

Within the buoyant jet the vertical pressure distribution is non-linear and there is a superelevation of the water surface above that of the ambient water (2). When the vertical density deficit profile is known, the superelevation can be computed from Eq. 10.

From Eq. 10 the pressure term in the cross-stream (y) momentum equation can be written as

$$\frac{1}{\rho^*}\frac{\partial p^*}{\partial y^*} = +\underbrace{\frac{1}{Fr^2}\frac{\partial}{\partial y^*}{}^*}_{\text{water surface gradient}} - \underbrace{\frac{B}{Fr^2}\int_{z^*}^{\eta^*}\frac{\partial}{\partial y^*}(\Delta\rho^*)\,dz^*}_{\text{density gradient}}. \qquad 11.$$

The order of magnitude of both components of the pressure term can be estimated by assuming a functional form for the vertical density deficit profiles. Adopting a simple linear decay from a maximum of $\Delta\rho_s$ at the free surface to zero at depth h, the bottom edge of the jet, the water surface superelevation can be estimated (2) as

$$\eta = \frac{1}{2}\frac{\Delta\rho_s}{\rho_a} h.$$

It follows directly that the non-dimensional water surface gradient and the non-dimensional integral (ie excluding the scaling factors B and Fr in both cases) in Eq. 11 are the same order of magnitude. Hence the ratio of the water surface gradient term to the density gradient term is of order 1/B. Established cooling water outfalls have buoyancy parameters of approximately 0.0025, which indicates that the density gradient component of Eq. 11 can be neglected, being at least two orders of magnitude smaller than the water surface gradient component. For surface discharges then the dynamic influence of buoyancy in the equations of motion is predominantly exercised through water surface gradients, via the intermediate mechanism of density-induced water-surface superelevations.

A recent theoretical study (6) has omitted the water-surface gradient term but retained the density gradient term. In this respect the physical basis of the proposed integral model would appear suspect, the pressure term being substantially underestimated. However it is not immediately clear that the computed predictions are particularly sensitive to this term as the forcing influence of the density deficit is also included as a boundary condition, via the vertical entrainment.

5. CONCLUSIONS

In studies of warm water discharges the isobaric compressibility β_a is normally assumed constant but the physical wisdom and mathematical necessity for such an assumption can be questioned. It is suggested that the conventional Taylor series equation of state is unsuitable in this context and that the elementary (essentially tabulated) equation of state (Eq. 5) is physically more realistic and almost as convenient.

A discussion of the non-dimensional parameters sufficient to describe a warm water discharge in the near-field region indicates that the source buoyancy parameter B should be retained as an independent parameter, contrary to some current practice in which the Froude number and the buoyancy parameter are combined into a single parameter, the densimetric Froude number.

For surface discharges of warm water the pressure term in the cross-stream momentum equation can be divided into two components, representing the water surface gradient and the density gradient respectively; it is through these terms that the dynamic influence of excess temperature is introduced into the equations of motion. An order of magnitude estimate indicates that the density gradient term can be neglected in comparison with the water surface gradient term; it is the density deficit however that is predominantly responsible for the water surface superelevation.

6. ACKNOWLEDGEMENTS

This paper was completed while the author was Research Engineer, Sonderforschungsbereich 80, University of Karlsruhe, West Germany. The financial support of the Deutsche Forschungsgemeinschaft, through the Sonderforschungsbereich 80, is gratefully acknowledged.

REFERENCES

1. Bird, R B, Stewart, W E and Lightfoot, E N "Transport Phenomena". New York, Wiley, 1960.

2. Engelund, F and Pedersen, F B "Surface Jet at Small Richardson Numbers". Jnl. of Hydraulics Division, ASCE, 99, 1973, pp405-416.

3. Harleman, D R F and Stolzenbach, K D "Fluid Mechancis of Heat Disposal from Power Generation". Annual Review of Fluid Mechanics, 4, 1972, pp7-32.

4. Hayashi, T and Shuto, N "Diffusion of Warm Water Jets Discharged Horizontally at the Water Surface". Procs, XII IAHR Congress, 4, 1967, pp47-59.

5. Jen, Y, Wiegel, R L and Mobarek, I "Surface Discharge of Horizontal Warm-water Jet". Jnl. of Power Division, ASCE, 92, 1966, pp1-30.

6. Landolt, H and Börnstein, R "Physikalisch-Chemische Tabellen". Vol. 1. Berlin, Springer, 1923.

7. Partheniades, E, Beechley, B C and Jen, Y "Near Field Temperature Distribution in Shallow Waters due to Submerged Heated Water Jets". Procs, XV IAHR Congress, 1, 1973, pp137-144. (See also Tech. Rep. No. 17, May 1973, Coastal and Oceanographic Eng. Lab., Univ. of Florida).

8. Stolzenbach, K D and Harleman, D R F "Three-Dimensional Heated Surface Jets". Water Resources Research, 9, 1973, pp129-137. (See also Rep. No. 135, February 1971 and Rep. No. 156, September 1972, R.M. Parsons Laboratory, MIT).

9. Turner, J S "Buoyancy Effects in Fluids". Cambridge U.P., 1973.

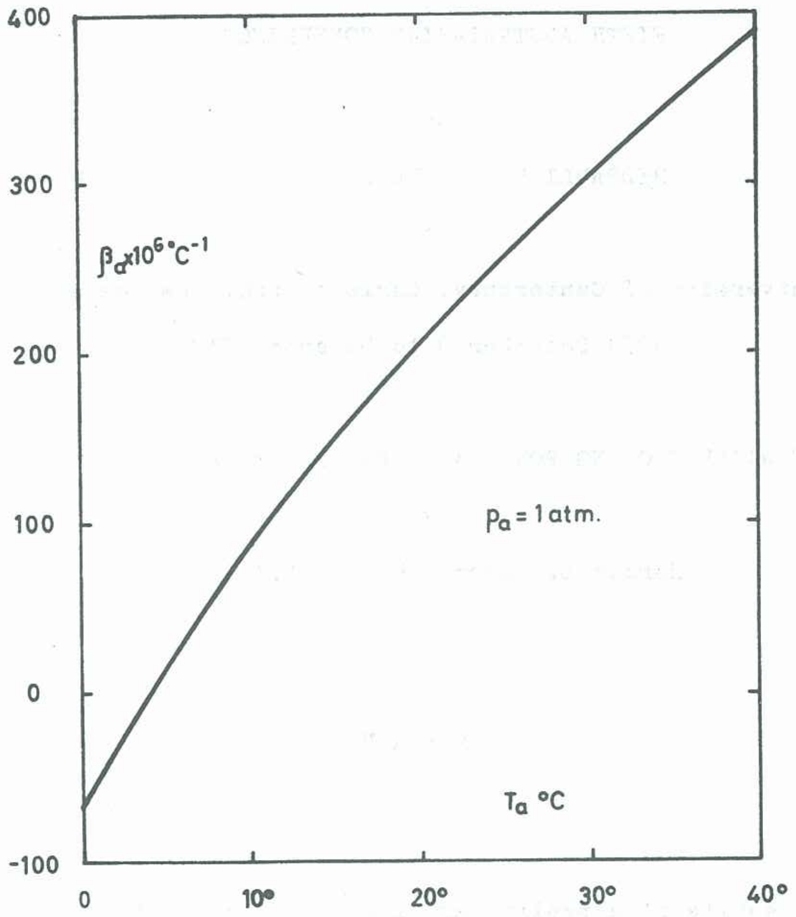

Fig. 1 - Isobaric Compressibility for Fresh Water

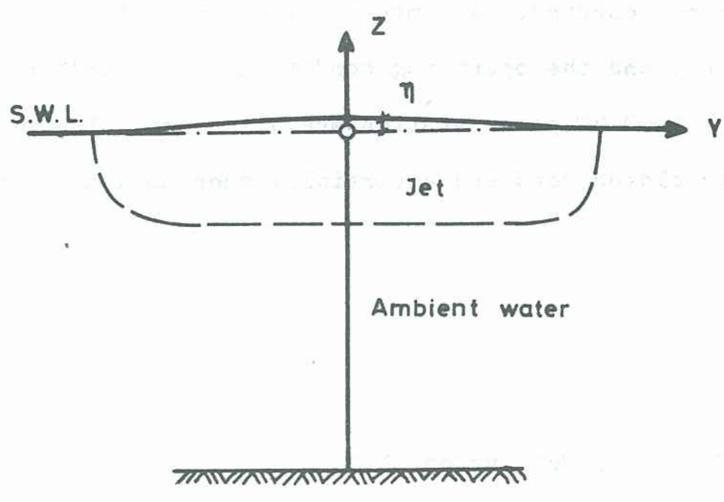

Fig. 2 - Warm Water Surface Jet