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WATER PARTICLE ORBITS IN DEEP - TO SHALLOW - WATER WAVES

by

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SUMMARY

No theory yet devised adequately predicts the orbital velocities of water particles as produced by wave propagation over the complete range of depth values required for design purposes. Experimental data indicate that for d/L ratios greater than 0.2 the linear theory gives maximum horizontal velocities at the still water and bed level accurately, but for shallower conditions these exceed the theoretical values. It is shown that for these smaller d/L values the dimensionless velocity $U_{max}/\sqrt{gd}$ varies directly with the H/d ratio. From the data presented a table is derived for still water and bed level maximum horizontal water velocities over a complete range of H/d and d/L. Relationships of similar nature could be derived for accelerations and vertical velocity when sufficient experimental data are available.

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INTRODUCTION

There is little need to stress the importance for accurate assessment of orbital velocities and accelerations in waves. The calculation of forces on structures depends directly on such variables. Drag forces vary with the square of any velocity and the inertia forces with the acceleration. Any error therefore is either directly proportioned or even squared.

Wave theories have been derived for celerities and profiles of waves and in this aspect have been reasonably successful. Their application to orbital motions of water particles have not been so successful and this criticism may be levelled more at the higher order solutions. Even without experimental verification, the results from many equations differ substantially from each other in the velocities quoted at SWL, bed or intermediate depths. Le Méhauté et al(1) have made a comprehensive comparison of maximum horizontal velocity, in which these differences are illuminated.

There have been a number of experimental investigations, the first of which appears to be that of Morrison and Crooke(2). Others of which the writer is aware are papers by Iwagaki and Sakai(3), Tsuchiya and Yamaguchi(4) and Goda(5). Some of these papers contain only a few results, mainly in the form of graphs of velocity versus water depth. But the comprehensive report by Goda(5) is based upon some hundred runs, which are tabulated in full.

The author(6) some years ago plotted the few results of the above authors, but including only the few graphed results in Goda's report. This graph used the dimensionless parameters $U_{max}/\sqrt{gd}$ and HL/d^2 (see notation at end of paper). These appeared to fall along a straight line (on a log-log scale) for the still water level (SWL) values and on a smooth curve for the bed horizontal velocities. The abscissa consisted of two ratios, namely $\frac{H}{d}$ and $\frac{L}{d}$, which is similar to but distinct from the Ursell parameter(7) HL^2/d^3. Plotting of the dimensionless maximum velocity against the Ursell parameter resulted in complete scatter.

Recently the author inspected Goda's paper(5) more thoroughly and found the tabulation of U_{max} for SWL and therefore plotted the complete set of results on the same basis. Differentiation was made between the d/L values, which then allowed isohyets of H/d to be drawn. The points grouped themselves along 45° lines which were parallel to the linear theory solutions. At the larger d/L values this theory predicted $U_{max}/\sqrt{gd}$ extremely well, but from d/L = 0.3 downwards the experimental velocities exceeded the theoretical, as would be expected from the cnoidal (8), hyperbolic(9) or solitary wave theories(10).

Vertical velocities and accelerations of water particles are not discussed in this paper. They warrant some investigation along similar lines to that set out below for maximum horizontal velocity, since the terms in the various theories are similar.

MAXIMUM HORIZONTAL VELOCITY AT SWL

The hundred and six results from Goda(5), who carried out tests with water depths ranging from 1.0 to 2.0 metres, are presented in Figure 1. The d/L ratios of the experimental points varied slightly from those quoted in the legend, but not by more than 1%.

It is seen at once that the results for d/L = 0.5 and 0.3 followed the lines for linear theory very closely. For smaller depth ratios a mean line has been drawn which is parallel to the theoretical curves. These deviate more from the latter as the d/L decreases. From the 45° slope of both sets of lines it can be concluded that

$$\frac{U_{max}}{\sqrt{gd}} = K\left(\frac{HL}{d^2}\right) \qquad .....(1)$$

where K is a function of d/L.

The breaking limit of H/d = 0.78 is shown dotted in Figure 1. The lower curve applies to the theoretical lines whilst the upper one refers to the experimental mean curves, which should be the more accurate. It is seen that all the results are within the stable zone of propagation

The straight line straddling the d/L lines is that given by Silvester(6) from his initial analysis of the problem. He inferred from his previous presentation that results of other workers should be preferred to those of Goda because the latter deviated from his line the most. This is now seen to be due to the wider range of d/L and H/d used by Goda, whose results are seen to be extremely consistent.

The intercept of the experimental mean curves with the line HL/d^2 = 1.0 (which is in the centre of the results) gave $U_{max}/\sqrt{gd}$ values for specific d/L values which are plotted in Figure 2. These points have been approximated by two straight lines. That for the lower d/L range is given by

$$\frac{U_{max}}{\sqrt{gd}} = \frac{2}{3}\frac{d}{L} \qquad .....(2)$$

indicating from equation (1) a complete relationship.

$$\frac{U_{max}}{\sqrt{gd}} = \frac{2}{3}\frac{d}{L}\;\frac{HL}{d^2} = \frac{2}{3}\frac{H}{d} \qquad .....(3)$$

The results were then replotted in Figure 3 as $U_{max}/\sqrt{gd}$ versus H/d, where it is seen that for d/L = 0.05 to 0.3 they fall very close to the line as indicated by equation (3). The ±16% curves of the velocity parameter are drawn, indicating a close agreement.

The few results for d/L = 0.3 to 0.5 fall outside the +16% limit as would be expected from the steeper line in Figure 2, which followed the relationship

$$\frac{U_{max}}{\sqrt{gd}} = 1.255\;(\frac{d}{L})^{1.5} \qquad ....(4)$$

indicating a complete function

$$\frac{U_{max}}{\sqrt{gd}} = 1.255(\frac{d}{L})^{1.5}\left(\frac{HL}{d^2}\right) = 1.255\;\frac{H}{d}\;(\frac{d}{L})^{0.5} \qquad .....(5)$$

which is the linear solution for deep-water conditions.

The results of the other three groups of workers quoted (1) (2) (4) are presented in Figure 4. The mean experimental curves from Figure 1 are included for comparison. The breaking limit also is derived for these lines. It is seen that the correlation with Goda's curves is consistent although more scattered than results in Figure 1. It is seen how the data are concentrated around the line previously assumed by Silvester(6).

MAXIMUM HORIZONTAL VELOCITY AT BED

The values of $U_{max}/\sqrt{gd}$ at the bed are plotted against HL/d^2 in Figure 5. By drawing mean lines through Goda's data at the same 45° slope as the theoretical curves and intercepting them on the ordinate $HL/d^2 = 1$, an equation can be derived of the form.

$$\frac{U_{max}}{\sqrt{gd}} = 2.92\;\frac{H}{d}\;\;(\frac{L}{d})^{0.21} \qquad .....(6)$$

Another approach is to graph the ratio or percentage of bed to SWL velocity against d/L as in Figure 6. The curve so drawn is derived from linear theory and is asymptotic to the 100% line at low depth ratios and decreases rapidly as deep water is approached. Concentrating on the experimental results of Goda(5) it is seen that down to d/L = 0.14 the theoretical relationship appears to predict the percentage reduction well. Below this depth ratio the bed motion is less than that predicted by linear theory. A mean curve is drawn through the points as a suggested reduction factor.

Also drawn in Figure 6 are the $U_{max}/\sqrt{gd}$ values for $HL/d^2 = 1.0$ and 10 against d/L. By choosing constant HL/d^2 values in steps of 10 the complete range of d/L can be examined to the limit of breaking waves. The ordinate values change by a factor of 10 for the same d/L scale, which has already been used for the percentage (bed/SWL) velocity.

It will be observed in Figure 5 that for any given HL/d^2 the theoretical traces increase until d/L = 0.2 is reached after which the velocity ratio decreases. This trend is exhibited more clearly in Figure 6 where for constant HL/d^2 an optimum value of $U_{max}/\sqrt{gd}$ is reached at d/L = 0.2. For smaller values of d/L the curves for SWL and bed velocities become asymptotic to each other.

From the upper dotted curve, representing Goda's experimental trend, the dimensionless bed velocity has also been drawn. This must be asymptotic to the linear theory trace at d/L = 0.2 and might be expected to become asymptotic to the hyperbolic theory for bed motion in shallow conditions. However the line drawn, following the Goda experimental percentage prediction, falls between the hyperbolic and linear curves.

CONCLUSIONS

1. For the depth ratios $d/L = 0.05$ to 0.3 the SWL dimensionless horizontal velocity is given by $U_{max}/\sqrt{gd} = 2H/3d$.

2. For depth ratios $d/L \geq 0.3$ the linear theory predicts the SWL horizontal velocity accurately, which can be expressed in the form $U_{max}/\sqrt{gd} = 1.255 \frac{H}{d} (\frac{d}{L})^{0.5}$.

3. The bed to SWL horizontal velocity ratio follows the linear theory for $d/L \geq 0.014$, but for shallower conditions is below this theoretical prediction but above the linear theory prediction

4. Horizontal accelerations and vertical velocities and accelerations should correlate in a similar manner with combinations of H/L and H/d for deep-water and shallow water conditions respectively.

5. Horizontal velocities at SWL and at the bed based upon the experimental evidence presented herein should be given within ±15% by Table I.

Table I - Values of $U_{max}/\sqrt{gd}$ at SWL (upper) and at BED (lower)

H/d	d/L	0.02	0.04	0.06	0.10	0.15	0.2	≤0.3	0.4	0.5	0.6	0.7	0.8
0.05	SWL	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.040	0.044	0.049	0.052	0.056
	BED	0.029	0.027	0.025	0.024	0.022	0.017	0.010	0.007	0.004	0.002	0.001	0.001
0.10		0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.079	0.089	0.097	0.105	0.112
		0.058	0.055	0.051	0.049	0.045	0.035	0.020	0.013	0.008	0.004	0.002	0.001
0.15		0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.119	0.133	0.146	0.157	0.168
		0.087	0.082	0.077	0.073	0.068	0.052	0.030	0.020	0.011	0.006	0.004	0.002
0.20		0.133	0.133	0.133	0.133	0.133	0.133	0.133	0.158	0.177	0.194	0.210	0.224
		0.116	0.109	0.102	0.097	0.090	0.070	0.040	0.026	0.015	0.008	0.005	0.003
0.25		0.167	0.167	0.167	0.167	0.167	0.167	0.167	0.198	0.222	0.243	0.262	0.281
		0.145	0.137	0.129	0.122	0.114	0.088	0.050	0.033	0.019	0.011	0.006	0.004
0.30		0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.238	0.266	0.292	0.315	0.337
		0.174	0.164	0.154	0.146	0.136	0.105	0.060	0.039	0.023	0.013	0.007	0.004
0.35		0.233	0.233	0.233	0.233	0.233	0.233	0.233	0.278	0.310	0.340	0.367	0.393
		0.203	0.191	0.179	0.170	0.158	0.122	0.070	0.046	0.027	0.015	0.009	0.005
0.40		0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.316	0.355	0.389	0.420	0.449
		0.232	0.219	0.206	0.195	0.181	0.140	0.080	0.052	0.031	0.017	0.010	0.006
0.45		0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.357	0.399	0.437	0.472	0.505
		0.261	0.246	0.231	0.219	0.204	0.158	0.090	0.059	0.035	0.019	0.011	0.007
0.50		0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.396	0.444	0.486	0.525	0.561
		0.290	0.273	0.256	0.243	0.226	0.175	0.100	0.065	0.039	0.021	0.013	0.007
0.55		0.367	0.367	0.367	0.367	0.367	0.367	0.367	0.436	0.488	0.535	0.577	0.617
		0.319	0.300	0.283	0.268	0.250	0.193	0.110	0.072	0.042	0.023	0.014	0.008
0.60		0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.476	0.532	0.583	0.630	0.673
		0.348	0.328	0.308	0.292	0.272	0.210	0.120	0.078	0.046	0.026	0.015	0.009
0.65		0.433	0.433	0.433	0.433	0.433	0.433	0.433	0.516	0.577	0.632	0.682	0.730
		0.377	0.355	0.333	0.316	0.294	0.227	0.130	0.085	0.050	0.028	0.016	0.009
0.70		0.467	0.467	0.467	0.467	0.467	0.467	0.467	0.556	0.621	0.680	0.735	0.786
		0.406	0.383	0.360	0.341	0.318	0.245	0.140	0.092	0.054	0.030	0.018	0.010
0.75		0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.595	0.665	0.729	0.787	0.842
		0.435	0.410	0.385	0.365	0.340	0.262	0.150	0.098	0.058	0.032	0.019	0.011

NOTATION

d	depth of water
g	acceleration due to gravity
H	wave height
K	function in equation(1)
L	wave length
U_{max}	maximum horizontal orbital velocity

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Figure 1 - Dimensionless velocity at SWL versus HL/d^2 over a range of d/L from Goda(5).

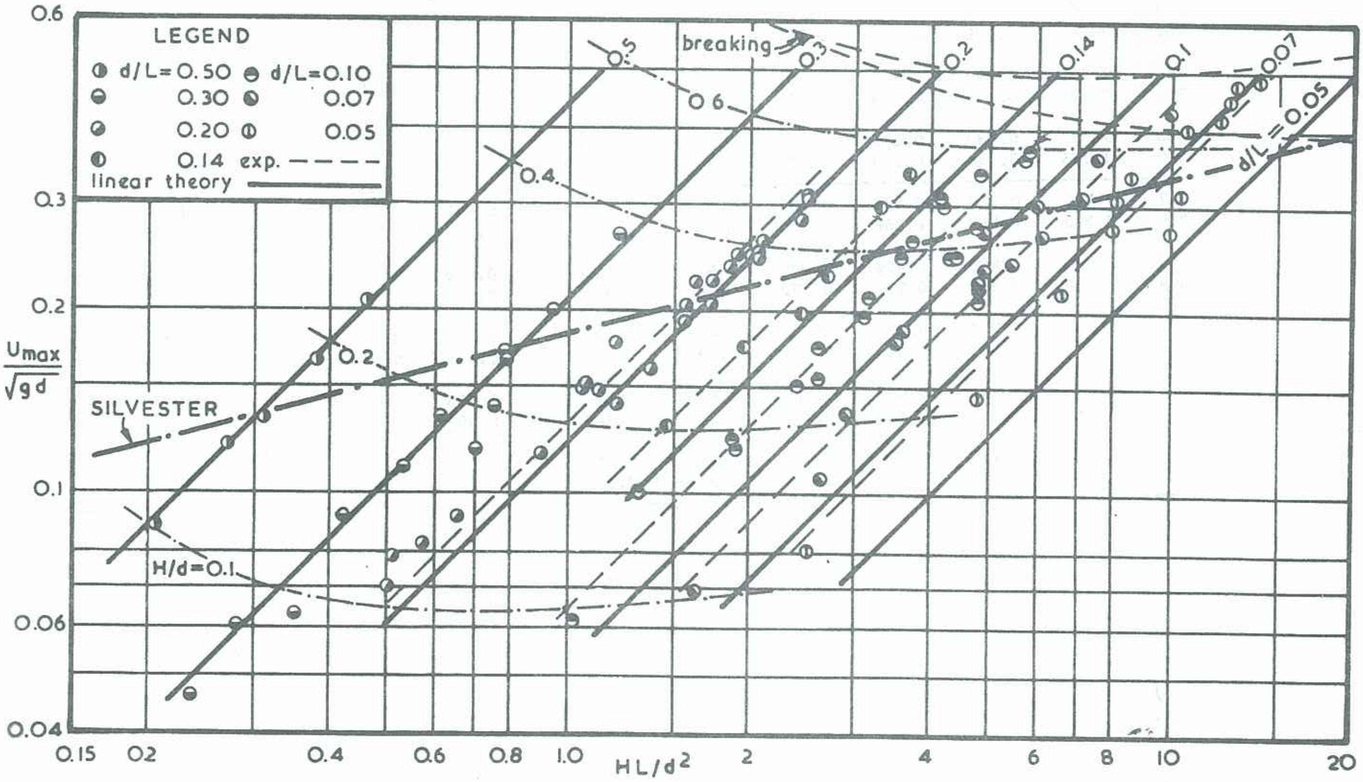
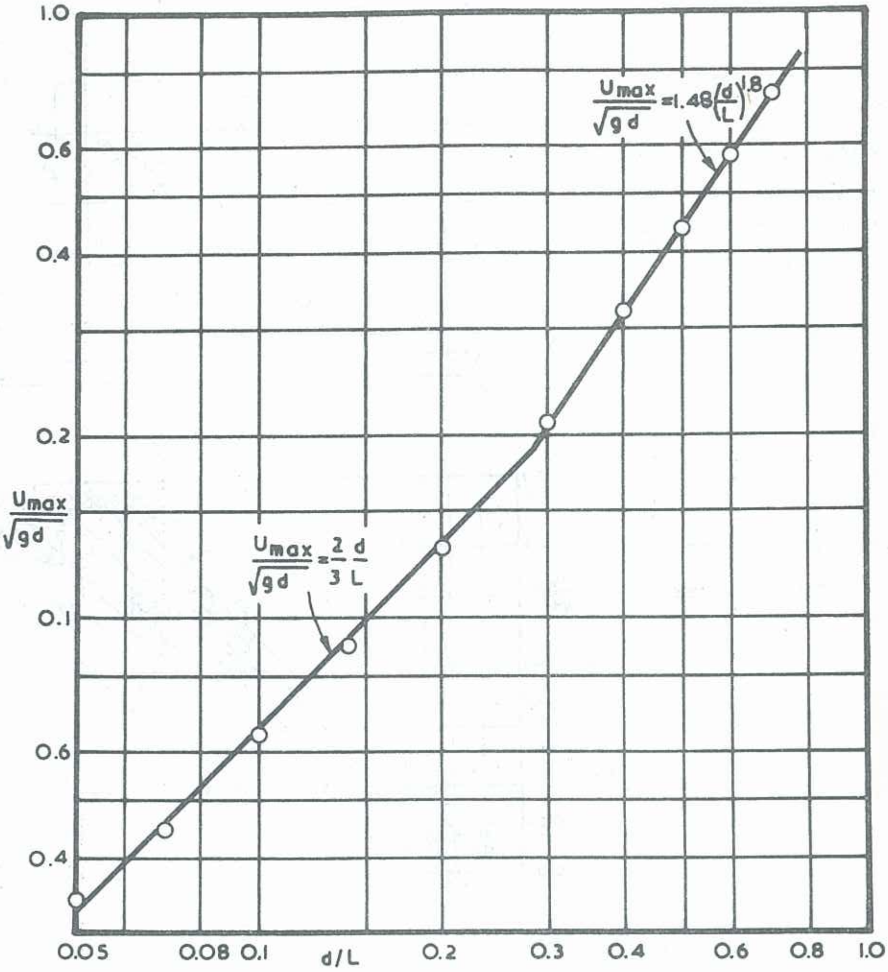


Figure 2 - Dimensionless velocity at SWL versus d/L for $HL/d^2 = 1.0$



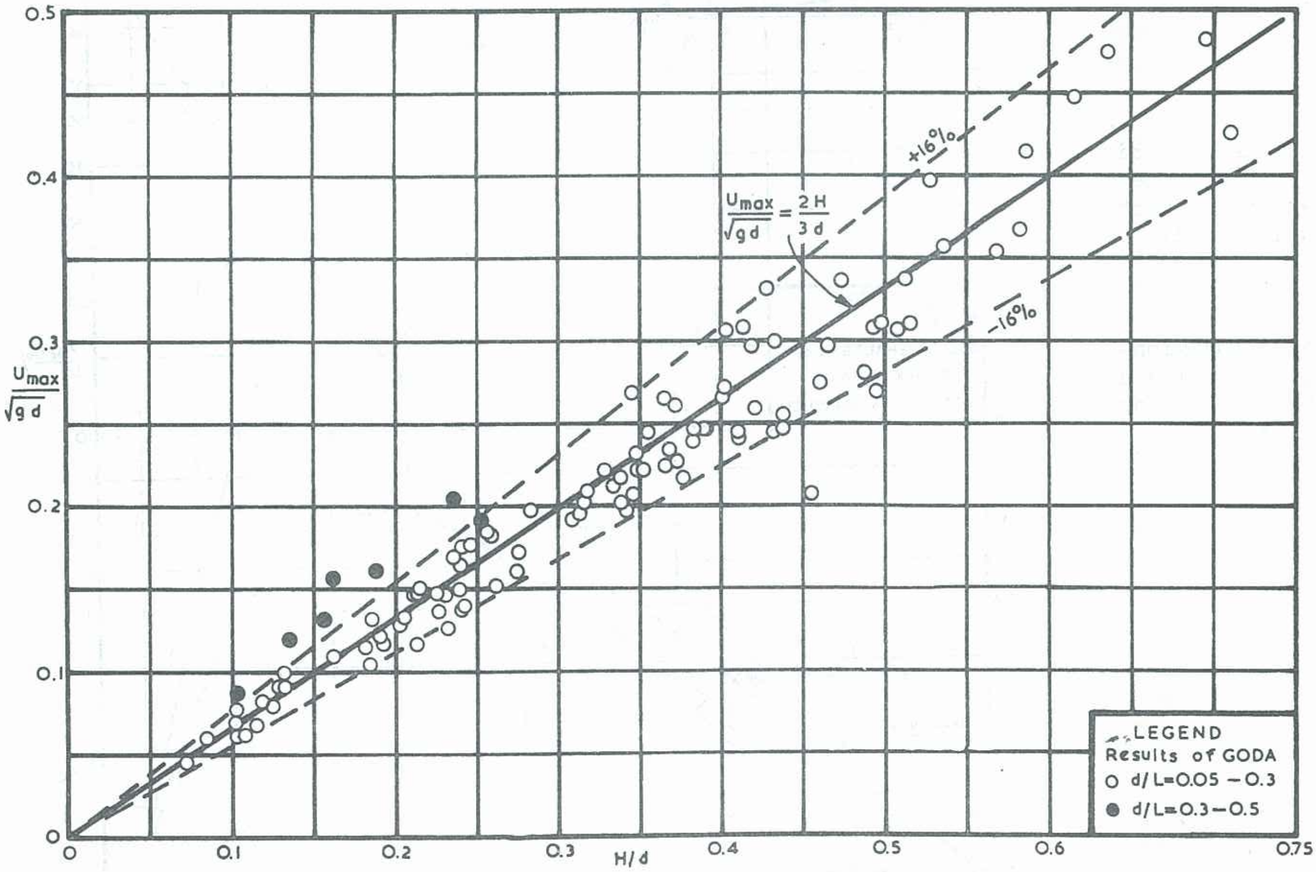


Figure 3 - Dimensionless velocity at SWL versus H/d for two specific ranges of d/L

Figure 4 -

Dimensionless velocity at SWL versus HL/d^2 from three sources (1) (3) (4) compared to experimental curves of Goda from Figure 1.

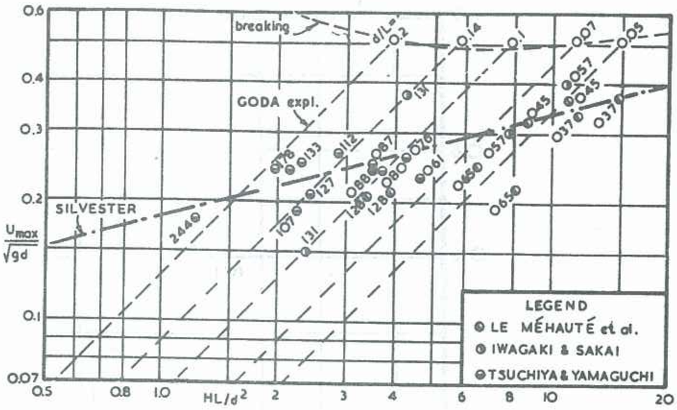


Figure 5 -

Dimensionless velocity at bed versus HL/d^2 from four sources (1) (3) (4) (5).

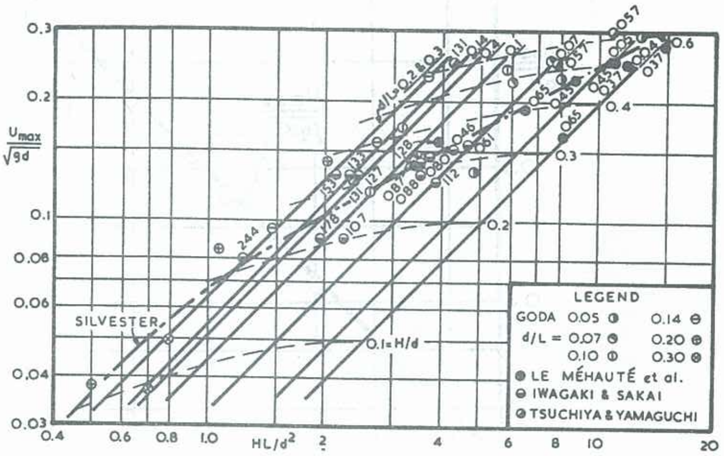


Figure 6 -

(a) Bed velocity as % SWL value versus d/L with experimental data (1) (3) (4) (5)

(b) Dimensionless velocity at SWL and bed for HL/d^2 = 1.0 and 10.0 versus d/L.