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NUMERICAL SOLUTIONS OF NON-LINEAR HYDRODYNAMIC FORCES DUE TO WAVE ACTION

by

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SUMMARY

The drag force acting on sea structures due to wave is often evaluated as is proportional to $u|u|$, where u is the velocity induced by waves. However, if the vibration of the structure is considered, the drag force must be proportional to $|u-\dot{x}|(u-\dot{x})$, where $\dot{x}$ is the oscillatory velocity of the structure. In this paper, the effect of $\dot{x}$ is investigated numerically and showed that the negligence of the interaction effect may cause a misevaluation of the wave forces, as well as of frequency responses.

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INTRODUCTION

It is now widely believed that Morrison's formula will work practically in order to estimate acting forces due to the wave action on piles and some other structures. Morrison's formula[4] is essentially a linear combination of the friction force and the form drag. It has been pointed out[6,7,8] that if the pile is flexible, which is naturally realistic enough, the movement of the pile must be taken into account. However, detailed analyses are rare, since in this case the hydrodynamic vibration system becomes non-linear. The aim of this paper is to find out the quantitative difference between non-linear and linear solutions.

STATEMENT OF THE PROBLEM

We consider the dynamic non-linear system such that

$$\ddot{x} + C\dot{x} + Kx + \alpha(\dot{x}-u)|\dot{x}-u| = F(t) \tag{1}$$

where x is the displacement, C is the linear damping coefficient, K is the spring coefficient, α is the non-linear damping coefficient, u is the velocity of the surrounding fluid and $F(t)$ is the other external force. All the variables are non-dimensionalized. The "mass" is unity, where the virtual term is included in this mass. The pressure term is included in $F(t)$. Morrison's formula is essentially the same as Eq.(1) if we set $\ddot{x}$, $\dot{x}$ and x equal to zero. However, not many people have checked whether this assumption is practically acceptable or not. One difficulty is, in this case, that the response of a non-linear system may depend on the external forces. Therefore, it is very difficult and almost impossible to derive a very general conclusion through such a kind of study. On the other hand, the methodology adopted here is still general to attack such non-linear problems at the moment, though $F(t)$ has a simple form.

We specify u and F respectively

$$u = \cos\Omega t \tag{2}$$

$$F(t) = A\sin\Omega t, \quad A,\Omega : \text{constants} \tag{3}$$

These assumptions are almost true in case of wave motion, though $F(t)$ may not be so simple.

The following three solving methods were employed and their results were compared.

i) Numerical solution of Eq.(1).
ii) A linearized solution where $\dot{x} - u \simeq -u$.
iii) Perturbation solution assuming $\alpha \ll 1$.

Only periodic solutions will be discussed for the cases above.

DETAILED DISCUSSIONS FOR THREE TECHNIQUES

1. Numerical Solution

Runge-Kutta-Gill method was employed. The computer used was HITAC-8700 (248K, virtual memory type) of Tokyo Institute of Technology. This solution will be referred as RKG solution in this paper. Initial condition was $x = 1.0$ and $\dot{x} = 0.0$ at $t = 0$.

2. Linearized Solution

$\dot{x}-u$ may be substituted by $-u$, if $|u| \gg |\dot{x}|$. Physically speaking, $|\dot{x}|$ is the vibrating speed of the pile. Therefore, this assumption will be valid if the water particle velocity is large compared with the oscillating velocity of the structure. On the other hand, the estimation of the validity is rather difficult since we do not know $\dot{x}$ in advance.

Anyhow, this method is so to speak the direct application of Morrison's formula and note that JSCE is recommending[1,2] this method, though the authors do not agree with the basic ideas of this method.

In this case, we obtain a linear differential equation as

$$\ddot{x} + C\dot{x} + x - \alpha\cos\Omega t\,|\cos\Omega t| = A\sin\Omega t \tag{4}$$

whose solution can be obtained easily.

3. Perturbation Solution where $\alpha \ll 1$

We transform the dependent variable in a form that

$$\dot{y} = \dot{x} - u \tag{5}$$

in Eq.(1), and we obtain

$$\ddot{y} + C\dot{y} + y + \alpha|\dot{y}|y = B\sin\Omega t - C\cos\Omega t \tag{6}$$

where
$$B = A + \Omega - 1/\Omega \tag{7}$$

Assuming that y can be expressed in a series form with respect to α, we obtain the first order solution by perturbation method.

$$y = y_o + \alpha y_1 \tag{8}$$

$$\ddot{y}_o + C\dot{y}_o + y_o = B\sin\Omega t - C\cos\Omega t \tag{9}$$

$$\ddot{y}_1 + C\dot{y}_1 + y_o = \mp\dot{y}_o^2 \tag{10}$$

The particular solution of Eq.(8) is Eq.(11)

$$y_o = p_1\cos\Omega t + p_2\sin\Omega t \tag{11}$$

where
$$p_1 = \frac{-C(1-\Omega^2)-CB\Omega}{(1-\Omega^2)^2+C^2\Omega^2} \tag{12}$$

$$p_2 = \frac{B(1-\Omega^2)-C^2\Omega}{(1-\Omega^2)^2+C^2\Omega^2} \tag{13}$$

The first order solution will be obtained by using y_o.

$$\ddot{y}_1 + C\dot{y}_1 + y_1 = \mp(q_1 + q_2\cos 2\Omega t + q_3\sin 2\Omega t) \tag{14}$$

where
$$q_1 = \frac{\Omega^2}{2}(p_1^2 + p_2^2) \tag{15}$$

$$q_2 = \frac{\Omega^2}{2}(p_2^2 - p_1^2) \tag{16}$$

$$q_3 = -\Omega^2 p_1 p_2 \tag{17}$$

Next, let us consider a function Y defined by Eq.(18)

$$Y = e\cos 2\Omega t + f\sin 2\Omega t + g \tag{18}$$

where
$$\left.\begin{aligned} e &= [(1-4\Omega^2)q_2 - 2C\Omega q_3]/[(1-4\Omega^2)^2 + 4C^2\Omega^2] \\ f &= [2C\Omega q_2 + (1-4\Omega^2)q_3]/[(1-4\Omega^2)^2 + 4C^2\Omega^2] \\ g &= q_1 \end{aligned}\right\} \tag{19}$$

The solution of Eq.(14) can be expressed as

$$\left.\begin{aligned} y_1 &= -Y \quad \text{where } \dot{y}_o - \dot{Y} \geqq 0 \\ \text{and} \quad y_1 &= Y \quad \text{where } \dot{y}_o + \dot{Y} < 0 \end{aligned}\right\} \tag{20}$$

Note that if $-\alpha\dot{Y} \leqq y_o \leqq \alpha\dot{Y}$, the solution is not valid, since in this case we are considering large α.

ACCURACY OF R-K-G METHOD

Since the analytical solution cannot be obtained in this case so far, the accuracy of R-K-G method which is the standard solution in this case must be checked. For this purpose, the analytic solution of a linear equation (21),

$$\ddot{x} + C\dot{x} + x = A\sin\Omega t \qquad (21)$$

is compared with the R-K-G solution. The result is satisfactory for the steady state.

The analytic solution of Eq.(4) can also be obtained. The R-K-G method showed a satisfactory result in this case. From these facts, the R-K-G solution is used as the standard solution.

THE ACCURACY OF THE LINEARIZED SOLUTION

The analytical solution of the linearized solution Eq.(4) is compared with the standard solution (R-K-G solution of Eq.(1)). Fig.1 shows the ratio of the amplitude of the linearized solution to that of the standard, E_1, which is a function of the force amplitude A. Relating with Fig.1 we could suggest the following:

a) For large A, E_1 becomes large. E_1 takes its minimum at $A \simeq 1.0$, which is $E_1 = 1.5$. However, E_1 increases again for small A. This means that the linearized solution is always over estimated with respect to the amplitude.

b) Perhaps, the physical interpretation is as follows: the terms $A \sin\Omega t$ and $u = \cos\Omega t$ are acting as the "external" forces and if A is large, the former becomes predominant and so to speak "locking" takes place. In this case $|\dot{x}|$ becomes large. If A becomes small, then u is predominant as $\dot{x}$ must be associated with u. Anyway, the result shows the refutation of the linearization in strict sense.

c) Fig.2 shows two oscillations for $A = 10$ and $\Omega = 0.707$, where the linearized solution showed that best result. We still observe a phase shift between two solutions.

ACCURACY OF THE PERTURBATION SOLUTION

As far as we use the perturbation method, the condition $\alpha \ll 1$ must be satisfied. Fig.3 shows the ratio of the amplitude of perturbation solution to that of R-K-G solution, E_2, and the ratio of linearized to R-K-G, E_3, in terms of α.

If $\alpha < 0.01$, the non-linear effect is practically nothing.

For the range of α where $0.01 \leqq \alpha \leqq 0.5$, the second order solution showed a good result and the error in amplitude is about 1%.

If $\alpha > 0.5$, the error reached to a considerable magnitude.

EFFECTS OF THE SPRING CONSTANT

The solutions obtained here are those for a unit spring constant or $K = 1$. We consider Eq.(1) again, where the effects of K must be studied. Fig.4 shows the ratio of the amplitudes of the non-linear and linear solutions E_4, where the variable is the spring constant K. In this case, if $K < 10.0$, the accuracy was unsatisfactory. Note, however, that the linear solution was always large. Though this conclusion may not be general, since we did not change all the variables, this information is important for practical purposes.

FREQUENCY RESPONSE CURVE

The frequency response curves showed some change for non-linear and linear cases. Fig.5 shows the amplitude-frequency relations. A significant change can be observed.

CONCLUSION

The analysis showed that the linearized solution may not be a good approximate solution always. The complete numerical experiment may not be appropriate since there are so many parameters involved. In this sense, the results or conclusions in this paper cannot be very general. However, it cannot be denied that the linearized solution should not be used without careful consideration. The linearized solution showed almost 50% of error in some cases, and the characteristic change of frequency response may be involved.

As far as this experiment concerned, if the coefficient α of the non-linear term is less than 0.01, the linearized solution is a good approximation, while the perturbation showed a reasonable results for $0.01 < \alpha < 0.5$. Note also that this is again not general. In any way, it is dangerous to use Morrison's formula without considering the structures response, since we can solve the non-linear equations without difficulty by computers nowadays, and the application of physical model may be appropriate.

BIBLIOGRAPHY

1. Minorsky, N., Non-linear oscillations, Van Nostrand, 1962.

2. Handbook of Hydraulics, 1971 ed., ed. by JSCE, 1971 (in Japanese).

3. Coastal structure design manual, JSCE, 1969.

4. Morrison, J.R., J.W. Johnson and M.P. O'Brien, Experimental studies of force on piles, Proc. 4th Conf. of Coastal Eng., 1953.

5. Goda, Y., Wave forces on a vertical circular cylinder, experiments and proposed method of wave force computation, Report of Port and Harbor Tech. Res. Inst., No.8, 1964 (in Japanese).

6. Shi-igai, H. and T. Kono, Oscillation of a sphere under wave action, 15th Conference of Coastal Eng. in Japan, 1968 (in Japanese).

7. Miyamoto, K., A study on the wave response of a tower structure, Proc. of Dept. of Civil Eng. of University of Tokyo, Vol.10, 1971 (in Japanese).

8. Malhotra, A.K. and J. Penzien, Analysis of tall open structures subjected to stochastic excitation, Dynamic Waves in Civil Engineering, Wiley Interscience, 1971.

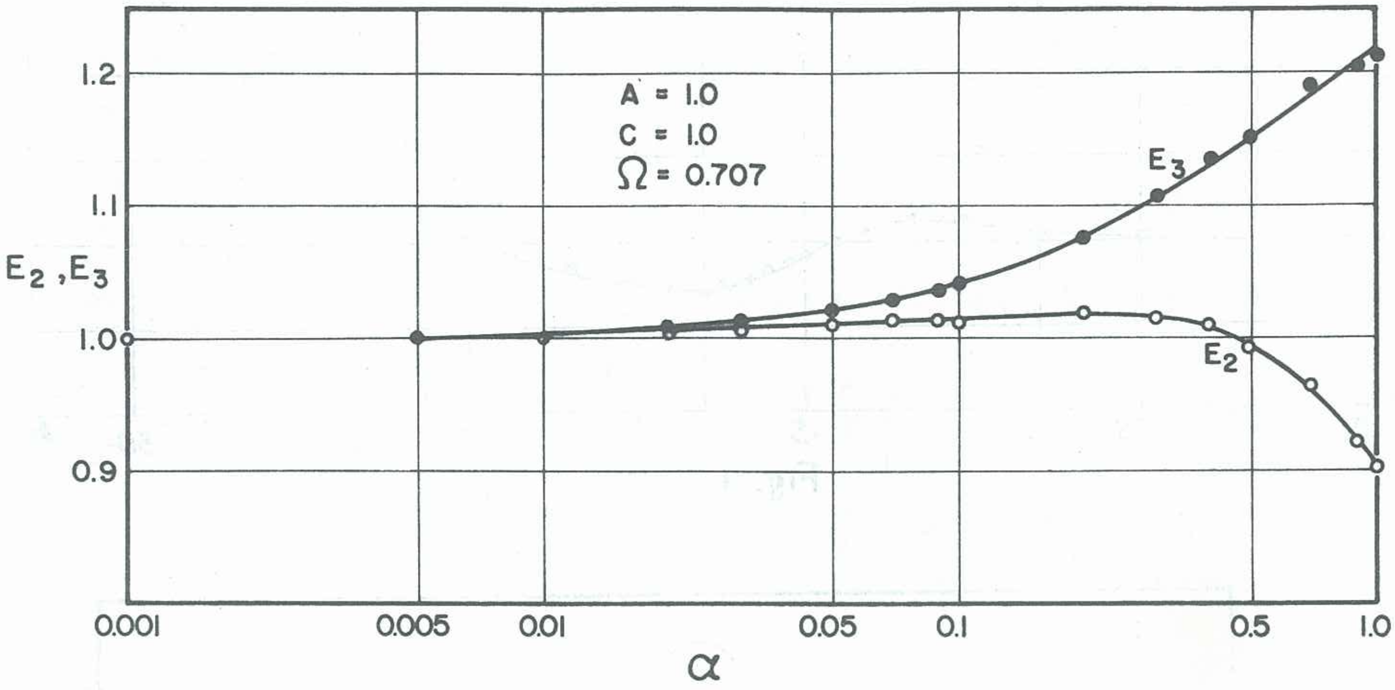
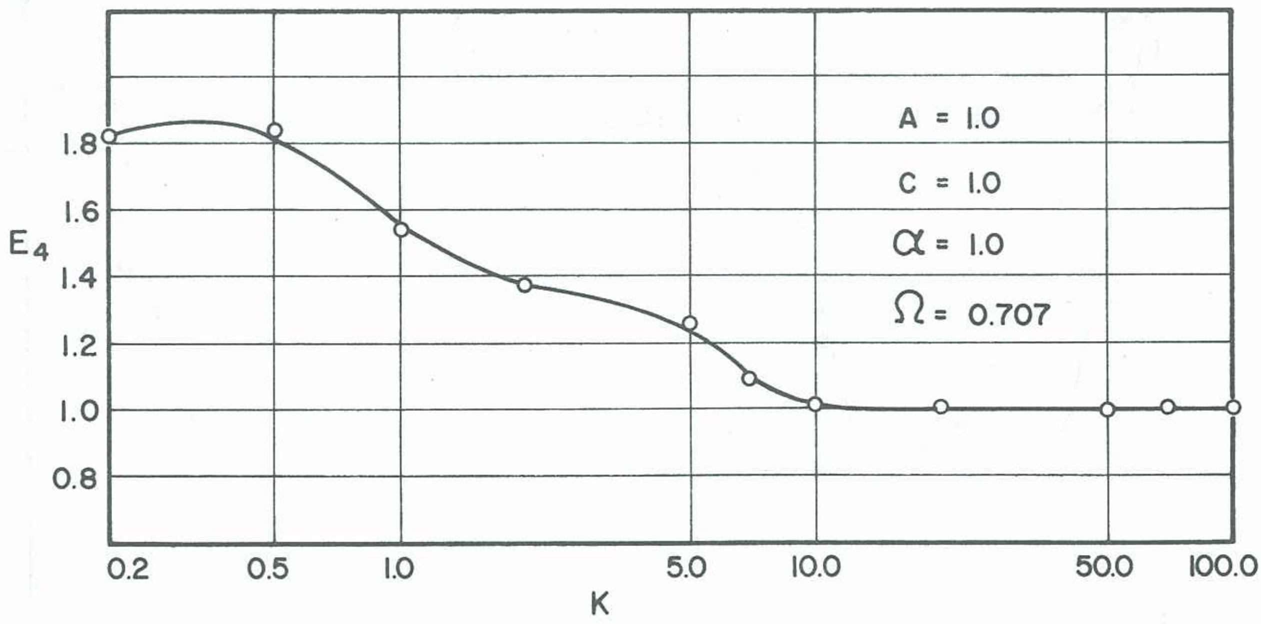

A = 1.0
C = 1.0
Ω = 0.707
E3
E2
E2 ,E3
1.2
1.1
1.0
0.9
0.001
0.005
0.01
0.05
0.1
0.5
1.0
α

Fig. - 3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C = 1.0
α = 1.0
Ω = 0.707
E4
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1.6
1.4
1.2
1.0
0.8
0.2
0.5
1.0
5.0
10.0
50.0
100.0
K

Fig. - 4

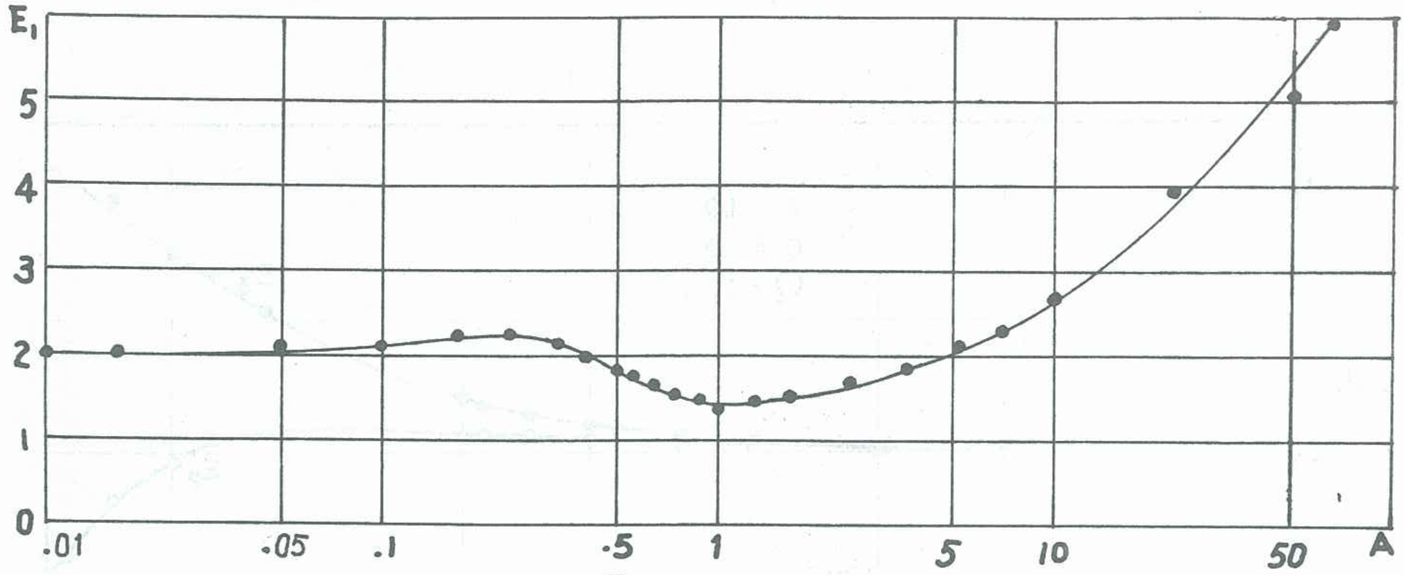

E1
5
4
3
2
1
0
.01
.05
.1
.5
1
5
10
50
A

Fig. 1

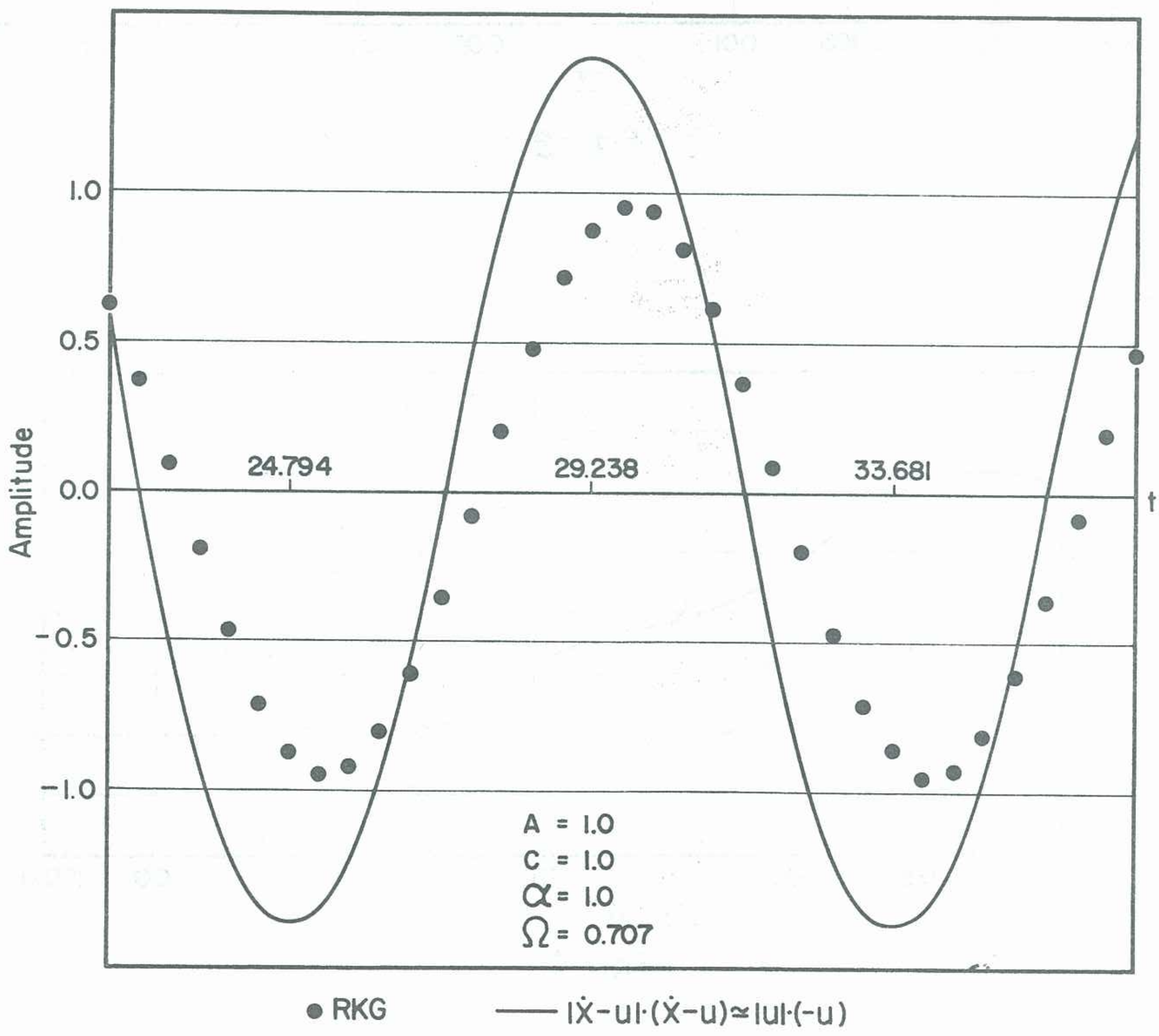
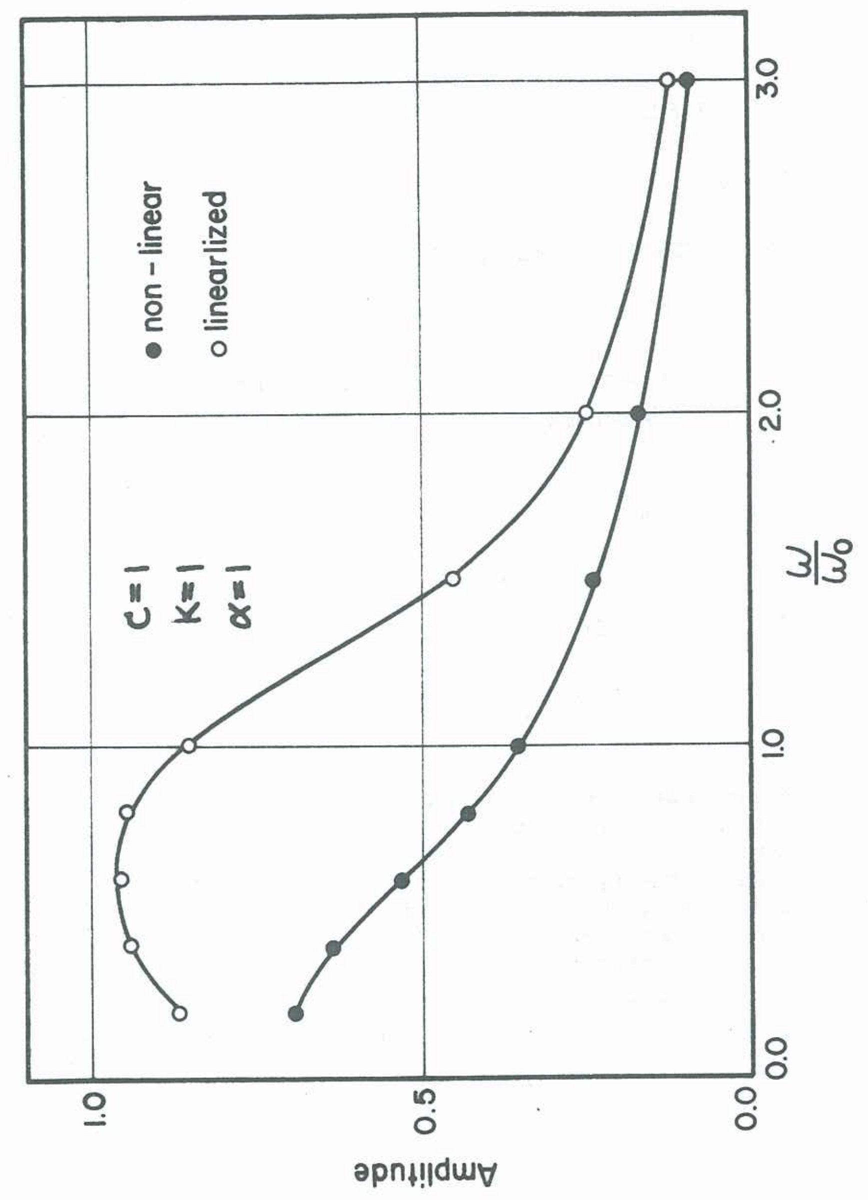

1.0
0.5
0.0
-0.5
-1.0
Amplitude
24.794
29.238
33.681
t
A = 1.0
C = 1.0
α = 1.0
Ω = 0.707
● RKG
—— |Ẋ−u|·(Ẋ−u)≃|u|·(−u)

Fig. − 2



C = 1
K = 1
α = 1
● non - linear
○ linearlized
Amplitude
0.0
0.5
1.0
0.0
1.0
2.0
3.0
$\frac{\omega}{\omega_0}$

Fig. - 5