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SOME ASPECTS OF LAGRANGIAN STUDIES OF SOLID PARTICLES IN TURBULENT PIPE FLOW

by

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The behaviour of large spherical particles in the central region of a turbulent flow within a horizontal pipe has been investigated. Lagrangian characteristics of the particle travel have been evaluated from high speed movie film records taken from a moving carriage.

Previous Eulerian studies of the particles had raised the possibility that relatively high diffusivities might be associated with particle movements in directions transverse to the pipe centreline. In the present paper the Lagrangian studies provide complementary evidence of particle diffusivity and clarify the earlier Eulerian results.

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LIST OF SYMBOLS

a	=	pipe radius;
C	=	concentration;
$C_L(\tau)$ or $C(\tau)$	=	true Lagrangian particle autocorrelation function;
$c(\tau)$	=	estimate of Lagrangian particle autocorrelation function;
D	=	particle diffusion coefficient;
N	=	number of sample records;
r	=	radial distance;
$R_L(\tau)$	=	Lagrangian particle correlation coefficient;
t	=	time;
T	=	length of sample record or total number of sample data points;
$\overline{U}$	=	mean pipe flow velocity;
u_*	=	shear velocity;
$V(t),\overline{V},v(t)$	=	particle, velocity, average velocity and fluctuating component respectively;
$\overline{v^2}$	=	variance of particle velocity fluctuation;
W_o	=	particle fall velocity in still water;
x_1,x_2,x_3	=	Eulerian Cartesian coordinates;
$y(t)$	=	Lagrangian coordinate of particle position after time, t;
α	=	dimensionless coefficient in eq.(1);
Δt	=	time interval;
$\varepsilon,\varepsilon_p$	=	fluid momentum (eddy viscosity) and particle turbulent mixing coefficients respectively;
$\mathfrak{J}$	=	Lagrangian particle integral time scale;
τ	=	time lag;
ρ,ρ_s	=	fluid and particle densities respectively;
Subscript: 1,2,3	=	components in x_1, x_2, x_3, directions respectively;
i	=	sample record index.

INTRODUCTION

The useful role of "single particle" experiments in the study of turbulent suspension flows was outlined by Batchelor (1) at the 1965 Conference. Reference was made to experiments by Binnie et al (2,3) in which the diffusivity of large solid particles in turbulent pipe flow appeared to be surprisingly large.

In previous Eulerian studies (4,5) of solid spherical particles in turbulent pipe flow, the present authors confined their considerations of particle diffusivity to the core region ($r/a \lessdot 0.5$) where diffusivity was uniform and it was established that radial forces on the particles, caused by velocity gradients, were negligible (Ref. 5). The experiments, using 2.54 mm diameter particles ($\rho_s \gg \rho$) in a 50.8 mm diameter pipe, indicated exponential vertical concentrations in the core of the horizontal pipe for all cases. This suggested the use of the well known diffusion-gravity balance equation which in the form given by Batchelor is

$$\varepsilon_p \frac{dC}{dx_3} + \alpha W_o C = 0 \qquad (1)$$

where α is a dimensionless number.

Traditionally α has not been included and the application of equation (1) with $\alpha = 1$ to suspended sediment in rivers has generally led to the approximate result that the particle diffusion coefficient ε_p approximates ε the eddy viscosity.

If α is assumed equal to unity for the authors' experiments, values of ε_p are indicated which are several times larger than the values of ε and some support for the possibility of large values of ε_p can be found in the data summarised by Householder and Goldschmidt (6).

It is clear that the results from studies of particles settling in vertically oscillating fluids cannot be directly applied to situations where similar particles are being transported, in permanent suspension, in turbulent flow. However, such studies (for example Houghton (7,8) and Field (9)) do illustrate an important point - that dissimilar particles can have the same fall-velocity, W_o, in still water but exhibit significantly different individual behaviour in pseudo turbulent fields. This adds support to Batchelor's suggestion that the value of α in equation (1) may depend to some extent on the size of particle and the scale of turbulence and is unlikely to be unity except where particle Reynolds number is less than unity.

The authors' Eulerian experiments suggest that α may depend upon particle density, particle size and W_o/u_*. Using equation (1) it was only possible to evaluate the ratio

ε_p/α. The question of whether large solid particles diffuse more rapidly than fluid particles can be examined, independently of the gravity-diffusion balance assumption, using results from the complementary Lagrangian experiments described in this paper.

LAGRANGIAN APPROACH TO TURBULENT DIFFUSION

This approach is usually based on the original work of Taylor (10) who considered the diffusion of fluid particles by continuous movements in a turbulent field which is homogeneous in both space and time. This theoretical work will be used as the basis for examining the transverse diffusion of the solid particles (relative density 1.035) in the core region of the pipe. Conditions in this region ($r/_a < 0.5$) are considered, as a first approximation, to be both homogeneous and isotropic. Support for this assumption is given by the constant values of ε_p/α obtained across this zone from the Eulerian studies. These studies also showed that the particles were not subject to significant radial forces (resulting from mean velocity gradients) in this zone.

For particle diffusion in the transverse direction of coordinate x_2 (Fig. 1) the particle displacement variance is given by Taylor as

$$\overline{y_2^2\,(t)} = 2\,\overline{v_2^2} \int_o^t (t-\tau)\, R_{L_2}(\tau)\, d\tau \qquad (2)$$

Here $R_{L_2}(\tau)$, the Lagrangian correlation coefficient is given by,

$$R_{L_2}(\tau) = C_{L_2}(\tau) \Big/ \overline{v_2^2} \qquad (3)$$

where $C_{L_2}(\tau)$, the Lagrangian autocovariance function

$$= \overline{v_2(t).v_2(t+\tau)} \equiv \lim_{T\to\infty} \frac{1}{T}\int_o^T v_2(t).v_2(t+\tau)\, dt \qquad (4)$$

For large lag times, τ, the integral of $R_{L_2}(\tau)$ approaches a constant value $\mathcal{J}_2$, the Lagrangian integral time scale associated with the v_2 particle fluctuations.

For large diffusion times,

$$\overline{y_2^2}\,(t) = 2\,\overline{v_2^2}\,\mathcal{J}_2\, t = 2D_2 t \qquad (5)$$

where $D_2 = \overline{v_2^2}\,\mathcal{J}_2$ is a diffusion coefficient.

EXPERIMENTAL PROGRAMME

The basic experimental flow circuit which has been described previously (4,5) incorporated an 18 m straight horizontal length of 50.4 mm diameter transparent pipe. For the Lagrangian experiments the measuring portion was 6 m long and was preceded by an 11 m approach length. The pipe was accurately aligned parallel to the rails of an adjacent towing carriage which has sophisticated speed control equipment. A high-speed 16 mm movie camera was mounted on the carriage, level with and about 1.5 m from the test pipe. To enable the position of a particle to be tracked in three dimensions, a mirror system (Figure 2) was used so that each film frame contained both a side elevation and plan view of the pipe and particle. The location of the mirror, the water trough used to minimize optical distortion and the position of a graduated scale are shown in Figure 2.

The experimental procedure involved setting up steady flow in the circuit and inserting sufficient particles for a number to be tracked simultaneously during each film run. Before filming each run, preliminary observations were made to determine the carriage speed corresponding to the mean velocity of an ensemble of particles. Film runs were made with the carriage speed remaining constant along the test length and the camera was started in sufficient time to reach the optimum film speed (approximately 300 frames per second). A large number of film records have been taken for 2.54 mm particles over a range of mean flow velocities from 0.76 to 1.30 m/s and for particle densities from 1035 to 1126 kg/m^3.

PROCESSING OF DATA

The time interval between successive film frames was determined from timing marks on the edge of the film. A film projector incorporating a frame counter was used to give a frame image of approximately 250 mm by 380 mm on the viewing screen of an XY digital reader. Pipe and particle positions were observed every second frame and were registered on a

Solartron data logger and recorded on punched paper tape.

For each film run it was possible to identify a number of particles which stayed both within the pipe core region and sufficient number of successive film frames, to make data conversion worthwhile. The punched paper tapes were converted to further tapes which were suitable for reading by the high speed paper tape reader connected to the Department's Interdata 70 mini-computer system.

ANALYSIS OF DATA

To investigate the solid particle diffusivity, the transverse position (i.e. in the x_2 direction) of particles at successive time intervals was examined within the framework of the Taylor relationship (Equation 2) outlined earlier for fluid particles. For a stochastic process such as this, the expected values of the statistical parameters may be estimated from ensemble averages of the behaviour of a number of individual particles or by time averages of a single particle realisation of considerable length.

The question of obtaining the best estimates of population parameters based on a number of samples N, each consisting of T data points, has been discussed by Hansen (10). A similar computational procedure has been adopted here in evaluating the quantities associated with the transverse position of the particles.

Considering the transverse direction, the velocity computation was

$$V_2(t) = (y_2(t+1) - y_2(t))/\Delta t \qquad (6)$$

Using the subscript, i, to refer to a single realisation, estimates of $C_2(\tau)$ were obtained from

$$c_{2,i}(\tau) = \frac{1}{T}\sum_{t=1}^{T-\tau} (V_{2,i}(t) - \overline{V}_2)(V_{2,i}(t+\tau) - \overline{V}_2) \qquad (7)$$

$$(\tau = 1,2,\ldots)$$

$$\text{where, } \overline{V}_2(t) = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} V_{2,i}(t) \qquad (8)$$

When a number of particles are used the appropriate expression is

$$c_2(\tau) = \frac{1}{N}\sum_{i=1}^{N} c_{2,i}(\tau) \qquad (9)$$

$$(\tau = 1,2,\ldots)$$

The velocity variance $\overline{v_2^2}$ was estimated from $c_2(o)$ which was obtained by extrapolating a linearised $c_2(\tau)$ v. τ plot. The Lagrangian time scale, $\mathcal{J}_2$, was obtained by fitting an exponential decay form to the correlogram of $R_{L2}(\tau)$, i.e. $R_{L2}(\tau) = \exp.(-\tau/\mathcal{J}_2)$ and the diffusion coefficient was obtained from

$$D = \overline{v_2^2}\,\mathcal{J}_2 \qquad (10)$$

An alternative procedure for computing the value of D, retains the form of $R_{L2}(\tau)$ assumed above and uses the value of $\overline{y_2^2}(t)$ to obtain the following,

$$\overline{y_2^2}(t) = 2\,\overline{v_2^2}\,\mathcal{J}_2\,(t - \mathcal{J}_2\,(1 - \exp.(-t/\mathcal{J}_2)) \qquad (11)$$

The values of $\overline{y_2^2}(t)$ were evaluated using the expression,

$$\overline{y_2^2}(t) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T-t+1}\sum_{j=0}^{T-t}(y_2\,(j) - y_2\,(j+t))^2\right) \qquad (12)$$

and the value of D was evaluated from equation (11).

COMMENTS AND CONCLUSIONS

The application of Taylor's equation was found to provide a suitable basis for describing the transverse (x_2 direction) diffusion of the solid particles in the core region in the present pipe flow experiments. Values of diffusion coefficient, D, obtained using equations (10) and (11) respectively agreed sufficiently to suggest that sample errors rather than shortcomings in Taylor's equations were responsible for any difference.

A typical result for a single particle realisation (tracking time $200\Delta t \approx 1\frac{1}{2}$ seconds) is shown in Fig. 3. The corresponding values of the diffusion coefficient, given here in the dimensionless form D/u_*a , were 0.0129 and 0.0113 when obtained using the velocity correlations (eq. 10) and the particle displacement variances (eq. 11) respectively. Evaluations of D using $\overline{y_2^2}$ (t) appeared to produce more consistent results.

Estimates of the appropriate value of ε/u_*a for the core region vary, but a typical value of 0.06 would appear reasonable. It is apparent in the present solid particle experiments that the particle diffusivity is significantly less than the fluid eddy diffusivity.

Although D has been determined from the particle displacement in the x_2 direction, the assumption that the turbulence in the core zone approximates homogeneous and isotropic conditions implies that D has a constant scalar value in this zone. Reference to the authors' previous Eulerian study (5) indicates a value of $\varepsilon_p/\alpha u_* a \approx .3$ corresponding to the present experimental conditions. The present Lagrangian studies clearly indicate that the surprisingly high values of ε_p/α are caused by small values of α in equation 1 rather than by $\varepsilon_p > \varepsilon$. The order of α is closer to 0.1 than to unity for the 2.54 mm particles and the need for caution in applying equation 1 is clear. Many more experiments are needed if it is required to establish the systematic variations in the value of α for different particles and flow conditions.

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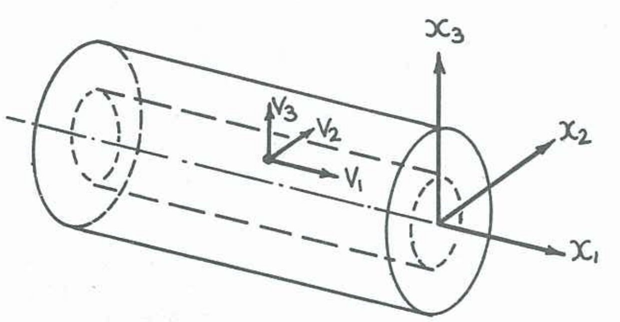

FIG I. COORDINATE DIRECTIONS

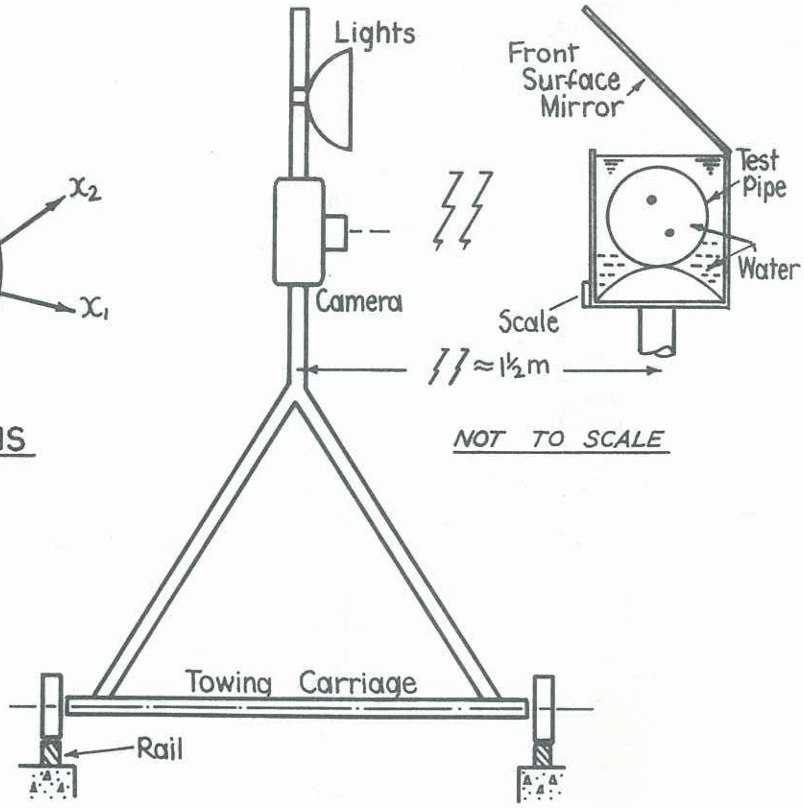

FIG 2. SCHEMATIC LAYOUT OF EQUIPMENT FOR PHOTOGRAPHY AND PARTICLE TRACKING.

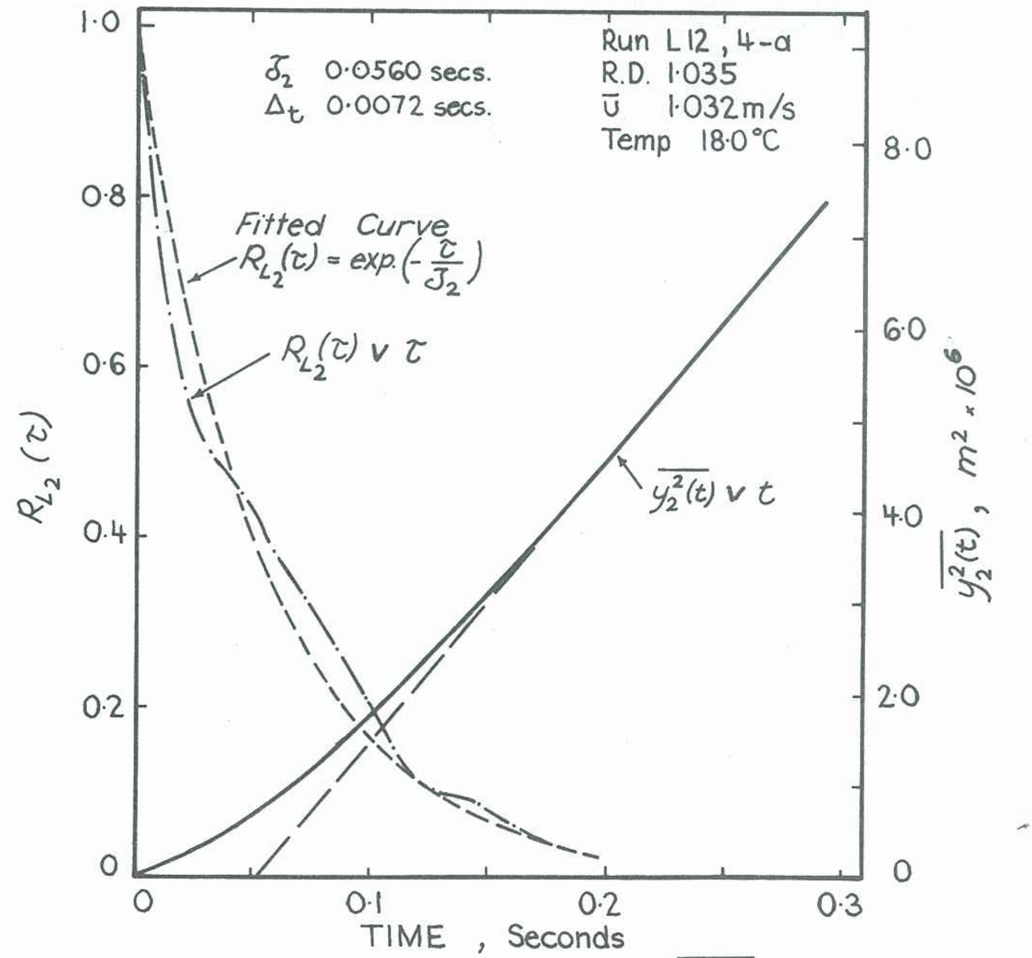

FIG 3. TYPICAL RESULTS FOR $R_{L_2}(\tau)$ AND $\overline{y_2^2(t)}$, FROM A SINGLE PARTICLE REALISATION.