FIFTH AUSTRALASIAN CONFERENCE
on
HYDRAULICS AND FLUID MECHANICS
at
University of Canterbury, Christchurch, New Zealand
1974 December 9 to December 13

TWO-POINT STATISTICS OF THE RANDOM PRESSURE FIELD BENEATH A HYDRAULIC JUMP

by

V.Sethuraman[1], M.H.Abdul Khader[2], K. Elango[3] and S. Sadasivan[4]

SUMMARY

Experimental results of the joint statistical characteristics of the fluctuating pressure field beneath a hydraulic jump are reported.

The results of the joint probability density reveal considerable departure from Gaussianity, in the violent region of the jump. The correlation measurements help understand the convected nature of turbulence in the flow direction. The cross-spectra indicate strong phase relationship for small separations in the flow direction.

These results are used in arriving at a semi-empirical equation for the pressure correlation function.

[1] Professor
[2] Asst. Professor
[3] Lecturer
[4] Senior Research Assistant

Hydraulic Engineering Laboratory
Indian Institute of Technology
Madras, India

GLOSSARY

A - Transducer - reference location
B - Transducer - correlated location
$C_{AB}(f)$ - Cospectral density
f - frequency
F_1 - incident Froude number
L_X, L_Y - macroscales along and across flow directions respectively
n, n_1, n_2 - empirical constants
p_A, p_B - Fluctuating pressure head values at locations A and B respectively
$\mathrm{Prob}(p_A, p_B)$ - Joint probability density
$Q_{AB}(f)$ - Quadrature spectral density
r.m.s. - root mean square value
$R(\Delta x, \Delta y, \tau)$ - normalized cross correlation
T - Average eddy life
U_c - Convection velocity
Y_1 - incident depth of jump
$\sigma\bar{p}$ - root mean square pressure head value
$\sigma\bar{p}'$ - pressure head value normalized with respect to p
$\Delta x, \Delta y$ - separations along and across flow directions respectively
$\Delta x', \Delta y'$ - separations normalized with respect to Y_1
τ - lag time
ω_o - radian frequency

I. INTRODUCTION

Better prediction procedures have been developed in recent times by the application of random process theory for the response of structures subjected to random loads. The prominent examples are the analysis of noise level and structural stresses induced by fuselage panel vibrations of aircraft and space vehicles excited by a turbulent boundary layer, response of tall structures to atmospheric turbulence, response of dams to earthquake forces and response of marine structures to winds and waves. Shinozuka [1] lists the various other applications. In hydraulic engineering, the stilling basin is an example where the floor and walls and also the appurtenances are prone to cavitation, fatigue or resonance damage because of exposure to intense pressure fluctuations. Uppal et al [2] and Huval and Neilson [3] report prototype structural failures due to such pressure fluctuations. Lesleighter [4, 5] studies the influence of appurtenances on the scale and intensity of the pressure fluctuations. The present paper discusses the joint characteristics of the pressure field beneath a simple hydraulic jump.

II. EXPERIMENTAL DETAILS

The jump was formed downstream of a model spillway in a 770 cm x 60 cm x 70 cm glass walled flume. Inductive type of transducers HBM P1/0.1 were mounted to the rigid base of the flume, their positions being as shown in Fig.1. The reference location was kept at 7.2 times the initial depth Y_1 from the toe of the jump with respect to which all joint characteristics were computed. The transducers were used in conjunction with a 6-channel carrier wave frequency amplifier by HBM and the pressure fluctuations were recorded on a He 86 strip chart recorder. The data acquisition system was earlier proved suitable for measuring the dynamic pressures in the present study.

The data processing was carried out in the digital mode, with the data obtained by manual digitization. Algorithms given by Bendat and Piersol [6] were used to write FORTRAN programs to compute the different statistical characteristics.

The results of single point measurements obtained in a previous study [7] showed similar characteristics for the incident Froude number F_1 between 4 and 7. Hence the present investigation was confined to $F_1 = 5.9$.

III. RESULTS AND DISCUSSIONS

The joint statistics computed include the joint probability density, the cross correlation function and the co- and quadrature spectral density.

The equiprobability contours of joint probability density for dimensionless separation $\Delta x' = \Delta x/Y_1 = 2.93$ in the flow direction is shown in Fig.2. There is considerable departure from Gaussianity and it is seen that pressure values between 2.5 and 3.5 times the r.m.s. value at the reference location A occur more frequently together with pressures in the range of 1.5 - 2.5 times the r.m.s. value at B. Results for other separations indicate that for given $\sigma p_A' = 2.0$, values of $\sigma p_B'$ for maximum probability generally increase with separation, as seen from Fig.3. For lateral separations same order of magnitude of pressures was found to occur as in the reference location.

Fig.4 shows the space-time correlation functions together with those computed using the model representation of the following form:

$$R(\Delta x, \Delta y; \tau) = \exp\left[-|\tau|/T\right]^{n} \exp\left[-n_1 |\Delta x| / L_x\right] \exp\left[-n_2 |\Delta y| / L_y\right] \cos\left[\omega_o\left(\tau - \frac{\Delta x}{U_c}\right)\right]$$

Herein

Δx - Separation in flow direction
Δy - Separation in lateral direction
τ - lag time
T - average eddy life
L_x - macroscale in the flow direction
L_y - macro-scale in the lateral direction
ω_o - a characteristic radian frequency
U_c - convection speed.

The model takes into account the temporal and spatial decaying nature and the convected pattern of the fluctuations in the flow direction. The form is analogous to the one employed by Maestrello et al [8] to represent the pseudo-sound field of a jet.

In the above model the following values of the parameters were used:

T = 0.03 sec
L_x = 2.82 cm
L_y = 5.00 cm
ω_o = 31.40 rad/sec
U_c = 141 cm/sec
$n = 0.5$, $n_1 = 0.1$ and $n_2 = 1.0$

The parameters T, L_x, L_y, ω_o and U_c which have physical significance were computed from experimental results. The eddy life was ascertained as the time at which the moving axes autocorrelation (envelope to cross correlation peaks) falls to the value $1/e$. The integral scales were computed approximately as the areas upto the first zero crossing from the cross correlation curves with zero time delay. The convection speed was taken as the average value of the time to peak divided by separation. The characteristic frequency was considered as the frequency at which the auto-spectral density at the reference location was maximum. The values of n, n_1 and n_2 were obtained empirically.

It may be seen that the correlation is small even for the smallest separation feasible with the transducers adopted. The model curves generally follow the experimental ones.

The co- and quadrature spectra obtained experimentally are shown in Fig.5 for separation in the flow direction. The ordinates become negligibly small beyond 20 hertz. That the co- and quadrature spectral ordinates are equally strong for smaller separations indicates strong phase relationship. However for lateral separations, the quadrature spectral ordinates were negligible in comparison to the co-spectral ordinates which again tapered off considerably beyond 20 hertz. These results point to possible wave like structure in the flow direction with strong macropulsations.

IV. CONCLUSIONS

Experimental study of the joint characteristics of the pressure fluctuations beneath a simple hydraulic jump brings out the non-Gaussian nature of the joint-probability density in the violent region of the jump. A model representation of the space-time correlation of the pressure fluctuations is presented with possible application in the computation of response of structures exposed to such a pressure field. Suitable choice of the model parameters is seen to lead to satisfactory agreement with experimental results. Existence of macropulsations is revealed by the experimental cross-spectra.

ACKNOWLEDGEMENT

The present study is under a continuing Central Board of Irrigation and Power sponsored project at the Hydraulic Engineering Laboratory, Indian Institute of Technology, Madras, India.

REFERENCES

1. Shinozuka, M., "Monte-Carlo Solution of Structural Dynamics", Computers and Structures, Vol.2, 1972.

2. Uppal, H.L., Gulati, T.D., and Sharma, B.A.D., "A Study of Causes of Damage to the Central Training Wall Bhakra Dam Spillway", Journal of Hydraulic Research, Vol.5, No.3, 1967.

3. Carl Huval and Frank Neilson, Discussions, Journal of Hydraulics Division, ASCE, Vol.96, No.HY 8, Aug.1970.

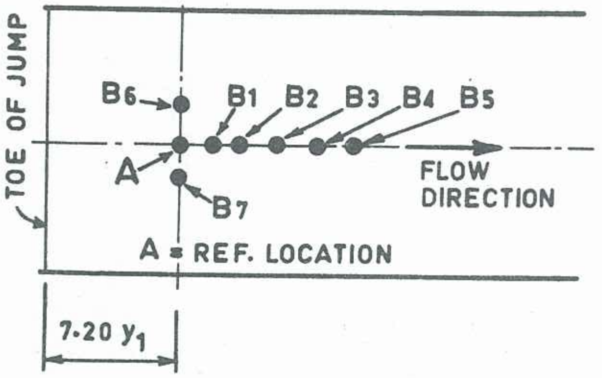
4. Lesleighter, J., Discussions, Journal of Hydraulics Division, ASCE, Vol.96, No.6, June 1970.

5. Lesleighter, J., "Pressure Fluctuations in large energy dissipation structures", Proceedings, IV Australasian Conference on Hydraulics and Fluid Mechanics, 1971.

6. Bendat S., and Piersol, G., "Measurement and Analysis of Random Data", John Wiley & Sons, Inc., New York, 1966, Ch.7, pp.295-300.

7. Abdul Khader, M.H., Elango, K., and Sadasivan, S., "Statistical characteristics of a random pressure field", Proceedings Third Conference on Fluid Mechanics and Fluid Power, Indian Institute of Technology, Kharagpur, India, 1972.

8. Maestrello, L., Gedge, M.R. and Reddaway A.R.F., "The response of a simple panel to the pseudo-sound field of a jet", Aerodynamic Noise, Proceedings, AFOSR-UTIAS Symposium, University of Toronto Press, Toronto, 1968, pp 189-208.



Separation	$\Delta x' = \Delta x / y_1$	$\Delta y' = \Delta y / y_1$
A – B1	2.93	–
A – B2	4.26	–
A – B3	6.38	–
A – B4	10.63	–
A – B5	14.89	–
A – B6	–	+ 2.93
A – B7	–	– 2.93

FIG.1 – TRANSDUCER LOCATIONS IN PLAN.

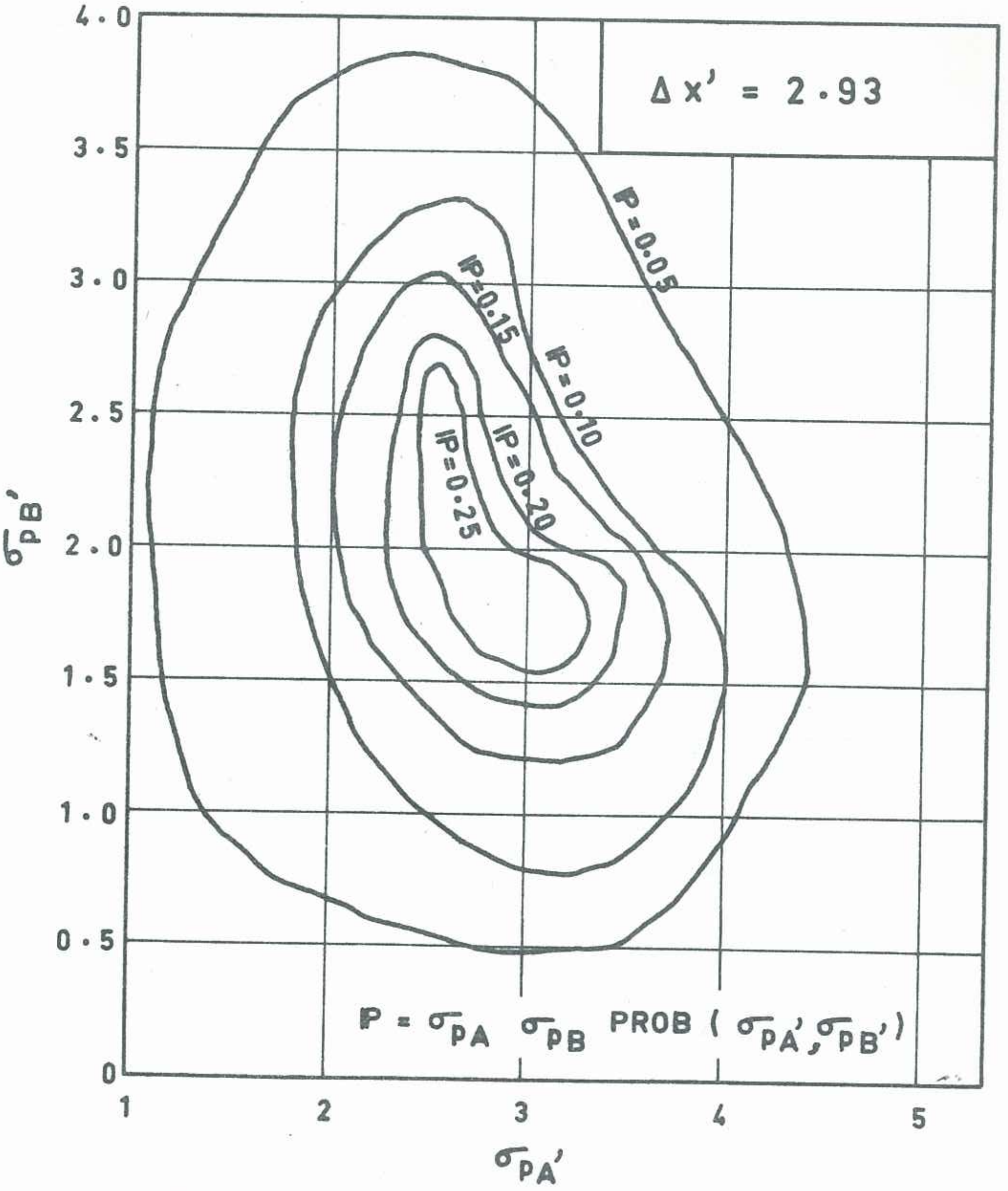


FIG. 2 – EQUI – PROBABILITY CURVES.

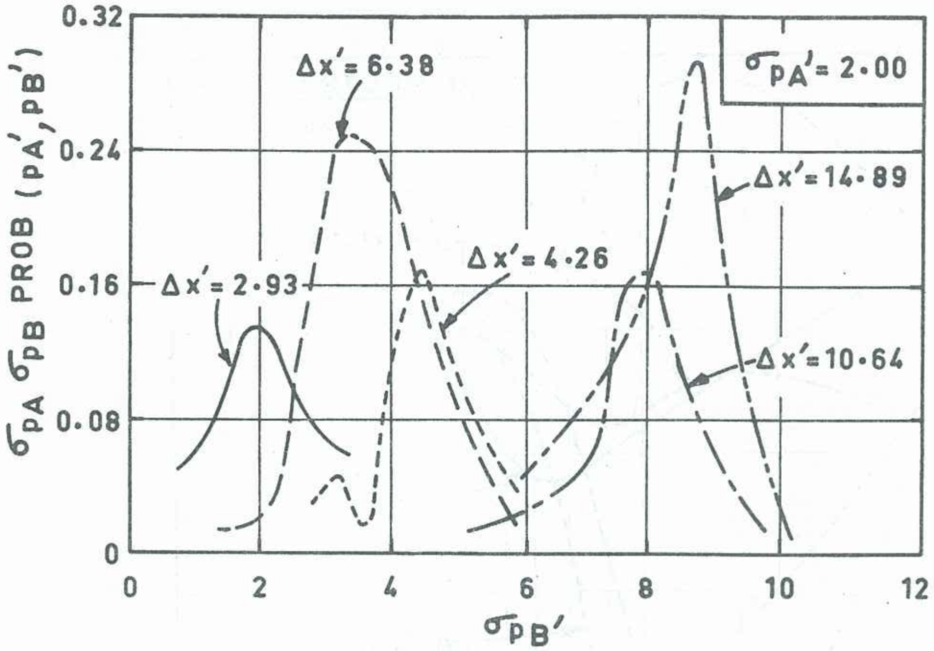


FIG.3 - CONDITIONAL PROBABILITY SECTIONS.

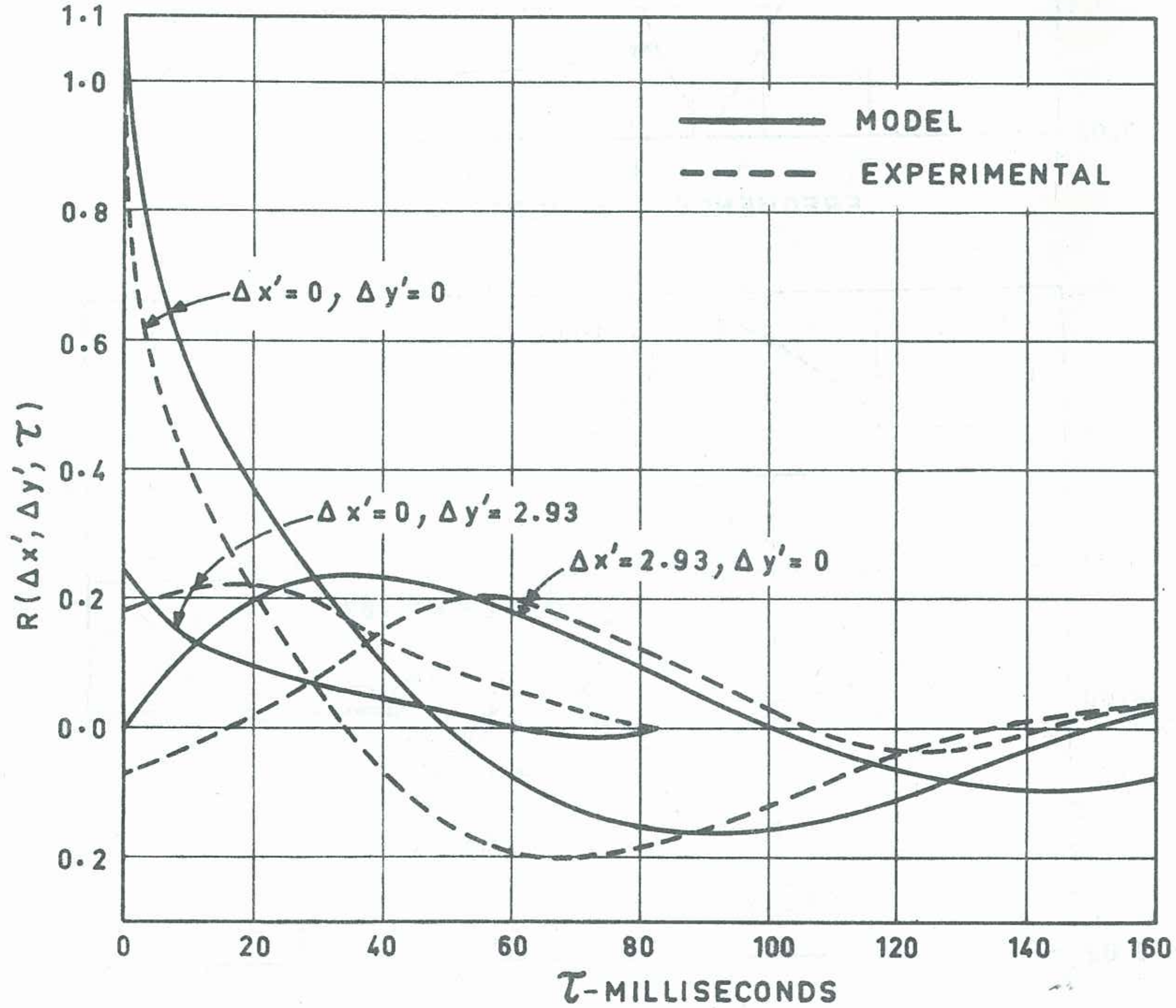


FIG. 4 - NORMALIZED CROSS CORRELATION OF THE
PRESSURE FIELD.

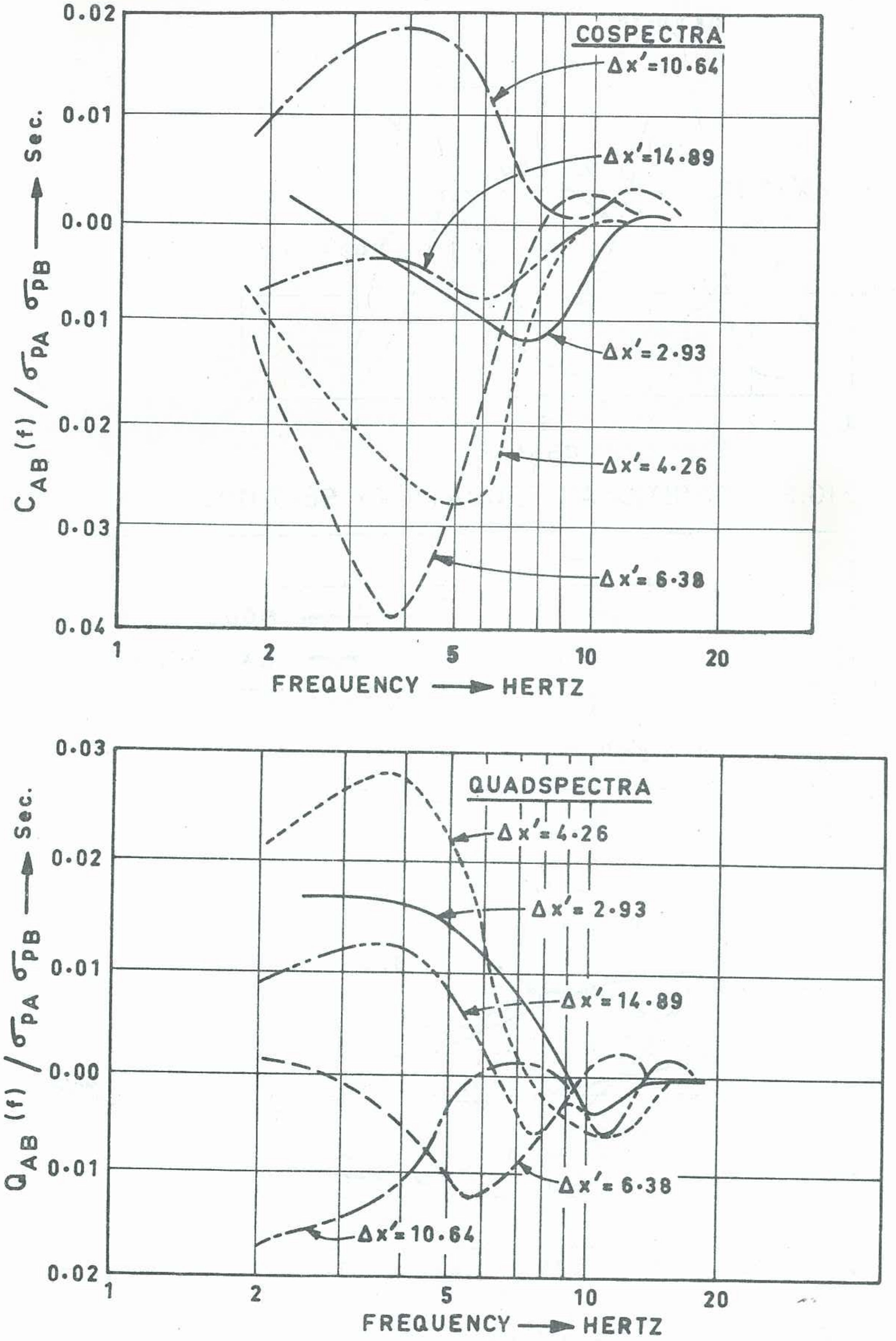


FIG.5-CROSS-SPECTRA WITH SEPARATION IN FLOW DIRECTION