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FUNCTION INVERTERS IN HOT-WIRE ANEMOMETRY

by

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S U M M A R Y

Many experimenters attempt to function invert (linearise) the hot-wire signal to improve the technique of estimating the slope of the static calibration curve, needed to determine turbulence sensitivity. This paper deals with the problems and pitfalls that accompany such processes generally and in hot-wire anemometry particularly when using an electronic lineariser.

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1. INTRODUCTION

Many practical transducers have non-linear performance characteristics. That is to say the functional relation which exists between the input and output is not linear. Specifically for hot-wire anemometers the approximate relation widely used is of the King's law form

$$E_o = A + BU^n \quad , \qquad (1)$$

where E_o is a voltage output, U is a velocity, A, B, n are supposedly constant.

Due to the highly non-linear nature of equation 1 there are strong incentives for function inverting it. Firstly there is the need to accurately estimate the local slope to determine turbulence sensitivity. Secondly at high turbulence levels errors of measurement due to instrument non-linearity become significant.

In principle function inversion is a reasonable and logical technique for overcoming both problems. With experimental scatter present, it is considerably more precise and less difficult to fit a straight line to the data than a curved one. Consequently the local slope of the calibration curve could be estimated with greater precision from such a straight line than from the curved non-linear response curve. The second problem can be eliminated by "unbending" the response curve provided the unbending process (function inversion) can be achieved with some precision.

In practice however there are some fundamental objections to function inversion when the function to be inverted is not known precisely as is the case with the hot-wire anemometer. Equation 1 is an empirical relation derived from empirical heat transfer laws and furthermore B and n are not constants but complicated functions of velocity U (Bruun (1)).

2. FUNCTION INVERSION

In order to appreciate the very real difficulties involved in the inversion of equation 1, it is appropriate to define the problem. Figure 1 shows the block diagram of the process involved. At this stage it is not important to state whether the function is inverted by a process such as plotting of the data on appropriately non-linear axes, or by an electronic analogue lineariser. Whichever the case the result will have some "wiggles" as shown on figure 2. These wiggles are a result of imprecise function fitting of the original response equation f with g . Experimental simplicity demands amplitude matching of the two functions in some range. At each matching point the two functions will cross, resulting in almost imperceptible wiggles on an amplitude plot. However these wiggles can cause significant errors in local derivative estimation, particularly when used at small turbulence levels. At high turbulence levels the global value of the local gradient will not be greatly in error.

3. ANALYSIS OF ERROR

The standard formula for numerical differentiation of a function f(x) known only by tabular values at intervals of $\Delta\chi$ is given by

$$df/dx|_{x=x_o} = 1/2\Delta x\left[f(x_o + \Delta x) - f(x_o - \Delta x) \right]$$

$$- (\Delta x)^2 /3!\left[f'''(x)\right] \qquad (2)$$

where f''' denotes the maximum value of d^3f/dx^3 in the range $x_o-\Delta x < x < x_o+\Delta x$ (2) . This equation rests on the assumption that f(x) is continuous and differential in the range to the third differential at least.

When using numerical techniques for differentiating experimental data two types of errors must be considered

(a) Truncation errors

In the above equation this is represented by the last term

$$ET(f) = -\frac{(\Delta x)^2}{3!} f'''(x)$$

It is clear that for (f) a second order function the value of ET will be zero, but not for higher order functions f.

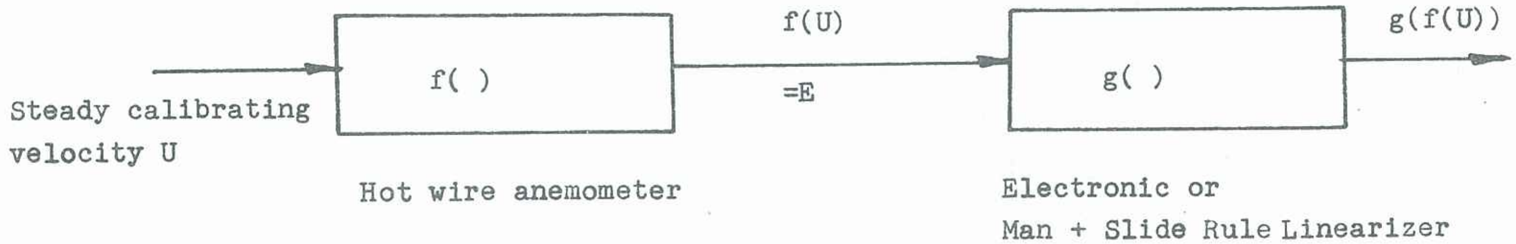

FIG. 1 Block Diagram of Linearizing Process

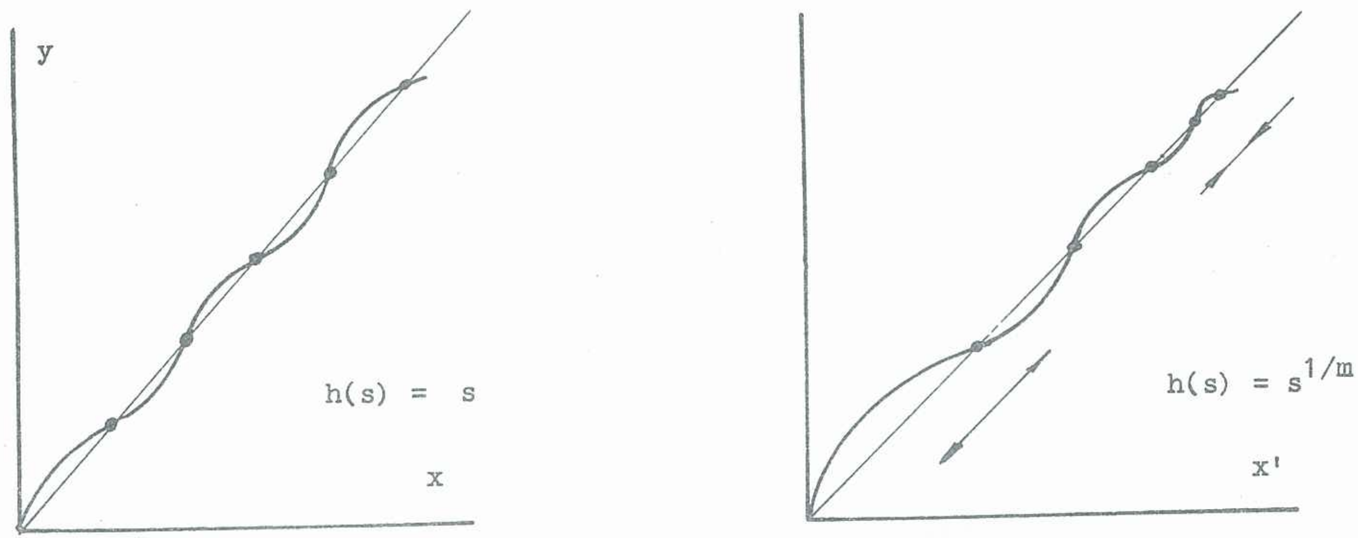

FIG. 2. h(g(f(U))) against h(U)

(b) Rounding errors.

To illustrate, consider a random error in f, denoted ξ then the error in the differential formula, equation 2, may be written as

$$df/dx = (f_1 \pm \xi_1 f_1 - f_{-1} \pm \xi_{-1} f_{-1}) / 2\Delta x$$

where $f_1 = f(x_o+\Delta x)$ and $f_{-1} = f(x_o-\Delta x)$. Consequently the worst limit error due to rounding is given by

$$ER = {}^1/2\Delta x \quad (\xi_1 f_1 + \xi_{-1} f_{-1})$$

It is evident that as the interval Δx is reduced so the rounding error ER will increase without limit.

In applying numerical or graphical differentiation methods, the user must select the differentiation interval so that both sources of error are minimised.

It now remains to evaluate such errors when numerical or graphical methods are applied to an approximately inverted function. The appropriate simple inverter for equation 1 is given by

$$g(f) = (f^2 - A)^m \tag{3}$$

and the second order gradient estimator to g is given by

$$dg/dx\Big|_{f = f_o} =$$

$$\{ [(f_1 + f_1 \xi_1)^2 - A]^m - [(f_{-1} + \xi_{-1} f_{-1})^2 - A]^m \} / 2\Delta x \tag{4}$$

In most practical situations $f^2 < A$, $\xi << 1$ and equation 4 may be reduced to an error term in dg/dx given by

$$\xi\Delta g \simeq 4mf^{2m} \xi [1 - {}^A/f^2]^{m-1} \tag{5}$$

Equation 5 rests on the assumption; that $\xi_1 = O[\xi_{-1}]$ and second order terms in ξ may be neglected, resulting in ${}^A/f_1^2 = O[{}^A/f_{-1}^2]$. Since we are only concerned with an estimate of the error all these assumptions are appropriate.

From equation 7 the amplification of rounding errors by a function inversion process is given by

$$\Gamma = \frac{\xi\Delta g/\Delta g}{\xi\Delta f/\Delta f} = \frac{m}{1 - A/f^2} \left[\frac{1 \quad f_{-1}/f_1}{1 - f_{-1}/f_1} \right] \tag{5a}$$

where the term in square brackets is of order 1 and using some practical values for the various terms results in $m = 2$, $A/f^2 \simeq 0.5$ and $\Gamma = 4$. The implication is, that <u>the function inversion process is an error amplifier</u>.

When an electronic inverter is used the output of the anemometer is direct coupled to the inverter. Under these conditions the only source of random error input to the inverter is that due to minor changes in the performance of components in the hot-wire system. These changes can occur due to changes in the dynamic operating conditions of the system. Although such changes may be quite small in the dynamic response of the anemometer, when amplified by the inverter they may be quite significant. In this context it is worth noting that equation 5a was derived using a binomial expansion on $(1-A/f^2)^{k-1}$. This expansion will only hold as long as A/f^2 can approach unity to within any desired degree of accuracy without contravening the requirements of the binomial expansion. Under these conditions Γ can increase without limit.

So far the analysis has been restricted to rounding errors. In order to investigate the effect of truncation errors the degree of approximation involved in the inversion process must be known. These errors may be predicted for a practical inverter (DISA. 55-D-10 lineariser) and an ideal hot-wire as proposed by Davies and Bruun (3). Davies and Bruun have calibrated an extensive set of similar tungsten wires with a constant temperature hot-wire anemometer built at Southampton. The results of their calibration provide a "universal" calibration for wires of similar geometry with an adjustable constant requiring only a one point calibration in practice. Once the constant has been determined the calibration may be scaled from a tabulated universal calibration.

The Davies and Bruun tabulation was curve-fitted and the following static response equation was found.

$$E_o^2 = A + Y(U)U^{R(U)} \quad ,$$

where

$$Y = 0.775 + 0.025\ (\log U)^4 \qquad (6)$$

and

$$R = 0.5 - 0.09\ (\log (U/10))^3$$

Equation 6 was function inverted according to equation 3 which is the mathematical model of the DISA. 55-D-10 lineariser. The inverted equation was differentiated analytically and the resulting equations for g and dg/dU are as follows

$$g = (A+YU^R-X)^m \quad , \qquad (7)$$

$$\frac{dg}{dU} = m(YU^R+A-X)^{m-1}\left[DY+Y.DR.\ln U + YR/U \right] U^R \quad ,$$

where

$$DR = -0.27 \left[\log (U/10) \right]^2 /U \ln(10) \qquad (8)$$

and

$$DY = 0.1 \left[\log U \right]^3 /U \ln(10) \quad .$$

Both equation 7 and 8 were evaluated for a large set of values of U in the range 3 to 30 m/sec. The values of m and X were adjusted to give less than 2% error in amplitude linearity. This decision rule was felt to be the most plausible one representing a "judgment" which would be used by the operator of the lineariser. A typical amplitude plot is shown on figure 3. The true differential evaluated in the same velocity range from equation 8 is shown on figure 4. As can be seen the errors are quite significant.

As a check of the overall performance of the simple inverter, the derivatives dg/dU were evaluated for a whole range of values of m and X. A typical set of curves are shown on figure 5, where the results have all been normalised at $U = 30$ m/sec.

4. EXPERIMENTAL LINEARISER PERFORMANCE

The DISA. 55-D-10 lineariser was tested experimentally using the constant temperature hot-wire anemometer of Perry and Morrison (4) with a normal DISA miniature probe type 55F15. The tungsten wire was replaced with a silver coated platinum wire (Wollaston wire) with a sensing length of 1 mm and a diameter of 4 µm. The probe was mounted in a shaker capable of inducing in the wire a precisely known velocity perturbation (5), and the output was processed on-line through an EAI type TR 48 analogue computer. In all tests a fixed resistance ratio of 2.0 was used.

Two kinds of output were observed.

(a) Static Calibration.

The lineariser was adjusted according to the Maker's specifications and the wire was calibrated statically. In performing this test it was observed that the linearised results drifted significantly more than the unlinearised data due to changes in tunnel temperature. This behaviour agrees with the prediction of Morrison (6), regarding the effects of temperature change on static calibrations. In order to overcome these effects the calibrating tunnel used in these tests was fitted with a temperature controller capable of maintaining tunnell temperature at some preset value to within 0.1°C. When using this controller there was negligible drift due to temperature changes.

(b) Dynamic Calibration.

Once reasonable static "linearity" was achieved the shaker was started and a dynamic

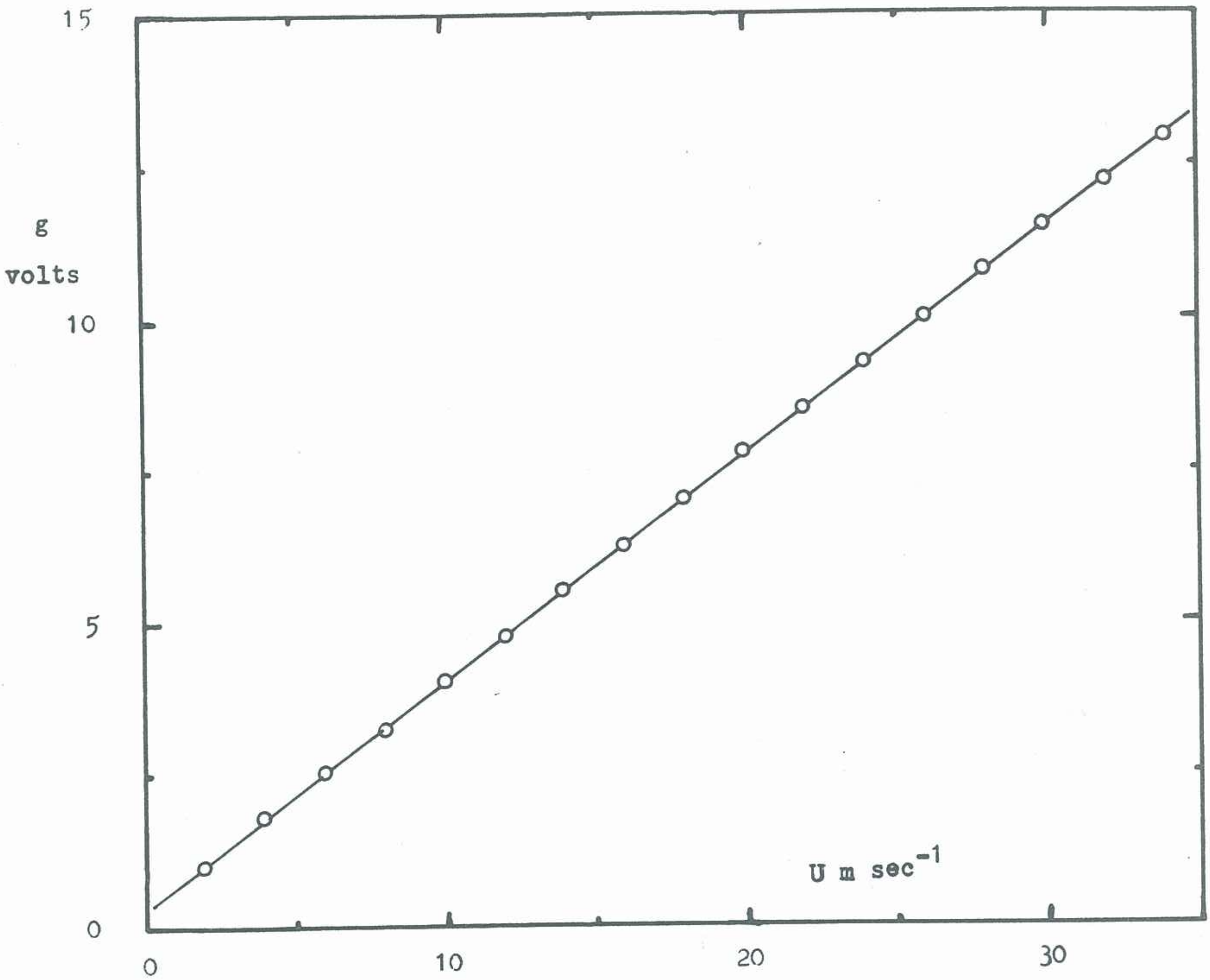

Fig.3 Amplitude Plot

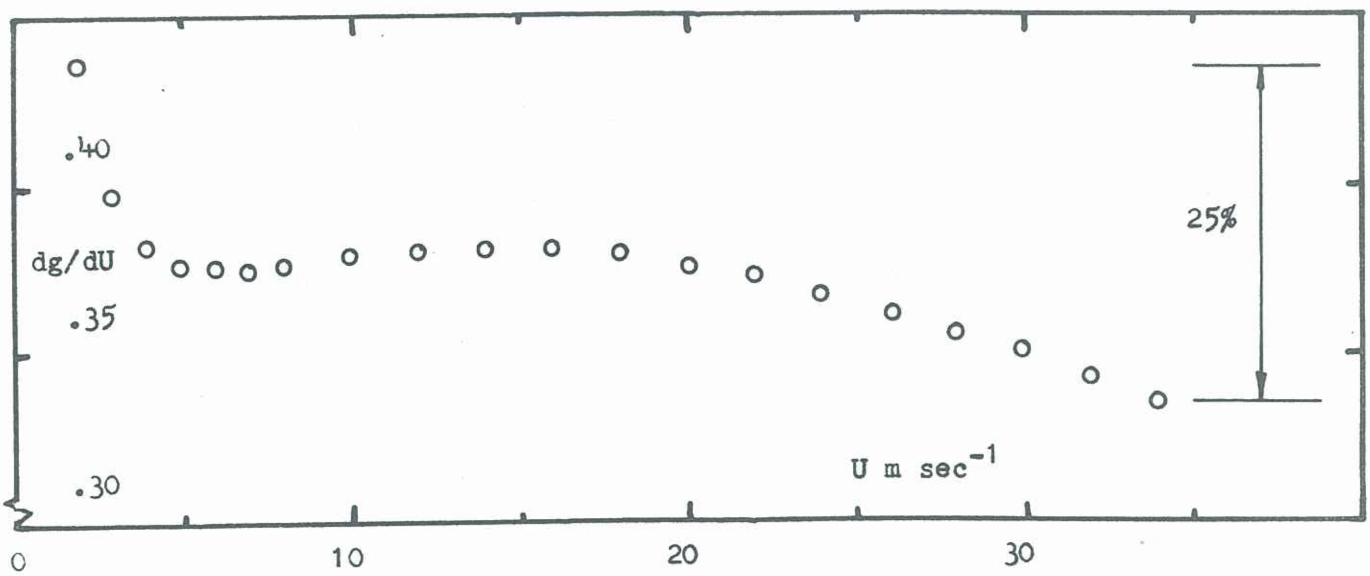

FIG. 4 Derivative Plot

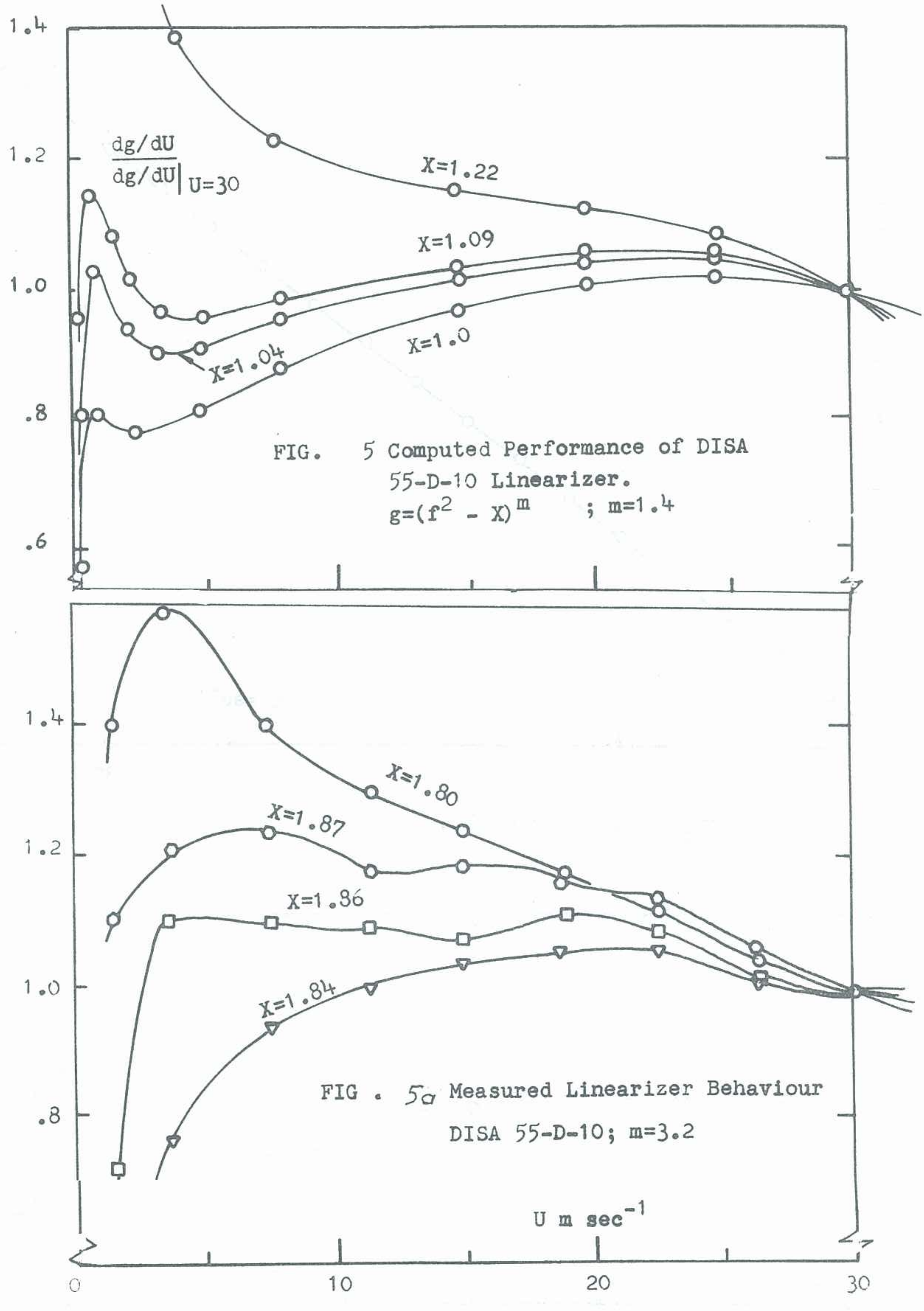

FIG. 5 Computed Performance of DISA 55-D-10 Linearizer.
$g=(f^2 - X)^m$; m=1.4

FIG. 5a Measured Linearizer Behaviour DISA 55-D-10; m=3.2

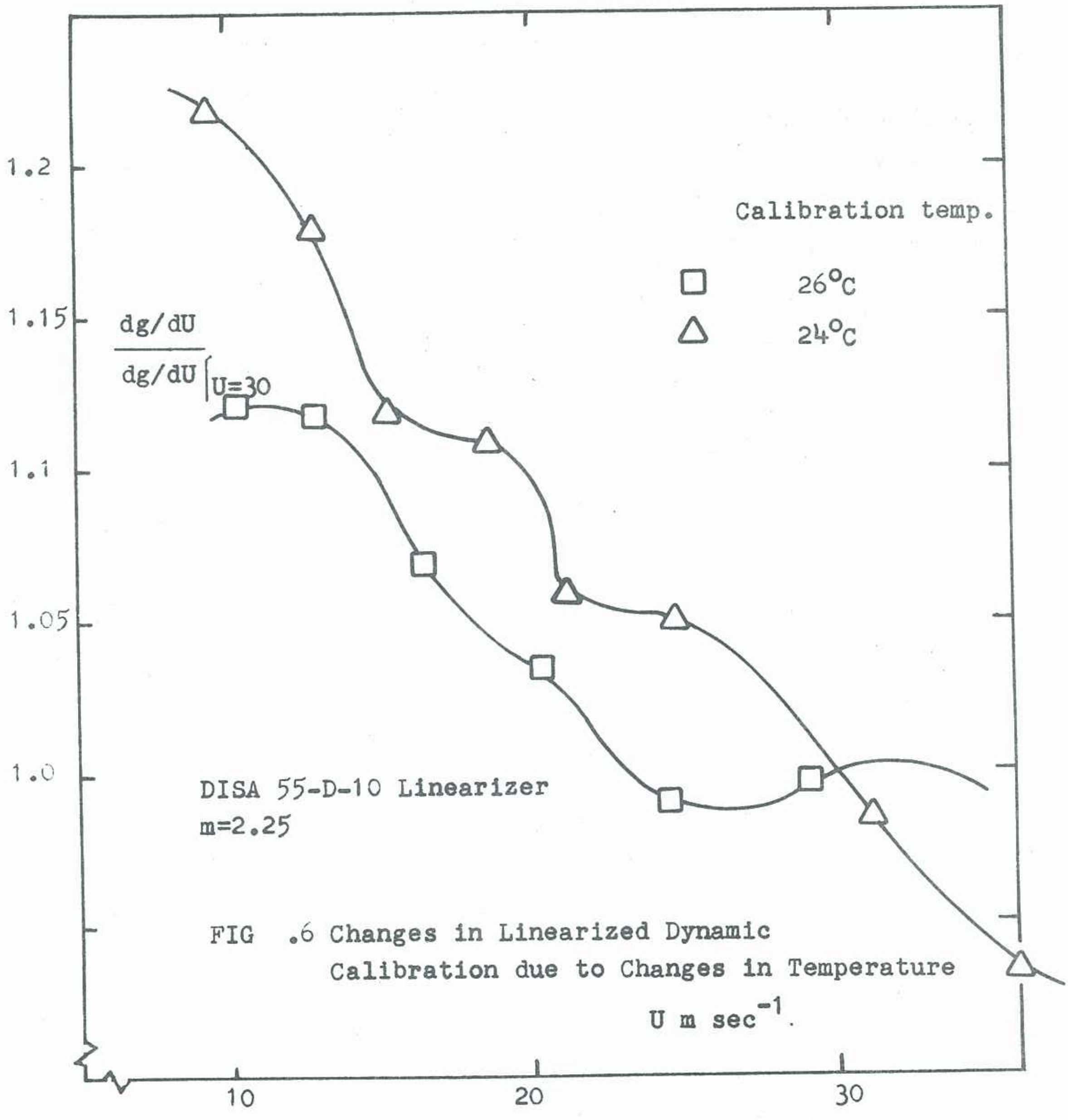

FIG .6 Changes in Linearized Dynamic Calibration due to Changes in Temperature calibration was carried out in accordance with the steps outlined in (5). For both types of calibration the output signal was integrated for 60 sec. to provide good repeatability. The consistency of the results was better than ½% . The linearised dynamic calibration was carried out for a series of values of lineariser adjustment and the results are shown on figure 5*a*.

As can be seen the shapes of the curves are similar to those predicted from the analytical study. The values of m and X are different. This difference may be due to the non-universality of the hot-wire performance equation proposed by (3) (equation 6) .

During these tests two types of uncertainties were observed in connection with the measured differential of the inverted hot-wire output. Firstly the "wiggles" due to the approximations in the inversion process. Secondly a change in position of these "wiggles" due to changes in testing temperature from one day to the next. This second kind of uncertainty is illustrated on figure 6.

5. COMMENTS AND CONCLUSIONS

There are significant errors involved when using function inversion to linearise experimental data which is later differentiated. These errors cast serious doubts on the use of such inversion processes. The existence of these errors has been demonstrated by the analytically predicted and real performance of the DISA 55-D-10 lineariser. In view of these errors the use of function inversion should be discouraged.

As the function inversion process investigated here is an error amplifier two interesting features of function inversion may bear investigation.

(a) Are function inverters generally error amplifiers?

(b) Can function inverters provide error attenuation?

Finally, the electronic lineariser should be used only in extreme cases, where high turbulence level prohibits the use of any other measurement technique.

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