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THE CORRECTION OF HOT WIRE MEASUREMENTS FOR FLUID TEMPERATURE VARIATION

by

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S U M M A R Y

The effects of fluid temperature variations on constant temperature hot-wire measurements are evaluated. The correction functions for temperature variations are derived both for turbulence and mean flow measurement. It is shown that the temperature variation corrections to turbulence measurements are more than four times as high as the corrections to mean flow measurements. The effect of temperature drift on hot-wire sensitivity is demonstrated by direct dynamic calibration of a hot-wire at two different temperatures.

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1. INTRODUCTION

The most commonly used technique for the measurement of turbulence intensity involves the calibration of the hot-wire for static behaviour. From this static calibration the dynamic behaviour of the hot-wire is inferred by graphical or numerical differentiation. Recently Perry and Morrison (1), and Perry, Morrison and Samuel (2) have proposed a method for determining the dynamic behaviour of the hot-wire directly, by inducing in the wire a known perturbation signal. Whatever the technique of calibration however the measurements are taken subsequent to the calibration procedure.

Changes in temperature may occur during calibration, between the times of calibration and measurement, as well as during measurement. In many flow situations, particularly where closed circuit tunnels are used, these temperature variations can be quite significant. Bearman (3) derived temperature correction equations for both linearized and non-linearized constant temperature anemometers. However the correction procedure proposed here is somewhat simpler to use as well as being independent of E_o the anemometer output voltage at zero velocity. The effect of using different wire materials is examined, and the derived correction procedure is verified by direct dynamic calibration of a hot-wire sensing element.

2. ANALYSIS

A schematic form of a constant temperature hot-wire circuit is shown in figure 1.

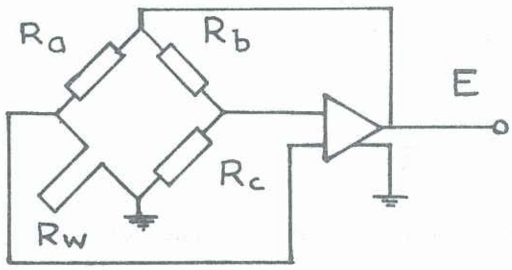

FIG.1 Schematic Constant Temperature Hot-Wire Circuit.

The most common form of heat transfer law assumed for such a system in a static calibration is given by

$$E^2 = (Ra + Rw)^2 \, (1 - Rg/Rw) \, (A + B\,U^n) \quad , \qquad (1)$$

where E is the d.c. output voltage,

U is the gas stream mean velocity,

Rw is the hot-wire resistance,

Rg is the wire resistance at gas temperature,

A, B and n are constants to be determined experimentally.

Bruun (4) has shown evidence that B and n are functions of U, the mean gas velocity. However, the results of the present analysis show that fluid temperature correction is only a weak function of A, B and n. Consequently in the range of interest of many experimental situations the correction function will be independent of A, B and n. The expressions for A and B used in the analysis are those given by Hinze (5) and n is taken as 0.5.

$$A = 0.42 \, (\gamma\pi\kappa_f \ell \,/\, \zeta R_o) \, (Pr)_f^{.20}$$

$$B = 0.57 \, (\gamma\pi\kappa_f \ell \,/\, \zeta R_o) \, (Pr)_f^{.33} (d/\nu_f)^{.5}$$

where γ is a conversion constant for electrical to thermal energy, κ_f is the gas thermal conductivity, $(Pr)_f$ = Prandtl number, ν_f = kinematic viscosity, ℓ = wire sensing length, d = wire diameter, ζ = thermal co-efficient of resistance, and R_o = the wire resistance at absolute zero of temperature.

All properties are taken at the film temperature $\theta_f (= \tfrac{1}{2}$ (Twire - Tgas)). To find the effect of temperature on the static calibration, equation 1 is differentiated with θ, the temperature, to give

$$\frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U = \frac{1}{2}\left\{\frac{1}{\kappa_f}\frac{d\kappa}{d\theta}\right.$$

$$+\frac{1}{1+G}\left[(0.2+0.33G)\frac{1}{(Pr)_f}\left(\frac{dPr}{d\theta}\right)_f\right. ,$$

$$\left.\left. -\frac{0.5G}{\nu_f}\left(\frac{d\nu}{d\theta}\right)_f\right] + \frac{1}{Rw/Rg-1}\;\frac{Rg}{Rw}\;\frac{d(Rw/Rg)}{d\theta}\right\} \qquad (2)$$

where $G = 1.36\ (Pr)_f^{.13}\ (Ud/\nu_f)^{.5}$

Using International Critical Tables (6), the fluid property terms have been evaluated for air over the range of θ from 100°C to 300°C and U in the range 3 to 30 m/sec.

Noting that $\dfrac{Rg}{Rw}\dfrac{d(Rw/Rg)}{d\theta} = -\dfrac{\zeta}{1+\zeta\theta}$

and $\zeta = O\ (10^{-3})$ reduces equation 2 in the above temperature and velocity range by order of magnitude argument to;

$$\frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U = \frac{1}{2}\,\frac{\zeta}{r-1} \qquad (3)$$

where $r = Rw/Rg$, the resistance ratio of the hot-wire sensing element.

Now define s , the dynamic sensitivity of the hot-wire system as

$$S = \left(\frac{\partial E}{\partial u}\right)_\theta \qquad (4)$$

and carrying out the appropriate differentiations and order of magnitude arguments results in

$$\frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_U = -\frac{1}{2}\,\frac{\zeta}{r-1} \qquad (5)$$

Both equations 3 and 5 have been previously derived by Bearman (3). Experimental verification of equation 5 is difficult, due to the lack of accuracy accompanying the differentiation of static calibration data. Although Bearman in his paper presents plots of velocity fluctuations obtained using both corrected and uncorrected values of $(\partial E/\partial u)$, it is impossible to establish without dynamic calibration of the hot-wire, which turbulence is the "real" one.

3. EXPERIMENTAL PROCEDURE

In general when the wire is calibrated, the three variables of interest are E, S, and U. Figure 2 shows the type of plot often used in static calibrations, while figure 3 shows a more general plot including the dynamic sensitivity S.

The usual type of static calibration curve is plotted on non-linear axis, as shown in figure 2. Under these circumstances the change in slope of the calibration curve, due to different temperatures, appears to be independent of whether the correction process is carried out at constant E , or constant U . However the slope of the calibration curve in figure 2 is given by

$$\left(\frac{\partial E^2}{\partial U^n}\right)_\theta = 2\,E\left(\frac{\partial E}{\partial \theta}\right)_U nU^{n-1} .$$

Thus the change in $(\partial E/\partial u)$ will depend on the process used for correction. Figure 2 illustrates the ambiguity of using non-linear scales for plotting calibration data.

The two approaches available for correcting experimental data are shown on figure 3. These

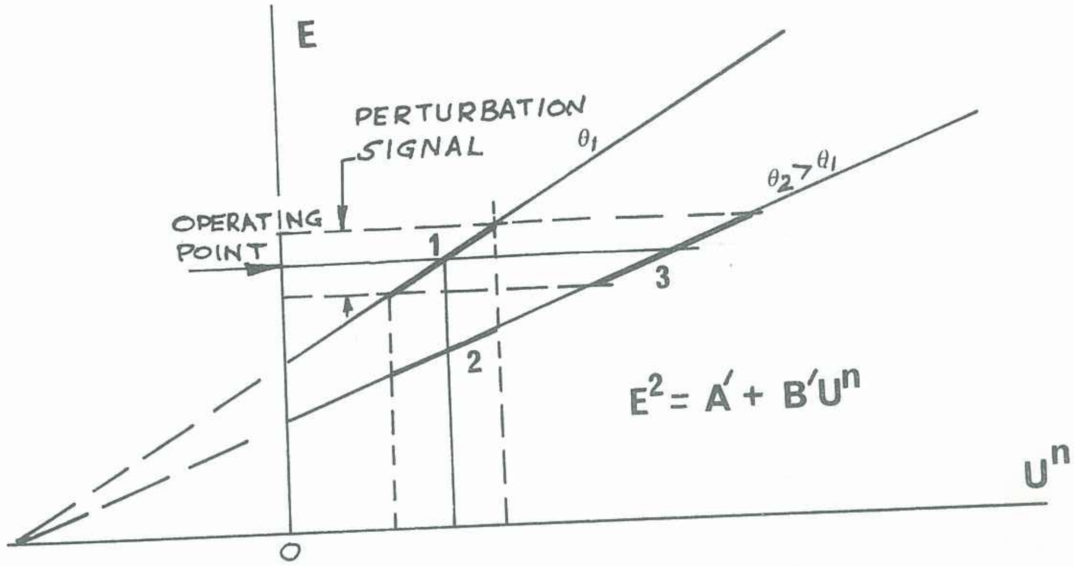

FIG. 2 Nonlinear Static Calibration Plot
(Note that although Slope 2 = Slope 3
$(\partial E/\partial U)_{\theta_2}|_2 \neq (\partial E/\partial U)_{\theta_2}|_3$)

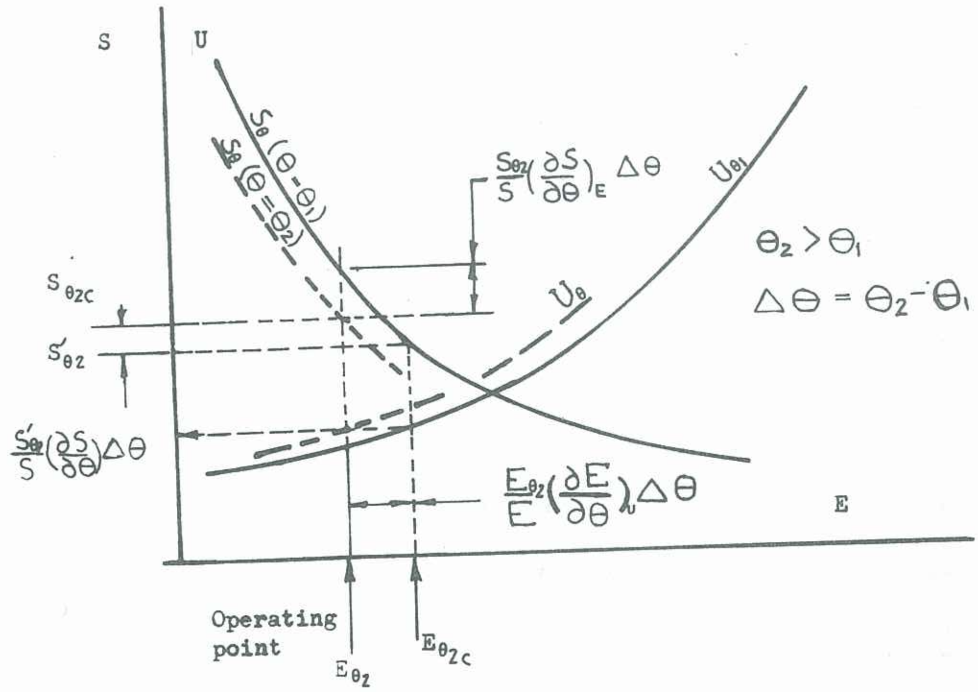

FIG. 3 Linear Calibration Plot Showing Dynamic Sensitivity S, and Velocity U Plotted Against E the Output Voltage. The two alternative procedures for correcting S are indicated.

approaches are equivalent and will be called "process 1" and "process 2" respectively. Each calibration point is corrected to some base calibration temperature θ_1 say. Then assume that during the actual test the temperature has changed to θ_2 , where $\theta_2 > \theta_1$. The following corrections are made at each operating point E_{θ_2} (refer fig. 3) .

(a) Correct E_θ to $E_{\theta_{2c}}$ by the relation ;

$$E_{\theta_{2c}} = E_{\theta_2}\left[1 + \frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U (\theta_1 - \theta_2)\right] ,$$

and either

(b) obtain $S_{\theta_{2c}}$ by ;

$$S_{\theta_{2c}} = S_{\theta_2}\left[1 + \frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_E (\theta_1 - \theta_2)\right] ,$$

or

(c) obtain $S_{\theta_{2c}}$ by ;

$$S_{\theta_{2c}} = S'_{\theta_2}\left[1 + \frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_U (\theta_1 - \theta_2)\right] ,$$

where S_{θ_2} is the value of $S_{\theta(\theta=\theta_1)}$ corresponding to E_{θ_2} , and S'_{θ_2} is the value of $S_{\theta(\theta=\theta_1)}$ corresponding to $E_{\theta_{2c}}$. Note that both values S_{θ_2} may be found from the original calibration curve $S_{\theta(\theta=\theta_1)}$.

Process 1 involves steps (a) and (b), while Process 2 involves steps (a) and (c) . Process 2 is simpler to apply as the correction functions, equations 3 and 5, are independent of the calibration constants, and depend only on the wire material properties and the resistance ratio used. It is relatively simple to show that the two processes are indeed equivalent.

Equation 1 may be written generally as:

$$f\left\{E,\ \theta,\ U\right\} = 0 \qquad , \qquad (6)$$

which is simply a state equation for the hot-wire system. This equation yields ;

$$\left(\frac{\partial E}{\partial \theta}\right)_U \left(\frac{\partial U}{\partial E}\right)_\theta \left(\frac{\partial \theta}{\partial U}\right)_E = -1 \qquad (7)$$

Furthermore

$$S = \left(\frac{\partial E}{\partial U}\right)_\theta = S\,(E,\theta) = S_1\,(U,\theta) \qquad . \qquad (8)$$

Some use of the above equations 7 and 8 results in ;

$$\frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_E = \frac{1}{S}\left(\frac{\partial S_1}{\partial \theta}\right)_U \left[1 - \frac{E}{S}\left(\frac{\partial S}{\partial E}\right)_\theta\right] \qquad . \qquad (9)$$

The R.H.S. of equation 9 may be transposed by the use of

$$\frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_U = \frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U$$

, to give

$$\frac{1}{S}\left(\frac{\partial S_1}{\partial \theta}\right)_U \left[1 - \frac{E}{S}\left(\frac{\partial S}{\partial E}\right)_\theta\right] = \frac{1}{S}\left(\frac{\partial S_1}{\partial \theta}\right)_U - \frac{1}{S}\left(\frac{\partial S}{\partial E}\right)_\theta \left(\frac{\partial E}{\partial \theta}\right)_U \qquad (10)$$

The second term of the R.H.S. of equation 10 is the correction due to the change in S corresponding to a change in E . This term represents the major part of the correction. Equation 10 effectively involves a two step procedure ("process 2"), firstly a correction in E and secondly a correction in S. Consequently the two processes are indeed equivalent.

Substituting equation 1 into 9 gives ;

$$\frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_E = \frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U \left[4 + \frac{2A}{B\sqrt{U}}\right] \qquad (11)$$

From equation 1 the second term in the bracket is given by ;

$$\frac{2A}{B\sqrt{U}} = \frac{E_o^2}{E^2 - E_o^2} \quad , \qquad (12)$$

where $E^2 = E_o^2$ as U tends to approach 0 , showing that the correction tends to very large values for low velocities.

Thus, as given by "process 2" ,

$$\frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_E = \frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U \left[4 + \frac{2E_o^2}{E^2 - E_o^2}\right] \quad , \qquad (13)$$

which shows that the correction to dynamic sensitivity, and hence to turbulence measurements, is more than four times as high as the correction to mean flow measurements.

To illustrate these temperature effects, a hot-wire probe (DISA type 55F 14) was calibrated statically and dynamically in air.

The procedure used the direct dynamic calibration of Perry and Morrison (1). Both d.c. and r.m.s. voltage measurements were integrated over a period of 60 secs. to provide good repeatability. The closed circuit calibration tunnel had a temperature controller installed for the test and the temperature level could be varied over a small range and kept constant at a set value to within ± 0.1°C. The wire used was a 25 μm diameter silver with a 5 μm diameter platinum core (Wollaston wire). Over the 1 mm working length the silver was etched away to give a length to diameter ratio of approximately 200. The wire resistance ratio was kept constant at 1.4 during the test, the low value was used in order to accentuate the temperature correction. The results and correction procedure are shown on figure 4.

$$\zeta \text{ for Platinum} = 3.5 \times 10^{-3} \text{ per } ^{o}C \quad .$$

From equation 3 over the 4°C range (18°C - 22°C) ,

$$\frac{1}{E}\left(\frac{\partial E}{\partial \theta}\right)_U = -0.0175 = \frac{1}{S}\left(\frac{\partial S}{\partial \theta}\right)_U \quad .$$

4. COMMENTS AND CONCLUSIONS

As can be seen from figure 4 the correction to S over the calibration range corresponds to approximately 10%, or 2½% per °C, considerably in excess of ½% per °C indicated by the correction to S at constant velocity $1/S\ (\partial S/\partial \theta)_U$ which would be the appropriate correction used during calibration.

With tungsten wires the temperature corrections are even more significant due to two effects. Firstly the temperature co-efficient of resistance of tungsten is higher than that of platinium $\zeta_{tungsten} = 5.2 \times 10^{-3}/^{o}C$.

Secondly the tungsten wires must be operated at lower resistance ratios than platinium, to avoid oxidation problems.

It has been demonstrated that in the velocity range 3 m/sec. to 30 m/sec. a simple temperature correction may be used to correct the sensitivity of the hot-wire to velocity fluctuations. The correction factor only depends on the temperature co-efficient of resistance of the wire material and the operating resistance ratio. Both of these terms are readily available to a high degree of accuracy. Attention has been drawn to the fact that the temperature correction to $\partial E/\partial U$ during calibration is significantly different from the correction necessary during measurement.

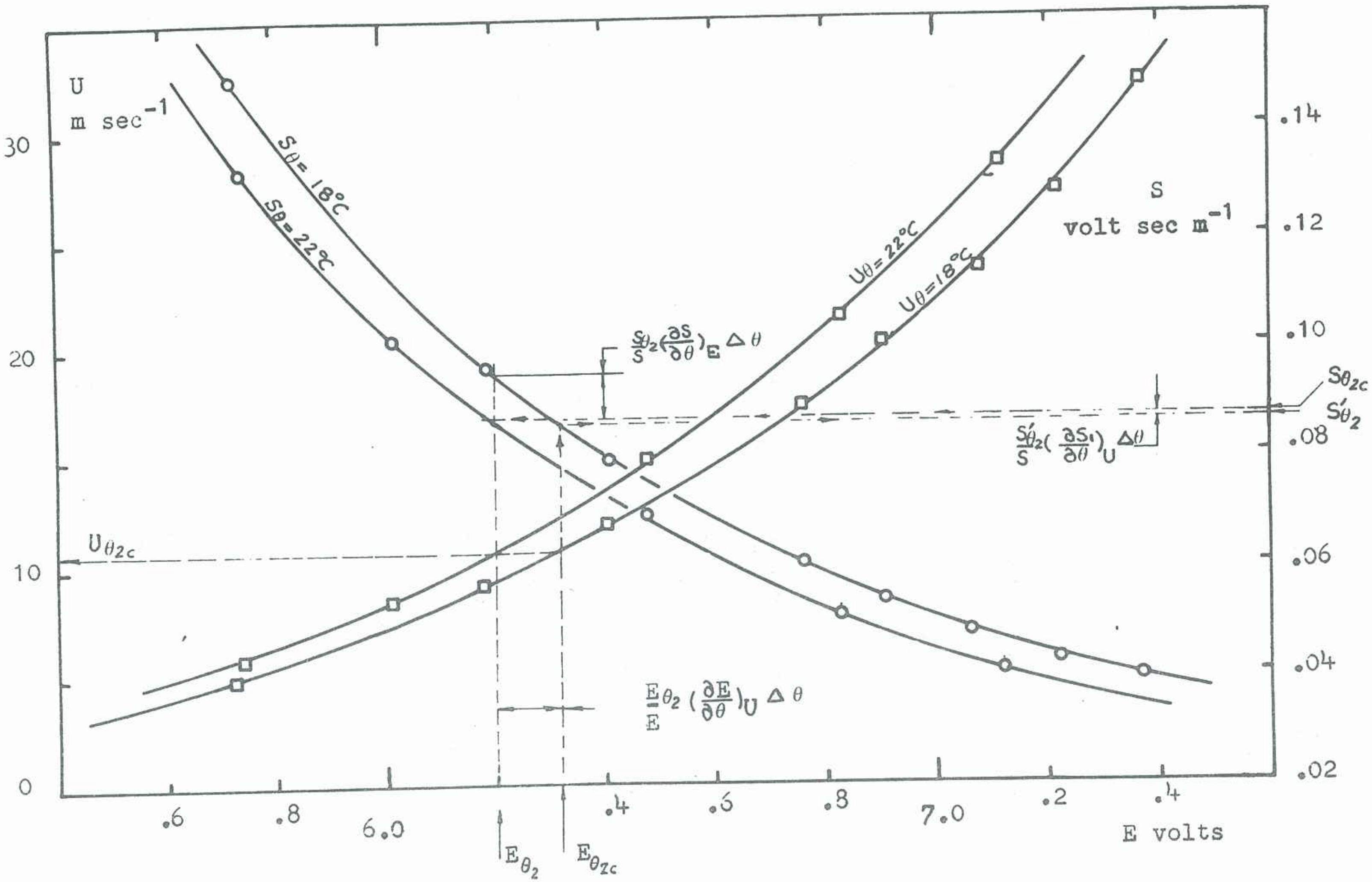

FIG.4 DYNAMIC CALIBRATION ;The chain dotted lines indicate the procedure for constructing points on the 22°C calibration from the 18°C calibration.

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