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THE SIMULATION OF WATER QUALITY IN THE WAIKATO RIVER

by

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SUMMARY

The prediction of water quality in the Waikato River requires the solution of the one-dimensional transport equation suitably modified by dispersion and decay terms. The flow rate and composition of the major effluents entering the river vary diurnally and as a result water quality in the river varies diurnally. It is necessary therefore to obtain transient solutions to the governing equations, rather than just steady state solutions.

Several explicit finite difference methods are described and a two-step scheme developed which appears suitable for solving the transient convection equation in waterways with unidirectional flow. The results of predictions of dissolved oxygen in the river are described and discussed.

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THE GOVERNING EQUATIONS

The equation governing solute concentration in a river may be written

$$\frac{\partial}{\partial t}(AC) + \frac{\partial}{\partial x}(AUC) = \frac{\partial}{\partial x}\left(AD\frac{\partial c}{\partial x}\right) + q \qquad (1)$$

where C is the average concentration, A the area, U the average velocity, D the coefficient of dispersion and q the rate of addition and/or removal.

In rivers the transport term dominates equation (1) and most of the errors encountered in its numerical solution are associated with the discretisation of this term. Errors of two kinds are encountered: amplitude errors (or numerical dispersion) which cause the amplitude of the solution to be inaccurately represented, and phase errors which cause the phase of the solution to be inaccurately represented. The errors associated with the schemes described below may be illustrated by looking at the solution of the equation

$$\frac{\partial c}{\partial t} = -U\frac{\partial c}{\partial x} \,. \qquad (2)$$

ONE STEP SCHEMES

SCHEME 1

A two-level explicit finite difference expansion of equation (2) using upstream differencing is:

$$\frac{c_j^{n+1} - c_j^n}{k} = -U\left(\frac{c_j^n - c_{j-1}^n}{h}\right) \qquad (3)$$

where k is the time step and h the mesh size.

Numerical stability requires that $|\alpha| = \frac{|U|\,k}{h} \le 1.$

The amplitude errors of Scheme 1 are zero when $\alpha = 1$ but increase as α decreases. Numerical dispersion is equivalent to a dispersion coefficient of magnitude [1,2]

$$D_p = \frac{U}{2}(h - Uk). \qquad (4)$$

When solving equation (1) it may be possible to reduce D by D_p provided that $D > D_p$ and thus improve the accuracy of the solution. Phase errors are not usually troublesome when using Scheme 1.

SCHEME 2

Using central space and time differences equation (2) may be written

$$\frac{c_j^{n+1} - c_j^{n-1}}{2k} = -\frac{U}{2h}\left(c_{j+1}^n - c_{j-1}^n\right) \qquad (5)$$

The amplitude errors associated with Scheme 2 are smaller than with Scheme 1 but lagging phase errors are noticeable in the upstream behaviour of the solutions.

Numerical dispersion [1,2] is of magnitude

$$D_p = \frac{U^2 k}{2} \qquad (6)$$

SCHEME 3

Fox[2] describes a scheme he calls the Fromm averaged phase error scheme which can be written

$$c_j^{n+1} = c_j^n + \frac{\alpha}{4} (c_{j-1}^n - c_{j+1}^n + c_{j-2}^n - c_j^n) + \frac{\alpha^2}{4} (c_{j-1} - 2c_j + c_{j+1})$$

$$+ \frac{\alpha^2 - 2\alpha}{4} (c_{j-2}^n - 2c_{j-1}^n + c_j^n) \qquad (7)$$

Amplitude errors are considerably smaller than with Scheme 1 and phase errors are not troublesome.

Stability requires that $|\alpha| \leq 1$.

Figure 1 compares Schemes 1, 2 and 3.

TWO STEP SCHEMES

Using Taylor series one can write

$$c_j^{n+1} = c_j^n + k \frac{\partial c}{\partial t}\Big]_j^n + \frac{k^2}{2} \frac{\partial^2 c}{\partial t^2}\Big]_j^n + 0\ (k^3) \qquad (8)$$

which is second order accurate. This may be written

$$c_j^{n+1} = c_j^n + k \frac{\partial}{\partial t} \left(c + \frac{k}{2} \frac{\partial c}{\partial t}\right)\Big]_j^n + 0\ (k^3) = c_j^n + k \frac{\partial}{\partial t} (c_j^{n+\frac{1}{2}}) + 0(k^3) \qquad (9)$$

SCHEME 4

The Lax-Wendroff[3] method involves writing

$$c_j^{n+\frac{1}{2}} = \tfrac{1}{2} (c_{j+1}^n + c_{j-1}^n) - \frac{\alpha}{4} (c_{j+1}^n - c_{j-1}^n) \qquad (10)$$

and $$c_j^{n+1} = c_j^n - \frac{\alpha}{2} (c_{j+1}^{n+\frac{1}{2}} - c_{j-1}^{n+\frac{1}{2}}) \ . \qquad (11)$$

Stability requires that $|\alpha| \leq 2$. Amplitude damping goes as α^2 and is only troublesome with solutions of large wave number. Phase errors however make the scheme unattractive for use in rivers in which flow is unidirectional.

If upstream differences are used in equations (10) and (11) the scheme becomes unstable.

SCHEME 5

Crowley[4] expanded equation (9) using

$$c_j^{n+\frac{1}{2}} = c_j^n - \frac{\alpha}{4} (c_{j+1}^n - c_{j-1}^n) + \frac{\alpha^2}{8} (c_{j+1}^n - 2c_j^n + c_{j-1}^n) \qquad (12)$$

and used equation (11) for the second step.

Stability requires that $|\alpha| \leq 2$.

Amplitude damping goes as α^4 and is smaller than with Scheme 4. Phase erorrs are again troublesome.

If upstream differences are used in equations (11) and (12) worse phase errors are introduced.

SCHEME 6

If equation (9) is expanded

$$c_j^{n+\frac{1}{2}} = c_j^n - \frac{\alpha}{2} (c_j^n - c_{j-1}^n) + \frac{\alpha^2}{8} (c_j^n - 2c_{j-1}^n + c_{j-2}^n) \qquad (13)$$

and the equation

$$c_j^{n+1} = c_j^n - \alpha \, (c_j^{n+\frac{1}{2}} - c_{j-1}^{n+\frac{1}{2}})$$

is used for the second step, the phase errors are reduced considerably. This happens at the expense of amplitude damping which is comparable with that encountered using Scheme 4. Stability requires $|\alpha| \leq 2$.

Figure 2 shows a comparison of Schemes 4, 5 and 6.

The two step schemes have the advantages over one step schemes that they are inherently second order accurate and that they can be used with larger time steps. Of the two step schemes available Scheme 6 appears to be most suitable for use in rivers.

The convection of a slug load is a severe test for numerical schemes, and the amplitude and phase errors associated in the solution vectors of smaller wave number are much smaller than those illustrated in Figures 1 and 2. Thus fairly accurate simulation is possible of the convection away from a diurnally varying source on the Waikato River using Scheme 6 with a mesh size of 1 km and time steps of about 0.25 hours.

WATER QUALITY IN THE WAIKATO RIVER.

Work is presently being undertaken to develop a mathematical model of water quality in the stretch of the Waikato River from Karapiro Dam to the sea.

The major sources of effluent appear to be the urban areas of Hamilton and Huntly, Dairy factories at Te Rapa, Taupiri and on various tributaries to the Waikato, and the AFFCO meat works at Horotiu. The outflow from Lake Karapiro and runoff from the heavily stocked pasture in the basin also have a considerable effect on water quality.

RIVER FLOW

Continuous flow recorders are in operation at Karapiro Dam, Ngaruawahia, Huntly and Mercer and there are staff gauges at several other locations.

Analysis of past records has enabled a good picture to be obtained of river flow patterns.

Cross sections have been surveyed at approximately 200 locations, and the results used to solve the open channel flow equations and predict stage and velocity profiles on a 500 m mesh at various flows.

DISPERSON

The coefficient of longitudinal dispersion has been estimated from detailed velocity measurements in the river at Hamilton using both Taylor's method and a time scale method[5]. These figures have been used to estimate a non dimensional coefficient

$$\frac{D}{RU^*}$$

from which the coefficient of dispersion in other stretches of the river may be evaluated.

D cm^2/sec	$\frac{D}{RU^*}$	Method
2.30×10^4	131.57	Time scale
5.13	164.33	Taylor
6.82	218.62	Time scale
6.97	398.71	Time scale
7.22	354.46	Taylor

R hydraulic radius
U* shear velocity
D coefficient of longitudinal dispersion.

TABLE 1 Dispersion Coefficients Predicted from Velocity Measurements in the Waikato River.

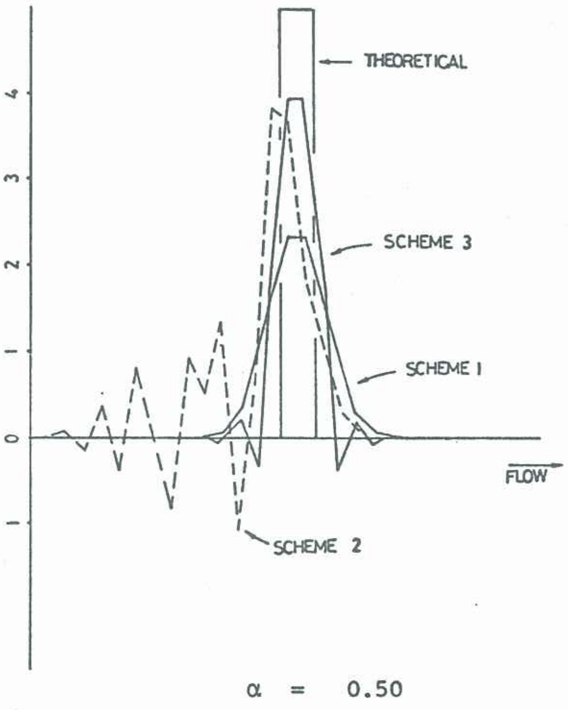

FIGURE 1

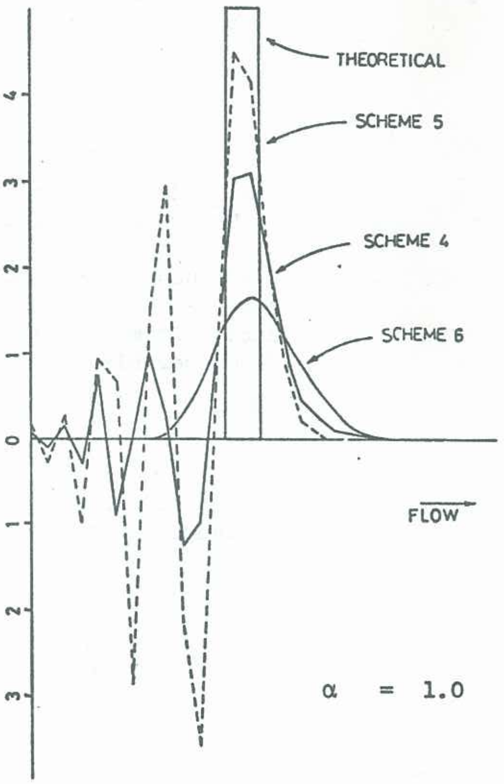

FIGURE 2

EFFLUENT DISCHARGE

Domestic and industrial effluent contains appreciable quantities of organic waste matter which stimulates the growth of bacteria and results in a drop in the concentration of dissolved oxygen in the waterway. Oxygen depletion is offset by the tendency of deficient waters to absorb oxygen from the air and by the activity of aquatic plants which at times make a net contribution to the concentration of dissolved oxygen in the waterway.

As a first step towards predicting water quality a model has been developed which uses the Biochemical Oxygen Demand (BOD) as the parameter of organic matter and which predicts the concentration of dissolved oxygen (DO) in the river. The amortization of BOD is assumed to be first order and the equations governing BOD and DO are

$$\frac{dC}{dt} = - k_1 C + q \tag{14}$$

$$\frac{dR}{dt} = - k_1 C + k_2 (S - R) + r \tag{15}$$

where C, R and S are the concentrations of B.O.D., D.O., and the saturation concentration of D.O. respectively; k_1 is the rate of amortization of BOD; k_2 is the rate of reaeration; q, r are the rates or addition of BOD and DO respecitvely.

k_2 the reaeration parameter has been estimated from the formula by Churchill et al.[6]

The contribution of the urban areas to BOD in the river has been estimated from the results of surveys of septic tank flow and composition carried out in Hamilton during 1966.[7]

Catchment Area	Population 1966	Load, kg/capita-day
Berescourt	4550	0.043
Fairfield	10500	0.067
Memorial Park	14000	0.045

TABLE 2

The figure for average BOD load quoted in the literature is 0.082 kg/capita-day.[8] The figures described above do not include the contributions from street runoff or from the weekly flushing of the municipal septic tanks into the river.

The reticulated population of Hamilton is approximately 64000, i.e. 80% of the total population.

The diurnal pattern of flow and composition has also been established from these surveys.

Extensive records of effluent flow are kept by the staff at the AFFCO Works at Horotiu; and some data has been collected on composition. From these records[9] a good picture has been obtained of the variation of BOD load both diurnally and seasonally.

The contribution from dairy factories has been estimated from production using average figures for volume and composition of effluent associated with the manufacture of unit quantities of various milk products[10]. See Tables 3 and 4.

The concentrations of BOD in several tributaries has been measured on odd ocassions[11] and these figures provide an estimate of the contribution of pasture runoff to organic matter in the river.

		October	February
Te Rapa	Milk powder	5000	1500
Taupiri	Butter	2000	500
Rukuhia	Cheese	100	100
Gordonton	Casein	200	100
Komakarou	Cheese	200	150
Rototuna	Cheese	150	120

Figures are tonnes/month.

TABLE 3 Average Monthly Production Figures for the Dairy Factories during October and February.

Product	Total process water m^3/tonne	Effluent m^3/tonne	Composition* BOD_5 mg/ℓ
Casein	40-45	30-32	2500-3000
Cheese	18-23	12	3000
Butter	16-18	4	3000
Milk powder	230	4-6	800-1000

* Soluble BOD . not including fats or grease.

TABLE 4 Volumes and Composition of Effluent Discarded per unit Production at the Dairy Factories.

The banks of the Waikato River are infested with aquatic macrophytes and during summer months the waters contain large numbers of diatoms and other motile algae. These plants cause a diurnal fluctuation of the concentration of dissolved oxygen most noticeable in the lower stretches of the river during summer low flow.

A survey of weed areas and predominant species was carried out by the Waikato Valley Authority[12] in 1969. Of the species encountered only four are thought to have the habitat and characteristics to affect the concentration of dissolved oxygen in the river water viz: Egeria densa, Elodea canadensis, Lagarosiphon major, Myriophyllum Robustum[15]. The rates of oxygen production and consumption by these macrophytes have been estimated from published data[13].

Algal counts are collected regularly by the Auckland Regional Authority[14] at Tuakau Bridge and by the Hamilton City Council at the water treatment station in Hamilton. These records have been used to obtain a rough estimate of algae biomass in the river. Oxygen production and consumption by these algae have again been estimated from published data.[16]

Macrophytes		
Production	1.00 gm/m^2 - hour	during sunlight
Respiration	0.25 gm/m^2 - hour	continuously
Algae		
Production	15 gm/m^2 - hour per mg/ℓ biomass	
Respiration	5 gm/m^2 - hour per mg/ℓ biomass	

TABLE 5 Macrophyte and Algal Respiration and Production

RESULTS

Several predictions have been made of dissolved oxygen in the Waikato River during a low flow period and using February production figures.

Figure 3 summarises the results of predictions made neglecting the effects of macrophytes and algae. It illustrates the fluctuations in dissolved oxygen which are attributable to diurnal variations of effluent flow and composition.

Figure 4 compares the results of predictions made both with without macrophyte and algae production with concentrations of dissolved oxygen observed in the river. The envelopes of solutions lie between the maximum and minimum concentrations of dissolved oxygen in the upper part of the river but lie outside the range in the lower part. This suggests that the rate of decay of BOD is higher than the value 0.015 hrs^{-1} (0.36 days^{-1}) employed. The decay rate 0.100 hrs^{-1} (2.4 days^{-1}) used in Figure 3 gives a profile which simulates the recovery observed in the lower part of the river. The decay rate is comparable with the high figure 0.150 hrs^{-1} (3.6 days^{-1}) observed in the Tarawera River and may be attributable to accumulation of aerobic bacteria in the porous pumice sediments.[17]

The figures for algal and macrophyte productivity quoted above are maximum values and were both measured in lakes rather than moving waters. Although the rates of production in the

Waikato River may be comparable with these figures a certain proportion of the oxygen liberated will be lost directly to the atmosphere.

In Figures 4 and 5 the dissolved oxygen profiles were predicted using the available data on weed areas, assuming a uniform concentration of 1.0 mg/ℓ algal biomass in the river and a 90% loss of the oxygen produced to the atmosphere.

The departure of solution profiles in Figure 5 from the observed figures suggests that either the polluting load has been overestimated or that either the rate of physical reaeration or the rate of reaeration by weeds and algae in the lower part of the river have been underestimated.

DISCUSSION

It was questioned right at the outset of the exercise whether the parameter BOD would be tractable as a tool for mathematical modelling. It has been observed that BOD measurements in the river water are frequently interfered with by algae and other aquatic organisms: and it is not possible therefore to relate predicted with observed BOD concentrations. Thus the accuracy with which the dissolved oxygen profile fits the observed data is the sole criterion for assessing the accuracy of the model. The concentration of dissolved oxygen is affected by several mechanicms about which little quantitative information has been collected and consequently cannot be used with any conviction to judge the effectiveness of the model.

The model however seeks only to summarise the available information on effluent discharge and river flow and proceed as far as possible with it. As more information becomes available then the model will be improved and hopefully more accurate predictions of dissolved oxygen profiles made.

Similarly it is hoped to model other parameters of water quality and in particular to look at the concentrations of coliform bacteria when the available data can be collected and summarised.

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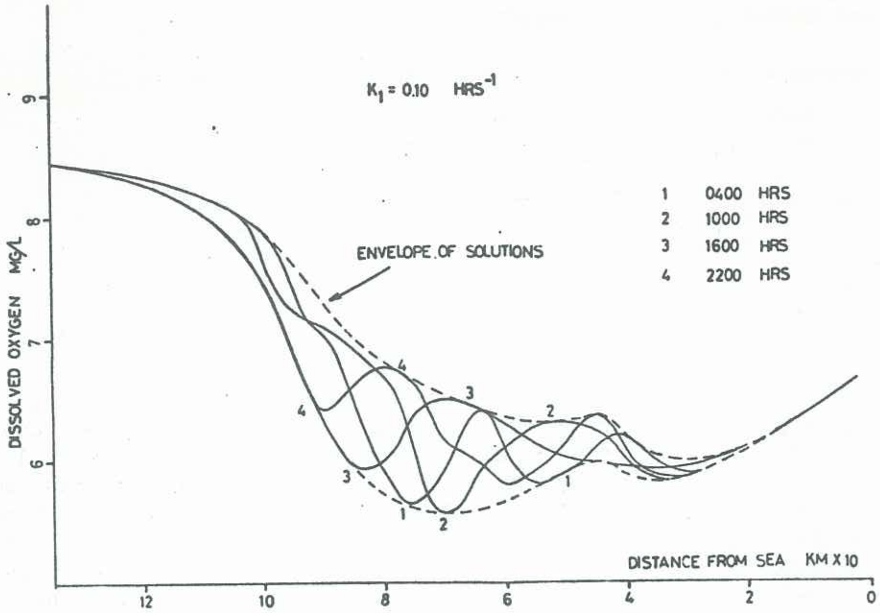

FIGURE 3

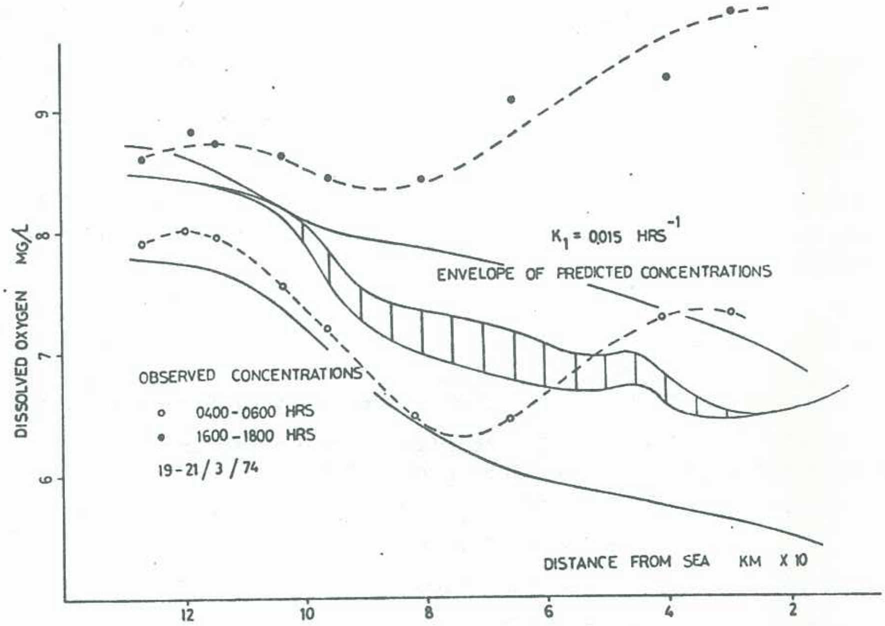

FIGURE 4

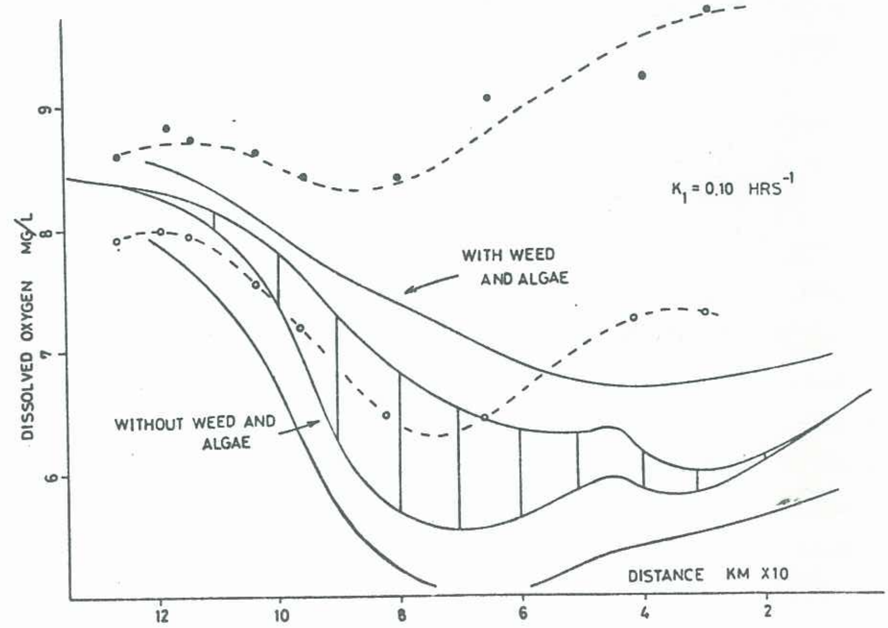

FIGURE 5