FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

SWIRL IN THE MANAGEMENT OF EFFLUENT DISCHARGE

by

D. G. Ross

SUMMARY

Swirl control of the rate of spread, mixing, and penetration of a jet suggests that similar control of these flow properties, in plumes emitted into a stratified environment, could be utilized to manage more efficiently effluent discharges such as the sea disposal of sewage. A model of an axisymmetric swirling turbulent plume projected vertically upward into an otherwise still, stable environment is developed to investigate these effects. The model is developed by using approximate forms of the integrated equations of motion in conjunction with an interpretation of the integrals in terms of profile averages. Closure of the system is achieved by means of an entrainment assumption made previously for a swirling jet.

The behaviour of a swirling forced plume in a stratified environment can be characterized by three dimensionless parameters, representing source mass flux, source angular momentum flux and environmental stability. Results are obtained which illustrate the effect of swirl on the maximum ascent height, rate of entrainment, and the decay of buoyancy along the axis, for the 'simple' plume (from a point source of buoyancy only). Of particular interest for the application above is the result that the effect of swirl on the maximum ascent height depends critically on the magnitude of the environmental stability parameter. The magnitude of the parameters appropriate to a wide range of source conditions and based on an actual outfall are determined, and the subsequent value of swirl control is discussed.

D.G.Ross, Department of Mathematics, Caulfield Institute of Technology.

1. INTRODUCTION

A swirling turbulent jet emitted into an otherwise stagnant environment is produced by discharging fluid with azimuthal as well as axial velocity from an orifice. The degree of swirl imparted to the jet can be characterized by the ratio of azimuthal to axial velocity at the orifice.

Theoretical and experimental studies of swirling turbulent jets (1-7) have determined the major changes in jet flow produced by varying the degree of swirl. They indicate that the presence of the azimuthal velocity field sets up radial and axial pressure gradients. In the case of a spreading flow the latter of these is an adverse gradient and causes axial deceleration. This combined with an increase in the initial rate of entrainment (8,4) produces an increase in the rate of spread and a reduction in penetration of the jet.

Swirl control of jets has proved valuable in a number of applications. Swirling jets are used in furnaces to provide flames of controlled length and stability and have also found application in various types of spray systems.

Similar control of these flow properties by swirl in plumes emitted into a stratified environment would provide an added controlling factor in managing the dispersion of pollutants. An example of a flow in which the type of control envisaged would be useful is an outfall for the sea disposal of sewage effluent. Before discussing this in detail however, it is valuable to list the following general objectives of such a discharge:

(i) to keep concentrations in the entire receiving water within specified limits,
(ii) to maximize mixing with the receiving water in order to obtain maximum dilution and, if possible, total submergence of the effluent field,
(iii) to predict and control the movements and characteristics of the effluent after discharge.

The modern-day submarine outfall consists of a long pipe to convey the waste to the point of discharge and a multiple-port diffuser situated on the bottom, to discharge the waste. The design of the diffuser seeks to satisfy objective (ii) by utilizing any stable density stratification of the receiving water as a barrier to prevent if possible the surfacing of the buoyant effluent. This is achieved by maximizing the initial mixing of the effluent plume with the cold bottom water in order to reduce the plume buoyancy. Thus the introduction of swirl to control this initial mixing and the resultant maximum height would be useful to ensure or at least prolong total submergence. In addition it could prove useful in satisfying objective (iii), since the added control would allow utilization of the best dispersive characteristics at a particular outfall site, and adjustment to their day to day variation.

The model developed below is intended to provide a framework for exploring the effects of swirl on buoyant plumes using approaches similar to those used previously for jets.

2. THE EQUATIONS OF MOTION

Consider the case of an axisymmetric swirling plume emitted vertically into an otherwise still, stable environment from a steadily-maintained source. Relative to cylindrical polar coordinates (r,ϕ,z) with origin at the centre of the source and Oz directed vertically upward, the velocity field has components $(u+u', v+v', w+w')$ where primes signify fluctuations and unprimed symbols signify ensemble mean values.

The plume density $\rho(r,z) + \rho'(r,\phi,z)$ is lower than the undisturbed ambient density $\rho_e = \rho_e(z)$ in the application considered but the following analysis is also valid for a negatively buoyant plume. The local buoyancy in the plume is given by,

$$\gamma(r,z) + \gamma'(r,\phi,z) = g\{\rho_e - (\rho + \rho')\}.$$

A neutral or linear stable stratification of the ambient fluid,

$$d\rho_e/dz = -\rho_o N^2/g \leq 0,$$

will be assumed where $\rho_o \equiv \rho_e(0)$ is taken as a reference density for the system. Further it will be assumed that density differences are small in comparison with ρ_o so that the Boussinesq approximation applies, namely that density variations may be neglected except in buoyancy forces.

The equations for conservation of mass, momentum and buoyancy, averaged over realizations for steady-in-the-mean, incompressible, axisymmetric-in-the-mean turbulent flow at large Reynolds number such that the viscous stresses (and diffusion of buoyancy) are negligible relative to Reynolds stresses (and transport of buoyancy) are taken as the governing equations for investigating the behaviour of such plumes.

Morton (6,9) presents an order of magnitude analysis of subsets of these equations suitable for a swirling jet and a non-swirling plume respectively. Simultaneous examination of these provides the details for the reduction of the equations above to approximate forms suitable for a swirling forced plume.

Integration of the reduced equations for conservation of mass, axial and azimuthal momentum and buoyancy over a cross-section z = constant, of the plume, with the necessary boundary conditions, and some further reduction leads to the following set of flux equations for the swirling plume:

$$\frac{d}{dz}\int_0^\infty \rho_o\, r\, w\, dr = -(\rho_o\, r\, u)_\infty , \tag{1}$$

$$\int_0^\infty \rho_o\, r^2\, vw\, dr = G , \tag{2}$$

$$\frac{d}{dz}\int_0^\infty (w^2 + \overline{w'^2} + \frac{p}{\rho_o})\, r\, dr = \int_0^\infty \frac{\gamma}{\rho_o}\, r\, dr, \tag{3}$$

$$\frac{d}{dz}\int_0^\infty w\,\gamma\, r\, dr = g\,\frac{d\rho_e}{dz}\int_0^\infty r\, w\, dr. \tag{4}$$

The equation for the conservation of radial momentum is more conveniently left in the reduced form

$$\frac{\partial p}{\partial r} = \rho_o\,(\frac{v^2}{r} - \frac{\partial \overline{u'^2}}{\partial r}). \tag{5}$$

Equation (1) indicates that the axial rate of increase of mass flux in the plume is equal to the rate of entrainment of mass or radial inflow of ambient fluid from large distances. It should be noted that r has more or less the character of an inner variable, and $r \to \infty$ does not imply $r/z \to \infty$, but merely that r is sufficiently large to include all the core region.

Equation (2) indicates that the angular momentum flux G is conserved.

Equation (3) indicates that the axial gradient of the "flow force" in the plume is equal to the total buoyancy force acting over a plume section, where the term representing the entrainment of ambient axial momentum inwards across the plume edge has previously been shown to be zero. The notation of Benjamin (10) has been adopted with the "flow force" per unit section taken as the axial momentum flux plus the dynamic pressure ($\rho_o\, w^2 + \rho_o\, \overline{w'^2} + p$). The role of p being significant for the case of strong swirl. Further we see that the sign of the axial gradient of "flow force" is that of the buoyancy, proportional to the deficiency of mean plume density compared to ambient density. If the ambient fluid is stably stratified the entrainment of fluid whose density is decreasing with height results in this deficiency continuously decreasing to zero, at a height referred to as the zero buoyancy level. The excess of momentum, at this level, carries the plume above this height until the reversed buoyancy reduces the "flow force" to zero at the maximum plume top height. Thus the axial "flow force" increases with height in the positive buoyancy region ($\gamma > 0$), and then decreases to zero at plume top in the region of reversed buoyancy ($\gamma < 0$).

Equation (4) indicates that in a stably stratified environment the buoyancy decreases with height, at a rate proportional to the gradient of the stratification.

Integration of equation (5) with respect to r yields a relationship for the dynamic pressure defect on the axis,

$$p_\infty - p_o = \rho_o \int_0^\infty v^2/r\, dr - \rho_o\, [\overline{u'^2}]_0^\infty , \tag{6}$$

which is valid only for strong rotation. This illustrates the additional effect of the azimuthal velocity field on the axial disturbance pressure (p_o). It indicates that swirl will influence the axial variation of p_o only if $v = v(r,z)$. A swirling plume undergoes progressive lateral spread of angular momentum by turbulent diffusion so it follows from equation (6) that strong swirl will produce a quite large adverse axial pressure gradient. This produces an associated axial deceleration and a consequent increase in the plume rate of spread.

After some reduction equation (6) can be used to eliminate the pressure term from equation (3) to obtain,

$$\frac{d}{dz} \int_0^\infty \rho_o \, (w^2 - \tfrac{1}{2} v^2) \, r \, dr = \int_0^\infty \gamma \, r \, dr \, , \qquad (7)$$

where $(\overline{w'^2} - \overline{u'^2})$ has been neglected relative to $(w^2 - \frac{1}{2} v^2)$ with moderate accuracy, in accordance with the approximations of Chigier and Chervinsky (3,4). Examination of equations (7) and (3) indicates the possibility of swirl as an added controlling factor in managing the dispersion of pollutants, since plume-spread and penetration depend on axial deceleration as well as entrainment.

Equations (1), (2), (4) and (7) are a set of flux equations in the dependent mean flow variables (w, v, γ, r). Although they contain no explicit dependence on the local turbulent structure there is an implicit dependence in the unknown entrainment. To obtain closure of the set a relationship is assumed in section 2.2 between this term and the mean flow variables. The closed set cannot be solved for lateral profiles of velocity and buoyancy as this information has been suppressed by integration, however solutions can be obtained using the Pohlhausen technique of assumed r-profiles.

2.1 The Choice of Profile Shapes:

For the simple model being considered the "integral-average" method is the most appropriate. It has the advantage that it involves no assumption of similarity and does not require the actual profiles of velocity and buoyancy to be specified. Consequently the choice will be valid for strong swirl conditions. It should be noted however, that the Pohlhausen technique is not very good when there is a possibility of flow reversal. Equation (7) indicates that a sufficiently large degree of swirl will produce zero net axial force. This plume "breakdown" with an axial stagnation point near the orifice and an associated downstream bubble of recirculating flow has been generated by Gore and Ranz (11) for a confined jet flow, and by Chigier and Beer (2), Chigier and Chervinsky (3,4) and Mathur and Maccallum (7) for unconfined jet flows. This "breakdown" can be taken as characterizing very strong swirl conditions, and the Pohlhausen technique will clearly be unsatisfactory for such a large degree of swirl.

In the case of the "integral-average" method the integrals are interpreted in terms of profile averages by introducing the profile mean values of axial velocity, w_m, azimuthal velocity, v_m, and mean buoyancy, $\rho_o \, \gamma_m$, and their corresponding radial scales r_m, s_m and d_m defined by

$$2\int_0^\infty r \, w \, dr = r_m^2 \, w_m \, , \qquad 2\int_0^\infty r \, w^2 \, dr = r_m^2 \, w_m^2 \, , \qquad 2\int_0^\infty r \, v^2 \, dr = s_m^2 \, v_m^2 \, ,$$

$$2\int_0^\infty r^2 v \, w \, dr = r_m^2 \, s_m \, v_m \, w_m \, , \qquad 2\int_0^\infty r \, \gamma \, dr = \rho_o \, d_m^2 \, \gamma_m \, , \qquad 2\int_0^\infty w \, \gamma \, r \, dr = \rho_o \, r_m \, d_m \, w_m \, \gamma_m .$$

This approach allows for the introduction of the axial velocity-buoyancy and axial-azimuthal velocity lateral spread parameters σ and λ given by $d_m = \sigma \, r_m$ and $s_m = \gamma \, r_m$ respectively.

2.2 Closure - The Entrainment Assumption:

Closure of the flux equations is achieved using the entrainment assumption,

$$-(ru)_\infty = E_s \, r_m \, \{w_m^2 + \beta^2 \, v_m^2\}^{\frac{1}{2}} \, , \qquad (8)$$

where E_s is taken as an entrainment constant to be determined experimentally, and β is an additional constant to be experimentally determined and which allows different mixing roles for the axial and azimuthal flows. This form was postulated by Morton (6) in a model for a swirling jet and was based on scaling obtained from an order of magnitude analysis.

A swirling jet or plume in a non-rotating environment consists of an interior region, where the angular momentum increases from zero on the axis to a maximum near the plume edge, and an outer sheath where the angular momentum decreases to zero. The entrainment models dependence on the mean azimuthal velocity field is an assumption based on the radial stability of these two regions. It assumes that the overall level of turbulence (and hence entrainment) is enhanced by the unstable outer annulus, where the circulation decreases with radial distance, even though the inner core may act as a sink of the energy transferred to fluctuations.

Due largely to the difficulty in obtaining accurate turbulence measurements in systems with high levels of turbulence there appears to be few measurements of turbulent structure in swirling flows from which to test this assumption. An alternative approach used by Lilley and Chigier (8) does however give indications of the relative order of magnitude of shear stress components, and their variation with swirl for a swirling jet. They use experimental measurements of the mean velocity fields in swirling jets made by Chigier and Chervinsky (4), together with a numerical technique for integrating the equations of motion. The results show that the level of turbulence increases in an "initial" region (at an axial distance of four orifice diameter), predominantely due to a swirl induced increase in the rz-component of the shear stress. In a "fully developed" region (at an axial distance of fifteen orifice diameters) however, the opposite was true with the shear stresses being lower than in a non-swirling jet. In this region the $r\phi$-components are negligible in comparison to the rz-components. It is not clear from these results as to whether the "initial" region considered is a source development region where the results may have been due to source effects. Also the term "fully developed" is perhaps misleading as the axial station chosen appears (from Chigier and Chervinsky's results) to be a region where the swirl has decayed to a moderate-weak value. Further results are needed for the region between the axial stations chosen in order to clarify these points, however the results do provide evidence that a higher level of turbulence, and hence rate of entrainment, is produced by a higher degree of swirl.

The entrainment assumption of equation (8) broadly models these overall features by producing a swirl induced increase in entrainment near the source, and a reduction after this region due to the rapid decay of the azimuthal velocity and the swirl induced axial deceleration. The arguments above indicate that the role of the mean azimuthal velocity is clearly less than that of the mean axial velocity in the production of turbulence, consequently the arbitrary choice of $\beta = \frac{1}{2}$ is made in the results which follow in order to display their general nature.

3. A MODEL FOR A STRONGLY SWIRLING FORCED PLUME

A model can now be developed to investigate a turbulent swirling forced plume projected vertically into an extensive environment of uniform or stably stratified fluid from a source generating mass flux ($\pi\, \rho_o\, m_o$), longitudinal momentum flux ($\pi\, \rho_o\, \ell_o^2$), buoyancy flux ($\pi\, \rho_e\, f_o$) and angular momentum flux ($\pi\, \rho_o\, a_o$). This is achieved by reducing equations (1), (2), (4) and (7) using the "integral average" method, obtaining closure using entrainment assumption (8), and expressing the system concisely in terms of the dependent flux variables:

$$m = r_m^2\, w_m\ ,\quad \ell = r_m\, w_m\ ,\quad f = \sigma\, r_m^2\, w_m\, \gamma_m\ ,\quad a = \lambda\, r_m^3\, w_m\, v_m\ ,$$

which are proportional to the axial flux of mass, the square root of longitudinal momentum flux, the buoyancy flux, and the angular momentum flux respectively, for constant σ & λ. No experimental results are available to obtain additional equations for the variation of σ & λ with height so in the results that follow we will take them as constant with $\sigma = 1$ & $\lambda = 1$. The further set of transformations to non-dimensional variables M, L, F, A and Z can be written in general terms as:

$$M = m/m_D\ ,\quad L = \ell/\ell_D\ ,\quad F = f/f_D\ ,\quad A = a/a_D\ ,\quad Z = z/z_D\ ,$$

where the suffix D is used to indicate a characteristic plume quantity or combination of quantities having the same dimensions as the appropriate flux variables or height.

The physical quantities which determine the behaviour of a swirling plume in a stratified environment are its flux values (m_o, ℓ_o, f_o, a_o) at source level and N (a measure of the stratification). Any two of these quantities form a sufficient basis for the reduction, because the flow can be completely characterized by one length scale and one time scale. The form chosen for γ_m gives it the dimensions of acceleration. Consequently there are ten different ways of selecting the reduction to non-dimensional form, and for each case three dimensionless parameters may be constructed, each involving a characteristic quantity not used in combination with the transformation base quantities. The aim of this analysis is to investigate the effect of a variable source angular momentum flux on a plume, and to examine any dependence of this effect on the environmental stability. Consequently a_o and N will not be included directly in the dimensional reduction. This leaves a choice of any two of the source fluxes (m_o, ℓ_o, f_o) and of these the transformation base (f_o, ℓ_o) will be adopted since it provides the natural basis for scaling a 'forced' plume. This base yields the transformations:

$$M_D = E_s^{1/2}\, \ell_o^{5/2}\, |\, f_o\, |^{-1/2},\quad \ell_D = \ell_o\ ,\quad f_D = |\, f_o\, |\ ,\quad a_D = 2G/\rho_o\ ,$$

$$z_D = 2^{-1}\, E_s^{-1/2}\, \ell_o^{3/2}\, |\, f_o\, |^{-1/2},$$ and the working equations:

$$\frac{dM}{dZ} = L\, \{1 + \frac{8}{5}|T|\, (\frac{KS}{ML})^2\}^{\frac{1}{2}},$$

$$\frac{dL}{dZ} = \frac{\sigma FM}{4L^3} - \frac{8}{5}|T|\, \frac{S^2}{2M^3}\, \{1 + \frac{8}{5}|T|\, (\frac{KS}{ML})^2\}^{\frac{1}{2}},$$

$$A = 1,$$

$$\frac{dF}{dZ} = -\Delta M,$$ where $K = \beta/\lambda$.

The transformations are based on the source conditions $m = m_o$, $\ell = \ell_o$, $f = |f_o|$, $a = a_o = 2G/\rho_o$, at $z = 0$, or alternatively $M = 2^{3/2}\, 5^{-1/2}|T|^{1/2}$, $L = 1$, $F = \operatorname{sgn} T$ and $A = 1$ at $Z = 0$, where

$$T = \frac{5m_o^2\, f_o}{8E_s\, \ell_o^5}\ , \quad S = \frac{a_o}{m_o\, \ell_o} = \lambda_o \left\{\frac{v_m}{w_m}\right\}_o\ , \quad \Delta = \frac{N^2\, \ell_o^4}{2f_o^2}\ .$$

Thus the solution of the working equations with these initial conditions can be used to describe plumes from a series of sources of specified (f_o, ℓ_o), with the parameters $T \propto m^2_o$, $S \propto a_o$ and $\Delta \propto N^2$ incorporating changes in the other source conditions and environmental stability.

The non-swirling forced plume solutions of Morton and Middleton (12) are a subset of this family for $S = 0$. Thus the parameter T represents non-swirling forced plumes from sources of strength (m_o, ℓ_o, f_o) in a uniform environment. It may be interpreted as the inverse square of a Froude number if expressed in the form $8E_s\, T/5\sigma = r_{m_o}\, \gamma_{m_o} / w_{m_o}^2$. For upward emission from the source ($\ell_o > 0$), $\operatorname{sgn} T = \operatorname{sgn} f_o$, since $m_o^2 \geq 0$. Thus T may be positive or negative, with Morton and Middleton showing that it exhibits the following basic regimes of behaviour:

(a) $T \leq 0$: negatively buoyant forced plumes generated by upward emission of fluid denser than its environment.

(b) $0 \leq T \leq 1$: positively buoyant forced plumes generated by upward emission of light fluid with excess velocity relative to the simple straight sided plume ($T = 1$) from a virtual point source of buoyancy only.

(c) $T \geq 1$: positively buoyant forced plumes generated by slow upward emission of light fluid relative to the equivalent simple plume.

For the case where (f_o, ℓ_o) are regarded as specified, an increase in T implies an increase in r_m together with decreases in w_m and γ_m. The scaling will break-down if $f_o = 0$ and/or $\ell_o = 0$ so that neither the neutral jet (swirling or non-swirling) ($f_o = 0$, $m_o > 0$, $a_o \geq 0$) nor the plume from an infinitely wide source of zero velocity ($\ell_o = 0$) belong to this class of solutions. The value $T = 0$ corresponds to the hypothetical case of a source of zero radius at $Z = 0$ (the actual source level) which has $m_o = 0$ necessarily and includes the limiting cases of (a) and (b) as negatively and positively buoyant forced plumes from "actual point sources".

With the introduction of swirl it is still convenient to consider T as defining these three basic regimes of behaviour and to examine the effects of swirl on characteristic members from each regime. We can then define the following plume quantities to illustrate the variation of flow type with T, S and Δ.

3.1 Characteristic Plume Quantities:

Length Scales : There are two distinct length scales (a) the scale height $d = E_s^{\frac{1}{2}} \ell_o^{3/2} |f_o|^{-\frac{1}{2}}$ which arose naturally in the reduction of the equations where $z/Z = d/2E_s$, and (b) the mean plume radius at the source $r_{m_o} = d(M/L)_o = 2^{3/2}\, 5^{-\frac{1}{2}} |T|^{\frac{1}{2}} d$. An observer would probably judge many flow phenomena in terms of the source radius so with this in mind Morton and Middleton define a vertical length scale in terms of r_{m_o} as,

$$z/Z = d/2E_s = r_{m_o}\, 2^{-5/2}\, 5^{-\frac{1}{2}}\, E_s^{-1} |T|^{-\frac{1}{2}}.$$

This also provides a non-dimensional height, $|T|^{-\frac{1}{2}} Z$, for comparing plumes from different sources directly.

Normalized mean buoyancy flux:

$$\gamma_{m_n} = \gamma_m/\gamma_{m_o} = (2^{3/2}\, 5^{-\frac{1}{2}}\, |T|^{\frac{1}{2}})\, F/M,$$

where γ_{m_o} is the mean buoyancy flux at source level $Z = 0$.

3.2 Results:

Results have been obtained for the particular case of the positively buoyant simple plume ($T = 1$) emitted into environments whose stability is represented by the stability parameter values

(i) $\Delta = 0.05$
(ii) $\Delta = 1$.

Figure 1 illustrates the variation of maximum ascent height Z_{max} with the swirl parameter S. It exhibits the critical dependence of this variation on the magnitude of the stability parameter, with Z_{max} decreasing with increasing swirl for $\Delta = 1$ but increasing for $\Delta = 0.05$. Results are only presented from the range $0 \leq S \leq 2.5$, since the development of the model rests partially on the assumption that the plume is a tall narrow flow, and it can be shown that this assumption is clearly invalid for $S > 2.5$.

Figures {2(i) and (ii)} and {3(i) and (ii)} for the variation with height of non-dimensional rate of entrainment dM/dZ and normalized buoyancy flux γ_{m_n} further illustrate this dependence on the environmental stability for $S = 0$ and $S = 2.5$. For both values of Δ we see that

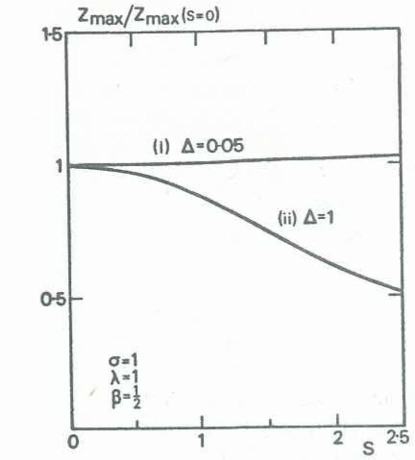

FIG.1. Non-dimensional curves showing the variation of maximum ascent height (normalized with respect to the non-swirling maximum ascent height) with swirl parameter S for the simple plume T = 1, and for values of the stability parameter:
(i) $\Delta = 0.05$ ($Z_{max}(S=0) = 6.642$);
(ii) $\Delta = 1$ ($Z_{max}(S=0) = 1.370$).

increasing swirl produces an increased rate of entrainment very near the source, but a decreased rate (in comparison to the non-swirling values) above this initial region as the axial and azimuthal velccities decay. This in turn produces a rapid decay of the plume buoyancy close above the source and a reduced rate of decay at greater heights. For the case $\Delta = 1$ the zero buoyancy level is little effected by the introduction and magnitude of the swirl since increased entrainment in the initial region compensates for reduced entrainment higher up, with the "overall mixing" being effectively unchanged. Consequently the observed decrease of maximum ascent height with swirl is effectively independent of the plume mixing, being produced by the swirl induced reduction in the excess momentum flux at the zero buoyancy level. Comparison of figures 2(i) and (ii) indicates that whereas the magnitude of Δ does not significantly effect the rapid decay of the increased rate of entrainment in the initial region, it does have a considerable influence above it. Hence at small Δ ($\Delta = 0.05$) the increased entrainment in the initial region is unable to compensate for the reduced entrainment above this level with consequent lower overall mixing, significant increase in the zero buoyancy level, and an increase in the maximum height of the plume with increasing swirl.

The critical dependence on Δ of the variation of Z_{max} with S at T = 1 suggest further examination for other values of T. Figure (4) illustrates the variation of the critical value of the stability parameter Δ_c with the source parameter T, where Δ_c is defined such that for $\Delta \geq \Delta_c$ Z_{max} decreases with swirl and for $\Delta < \Delta_c$ Z_{max} increases with swirl.

FIG.2. Non-dimensional curves showing the variation of rate of entrainment dM/dZ with vertical height Z for the simple plume T = 1, presented for two values of the swirl parameter S, and for stability parameter values:
(i) $\Delta = 0.05$; (ii) $\Delta = 1$. ($\sigma=1$, $\lambda=1$, $\beta=\frac{1}{2}$).

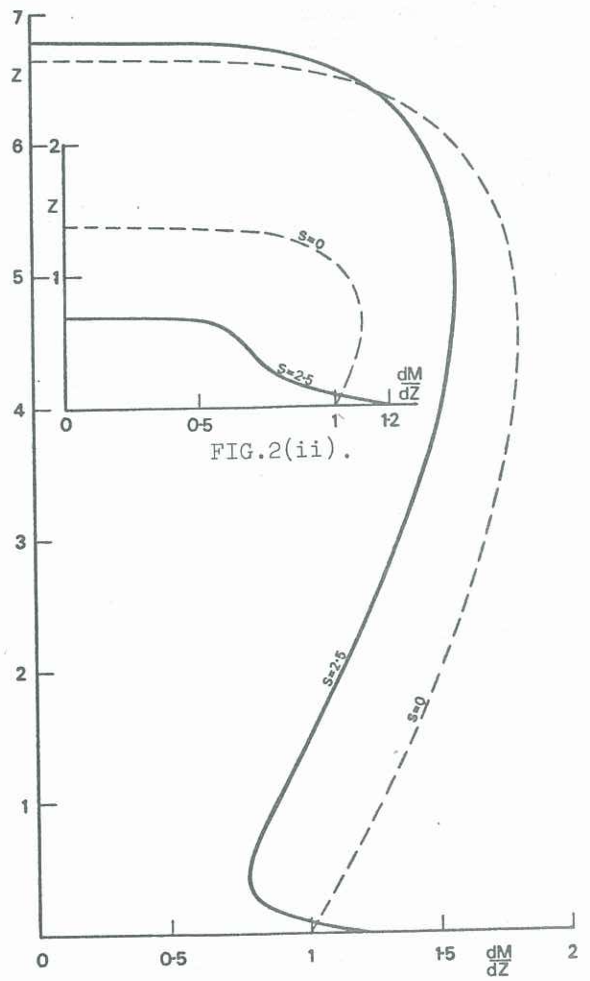

FIG.2(i).

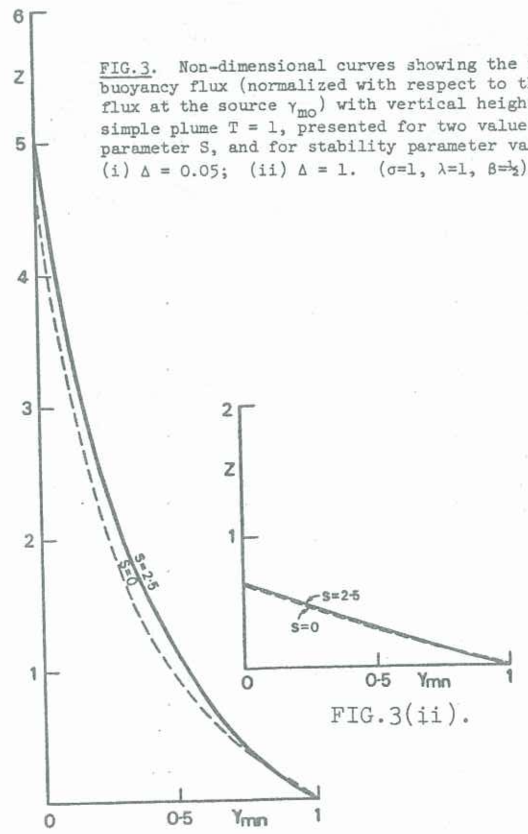

FIG.3. Non-dimensional curves showing the variation of buoyancy flux (normalized with respect to the buoyancy flux at the source γ_{mo}) with vertical height Z for the simple plume T = 1, presented for two values of swirl parameter S, and for stability parameter values:
(i) $\Delta = 0.05$; (ii) $\Delta = 1$. ($\sigma=1$, $\lambda=1$, $\beta=\frac{1}{2}$).

FIG.3(i).

4. APPLICATION TO THE SEA DISPOSAL OF SEWAGE EFFLUENT

The model has been developed to investigate the use of swirl as a controlling factor in managing the dispersal of pollutants, particularly in an outfall for the sea disposal of sewage effluent. The general objectives of such an outfall listed in the introduction indicated that control of effluent plume penetration, mixing, and rate of spread would be useful in adjusting to variations in the best dispersive characteristics at a particular outfall site. The major advantage however, would be gained by a swirl induced reduction in maximum height as this would help ensure or at least prolong total submergence of the effluent field. The results in Section 3.2 indicate the critical dependence on the environmental stability of swirl variation in maximum height. Consequently it is necessary to estimate practical ranges of the parameters Δ and T.

Table 1 below gives values of Δ and T for a wide range of source volume fluxes and discharge velocities. In each case the specific gravity of the sewage effluent and the bottom water are taken as 0.9987 and 1.0257 respectively. The specific gravity of the sea water is taken as $\rho_o = 1.0250$ in places where it does not appear as a difference, and the density gradient $-d\rho_e/dz$ is taken as $3 \times 10^{-6}\ m^{-1}$. These values are representative of the outfall at West Point in Puget Sound for Seattle, Washington, U.S.A. (Brooks and Koh (13)). The broadly representative value of $E_s = 0.1$ is taken for the entrainment constant.

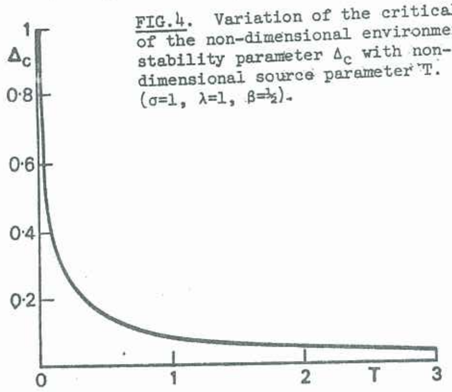

FIG.4. Variation of the critical value of the non-dimensional environmental stability parameter Δ_c with non-dimensional source parameter T. (σ=1, λ=1, β=½).

TABLE 1

w_{mo} (m/sec)	T: q_o (m^3/sec) 0.01	0.05	0.1	0.5	1	3	Δ
0.5	0.52	1.15	1.70	3.67	5.15	8.93	0.00005
1	0.09	0.20	0.30	0.65	0.91	0.58	0.0002
2	0.02	0.04	0.05	0.11	0.16	0.28	0.0009
4	0.003	0.006	0.009	0.02	0.03	0.05	0.003
8	0.0005	0.001	0.002	0.004	0.005	0.009	0.01
16	0.00009	0.0002	0.0003	0.0006	0.0009	0.002	0.6

Values of the dimensionless source parameter T and environmental stability parameter Δ, for a wide range of source discharge velocity w_{mo} (m/sec) and source volume flux q_o (m^3/sec). In each case the characteristics of the effluent plume and the receiving environment have been selected as representative of an actual outfall (13).

It is apparent from table 1 and figure 4 that at any point of this range of Δ and T the maximum height of plume will increase with increasing swirl. Thus, swirl control would not be of great value in helping to ensure submergence of the effluent field. Its only use could be in environmental conditions where an increase in the plume penetration might enhance natural mixing and dispersion. For example, in taking advantage of wind induced surface currents.

Suppose however we invert the system. This now models a heavy or cold effluent plume projected vertically downward into receiving waters whose temperature is decreasing with depth. In this case swirl control might be useful in reducing the overall mixing with the receiving water and increasing the depth of the stabilized effluent field.

5. ACKNOWLEDGMENT

I should like to thank Professor B.R. Morton, for his help and advice regarding this work.

REFERENCES

1. ROSE,W.G. (1962). - J.Appl.Mech., 29, Trans. ASME. 84, Series E, pp. 616-625.
2. CHIGIER,N.A. & BEER,J.M. (1964). - J.Basic Engg., 86, Trans. ASME. 4, Series D, pp. 788-796.
3. CHIGIER,N.A. & CHERVINSKY,A. (1966). - Israel J Tech., Vol 4, No.1, pp. 44-54.
4. CHIGIER,N.A. & CHERVINSKY,A. (1967). - J.Appl.Mech., 34, Trans. ASME. 2, Series E, pp. 443-451.
5. LEE,S.L. (1965). - J.Appl.Mech., 32, Trans. ASME. 2, Series E, pp. 258-262.
6. MORTON,B.R. (1968). - Mech and Chem Engg Trans., Paper No.2615, (Nov.), pp. 241-246.
7. MATHUR,M.L. & MACCALLUM,N.R.L. (1967). - J.Inst Fuel., 29, (May), pp. 214-224.
8. LILLEY, D.G. & CHIGIER,N.A. (1971). - Int J Heat, 14, pp. 573-581.
9. MORTON,B.R. (1968). - Geophysical Fluid Dynamics Laboratory, Paper 7, Monash University.
10. BENJAMIN,T. BROOKE. (1962). - J.Fluid Mech., 14, 593.
11. GORE, R.W. & RANZ,W.E. (1964). - J.A.I.Ch.E., 10, 1, (Jan.), pp. 83-88.
12. MORTON,B.R. & MIDDLETON,J. (1973). - J.Fluid Mech., 58, Part 1, pp. 165-176.
13. BROOKS,N.H. & KOH,R.C.Y. (1965). - Int Assoc Hyd Res., Xl Congress, Paper 2-19, Leningrad.