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MECHANICS OF FLOW IN MULTIPORT OUTLETS

by

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SUMMARY

The paper deals with the mechanics of flow related to a series of sharp-edged orifices distributed along the length of a circular pipe, forming a multiport system. For the complete solution of the multiport flow system it is shown that the variation of the coefficient of discharge C_d with the velocity factor ζ is essential. The various factors affecting C_d are discussed. A simple analysis is presented to obtain the variation of C_d with ζ. The results of the present analysis compare very well with the available theoretical result for two-dimensional flow and the emphirical equations for pipeflow.

The present proposition related to the variation of C_d, together with the basic differential equation for manifold flow, provide relevant expressions for the solution of various problems associated with multiport systems.

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INTRODUCTION

Multiport outlets are extensively used in hydraulic structures disposing power plant discharges and industrial effluents. They also find applications in fluid systems associated with sprinkler and drip irrigation, Navigation locks, waste water troughs and gas burners. Typically, in these applications a bank of orifices or laterals are distributed along the length of the main pipe to obtain a desired flow distribution. The commonly encountered design problems are:

(a) Determination of the distribution pattern of the outlets (size and location) for pre-determined outflow pattern along the length of the main pipe;

(b) Determination of the discharge and head distribution at the outlet locations for a selected geometric pattern of outlets.

Berlamont and Beken (1) have recently published an excellent review of previous work on flow through multiports.

The flow in a multiport device is a spatially varied phenomenon. The flow from an outlet will be governed by the transverse pressure head gradient (Δh) across the outlet and the local main flow velocity (v), in addition to the other geometric and fluid parameters. Consequently, the coefficient of discharge (C_d) for each outlet could be expected to vary along the length of the main pipe depending upon the parameter $\zeta = v^2/(2g.\Delta h)$. Most of the earlier investigators have overlooked the dependence of C_d on ζ. Noseda's (3) limited experiments do indicate this dependence of C_d on the velocity factor, ζ. Based on an experimental study of the outflow from small orifices in the corrugated side of an open channel, Vigander (4) has presented an emphirical equation for the variation of C_d with ζ.

In the present study, the primary variables affecting the coefficient of discharge of an orifice type outlet in the wall of a conduit are identified and an expression for the variation of C_d with ζ is presented. For meaningful solutions for flow through multiport systems, this information is to be used together with the basic differential equation.

ANALYSIS

Fig. 1 shows a typical multiport flow device. The differential form of the equation of motion is:

$$\frac{dh}{dx} + \frac{d}{dx}(\alpha\frac{v^2}{2g}) + \frac{dh_f}{dx} = 0 \qquad (1)$$

in which h = piezometric height at station x of the conduit above a reference datum, x = distance along the axis of the pipe, v = mean velocity at station x, α = kinetic energy correction factor, g = acceleration due to gravity, h_f = head loss due to energy dissipation. It is known that:

$$\frac{dh_f}{dx} = f.\frac{1}{D}.\frac{v^2}{2g} \qquad (2)$$

where f = friction coefficient and D = diameter of the multiport pipe at station x. For an orifice type outlet distributed along the length of the pipe:

$$-\frac{dQ}{dx} = q_x = C_d\ a\sqrt{2g\ h} \qquad (3)$$

in which Q = discharge in the main pipe, q_x = lateral outflow per unit length at station x, a = perforation or slot area per unit length at station x, and C_d = discharge coefficient. It is seen that in order to solve Eq. 1, together with Eqs. 2 and 3, information about f and C_d are needed. This paper deals with a study of the variation of C_d.

DISCHARGE COEFFICIENT, C_d

Consider a single, sharp-edged, circular orifice in the side wall of a smooth pipe, Fig. 2. The discharge q through the orifice is given by:

$$q = C_d . a_o . \sqrt{2g\ \Delta h} \qquad (4)$$

where Δh = pressure head differential across the orifice of area a_o. By dimensional analysis the variation of C_d can be expressed as:

$$C_d = F(\frac{d}{D}, \frac{d}{t}, R_e, \zeta) \qquad (5)$$

where $R_e = \frac{VD}{\nu}$ = pipe Reynolds number and $\zeta = v^2/(2g.\Delta h)$ = velocity factor. The effect of Reynolds number, R_e, on the discharge coefficient under turbulent flow range normally found in practice, may be considered to be negligible. Further, for very small ratios of orifice-to-pipe diameters, such as d/D < 0.1, the orifice discharge is unlikely to affect the main flow pattern itself, and as such, its effect on C_d can be considered to be not very significant. For large d/t values, i.e., for orifices in thin-walled pipes, the control for orifice flow will be at the sharp-edged entrance. Consequently, the parameter d/t does not have any influence on C_d. Hence, for small, individual, sharp-edged orifices in a thin-walled pipe:

$$C_d = F(\zeta) \qquad (6)$$

Consider a cross-section of such a lateral orifice as shown in Fig.3. The jet issuing from the orifice will have a velocity:

$$v_j = \sqrt{2g\ \Delta h} \qquad (7)$$

The jet will be deflected in the direction of the pipe flow due to interaction of the main pipe velocity V. As a result of this, the cross-sectional area a_n, normal to the jet velocity vector gets reduced. Also, the nominal size of the jet will be reduced by an amount controlled by the coefficient of contraction. Hence the discharge equation, considering the normal area of the orifice, can be written as:

$$q = C_d^*\ a_n \sqrt{2g.\Delta H} \qquad (8)$$

where C_d^* = a discharge coefficient which is essentially a coefficient of contraction. The normal area a_n is given by:

$$a_n = a_o \sin\theta \qquad (9)$$

where θ = inclination of the jet velocity with the pipe axis (Fig.3). Hence, from Eqs.(8) and (9)

$$C_d = C_d^* \sin\theta \qquad (10)$$

and C_d^* = value of C_d for $\theta = 90^o$. The inclination θ, for a first approximation can be obtained by consideration of the velocity vector diagram shown in Fig.(3) from which

$$\sin\theta = \sqrt{1 - \frac{v^2}{v_j^{\,2}}} \qquad (11)$$

or

$$\sin\theta = \sqrt{1 - \zeta^2} \qquad (12)$$

Hence

$$C_d = C_d^* \sqrt{1 - \zeta^2} \qquad (13)$$

where C_d^* = discharge coefficient corresponding to $\theta = 90^o$, i.e., $v = 0$. Eq.(13) can be expected to give the variation of C_d for $0 \leq \zeta \leq 1.0$.

To verify the above proposition no satisfactory experimental data were available. Hence, the theoretical work of McNown and Hsu (2) was considered for study. It is recognized that the work of McNown and Hsu (2) is for the case of lateral flow from a slot of width d', in a two-dimensional channel of width B. However, it is hoped that the case of small d'/D would approximate to conditions of the present study. Fig. (4) shows a plot of this data plotted as C_d vs. ζ. It is seen that Eq. (13) adequately predicts the variation of C_d with ζ for values of d'/B up to 1. Observe that the value of C_d^* for such a case is 0.61.

It is interesting to note that the empirical equation

$$C_d = 0.63 - 0.58 \frac{v^2}{2gE} \qquad (14)$$

where $E = \Delta h + \frac{v^2}{2g}$, supposed to have been suggested by Rawn et al (4) for small orifices in a pipe, is essentially the same as Eq. (13) with $C_d^* = 0.63$. Similarly, Vigander's (4) emphirical equation for C_d of an orifice in a corrugated pipe (guided flow) can be expressed satisfactorily by Eq. (13) with $C_d^* = 0.675$.

It is thus concluded that Eq. (13) adequately expresses the effect of the velocity factor ζ on the variation of C_d for a small lateral orifice in a smooth thin pipe. In a practical case, a multiport device is composed of a set of orifices arranged in specific disposition. Recognizing that C_d is essentially a coefficient of contraction, the effect of other parameters can be expressed as,

$$C_d = C_d^* \sqrt{1 - \zeta^2} \qquad (13)$$

and

$$C_d^* = F[\frac{d}{D}, \frac{d}{t}, R_e, \sigma_1, \sigma_2, \ldots] \qquad (14)$$

where σ_1, σ_2, ... are non-dimensional parameters to represent the geometrical configuration of orifices, the velocity profile of the approach flow, etc. Experimental study is needed to clearly delineate the various effects. However, under normal ranges of application the effects of various parameters shown in Eq. (15) is likely to be insignificant (4) and C_d^* is likely to be a constant.

MULTIPORT FLOW WITH ORIFICE TYPE OUTLETS

In the absence of further details, the variation of C_d can be obtained from Eq. (13) with $C_d^* = 0.61$. Including this information in Eq. (2)

$$(\frac{dQ}{dx})^2 = C_d^{*2} a^2 (2gh - v^2) \qquad (15)$$

Noting that $A = \frac{\pi D^2}{4}$ = area of main pipe and differentiating

$$2 \frac{dQ}{dx} . \frac{d^2Q}{dx^2} = C_d^{*2} a^2 (2g \frac{dh}{dx} - \frac{2Q}{A^2} . \frac{dQ}{dx}) \qquad (16)$$

Substituting the value of $\frac{dh}{dx}$ obtained from Eq. (16) in Eq. (1), and noting that f in Eq. (2) is a function of Reynolds number, the differential equation for multiport flow, in terms of Q and x can be obtained. This equation, being essentially non-linear, will have to be solved using numerical techniques. However, for the particular case of a short smooth pipe, the friction term in Eq. (1) can be neglected. For such cases, Eq. (1) when used, together with Eq. (16) reduces to a simple form

$$\frac{d^2Q}{dx^2} + K Q = 0 \qquad (17)$$

where

$$K = \frac{C_d^{*2} a^2}{A^2} (1+\alpha)$$

From this the solutions for the variation of Q with x - and hence the variation of h with x - for various boundary conditions can be obtained easily.

CONCLUSIONS

A study of the coefficient of discharge C_d of an orifice in the wall of a pipe, subjected to the action of lateral velocity is reported. Complete solution of multiport flow with orifice type outlets, the information on the variation of C_d is necessary. A simple analysis yields an expression (Eq. 13) for the variation of C_d with the velocity factor ζ. The various parameters affecting C_d are discussed and it is concluded that Eq.(13) with a value of $C_d^* = 0.61$ could be used to adequately represent the variation of C_d in normal practical multiport outlet systems. Extension of the analysis to solve the multiport flow problems is indicated.

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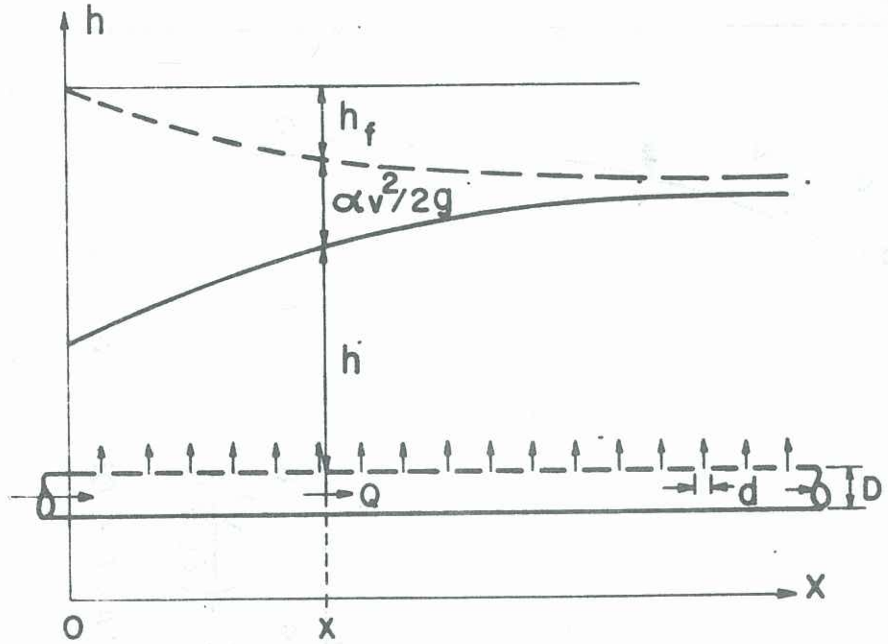

FIG. 1 TYPICAL MULTIPORT OUTLET

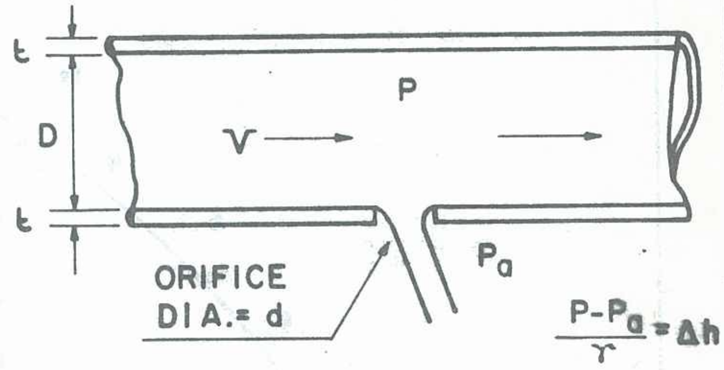

FIG. 2 DEFINITION SKETCH

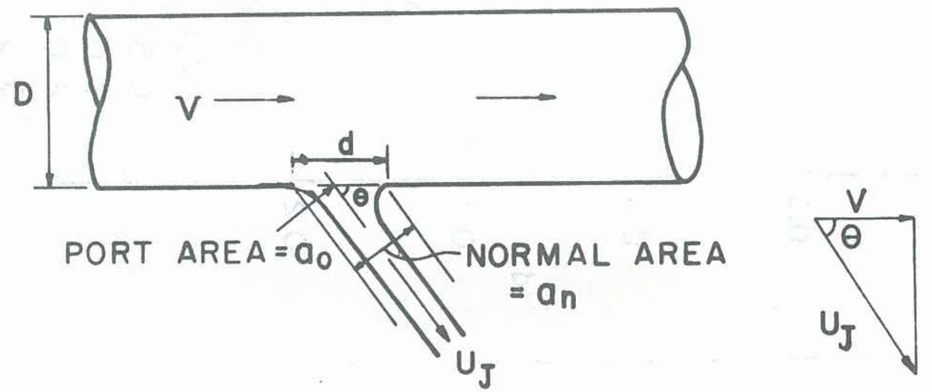

FIG. 3 FLOW IN A LATERAL ORIFICE

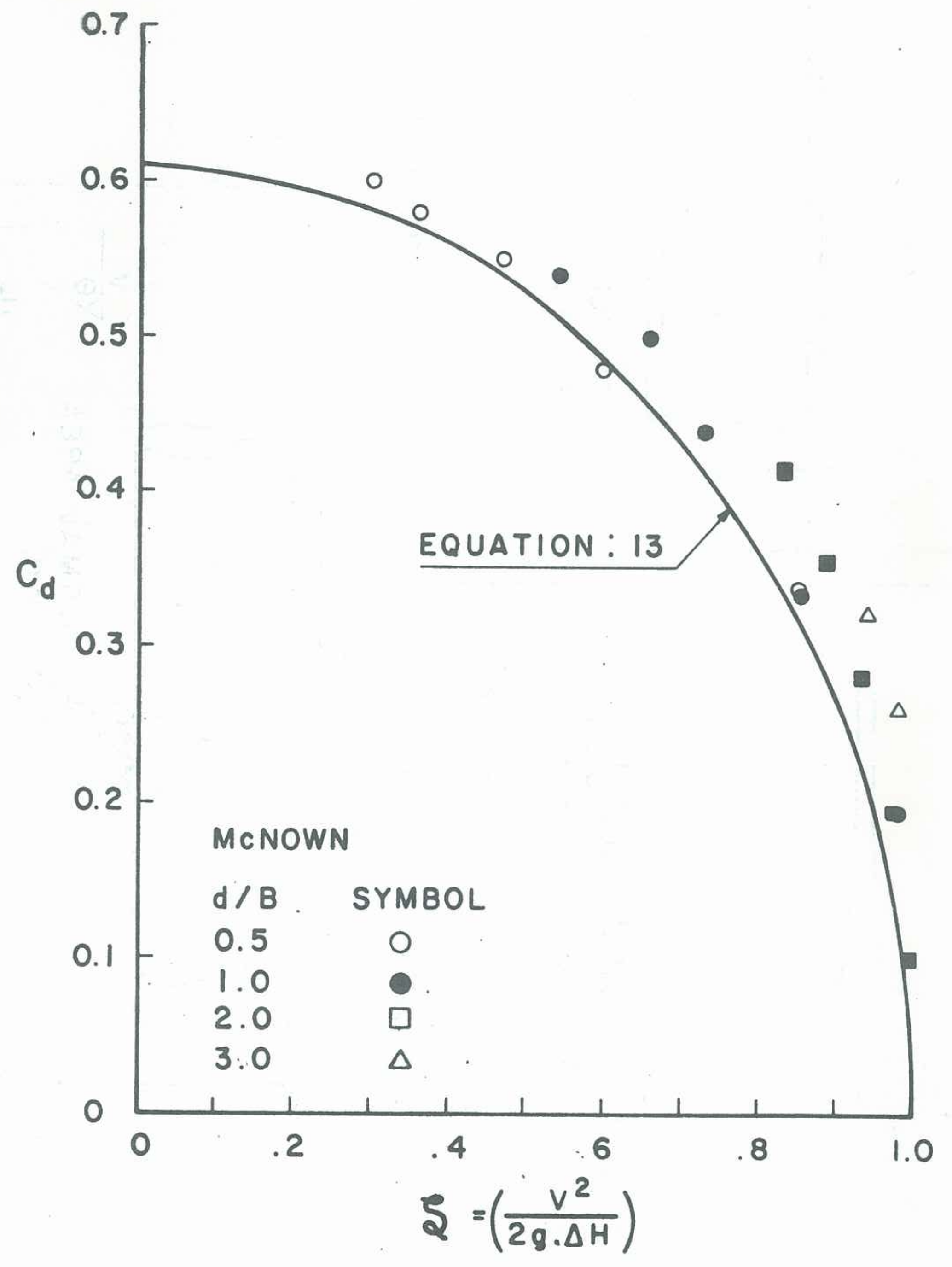

FIG. 4 VARIATION OF C_d WITH ξ