FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

AN INVESTIGATION OF A TURBULENT CYLINDRICAL WALL JET

by

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S U M M A R Y

The investigation presented here describes a study of a turbulent cylindrical wall jet in quiescent surroundings. A similarity analysis is presented which predicts the gross features of the outer part of the cylindrical wall jet. Measurements of mean velocities and variations of $y_{m/2}$, y_m and U_m with downstream distance from the jet exit are given. These measurements are compared with those of Starr and Sparrow[4]. The experimental results reported here substantiate the similarity analysis in that the outer part of the cylindrical wall jet grows linearly with the downstream distance and the maximum velocity, U_m, varies inversely with the downstream distance.

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1. INTRODUCTION

There are many practical flow situations in which a jet of fluid interacts with a solid boundary. Such flow fields are generally referred to as wall jets. Some of the common examples of the wall jet in quiescent surroundings are shown in Figure (1).

In recent years considerable effort has been devoted to both a plane wall jet and a wall jet on curved surfaces. A common feature of these wall jets is that a plane jet interacts with a solid boundary. In a plane wall jet the surface curvature is absent whereas a wall jet on a curved surface is influenced by the surface curvature and by the adverse pressure gradient.

In a radial wall jet a round jet impinges on a flat surface placed at right angles to the jet axis. For a cylindrical wall jet an annular jet is blown along the cylinder axis. As shown in Figure (1) a variety of cylindrical wall jets may be produced however, for the present an annular jet flowing outside a cylinder is considered. In comparison with other cylindrical wall jets this case is simpler because the pressure gradient is absent.

There exists a considerable knowledge for plane wall jets in quiescent surroundings. However, the availability of information for both radial and cylindrical wall jets is parse. The radial wall jet has been investigated by Bakke[1], and by Bradshaw and Love[2]. The experimental results on cylindrical wall jets have been reported by Lawrence[3], and by Starr and Sparrow[4]. Lawrence has presented only non-dimensional velocity profiles whereas Starr and Sparrow have measured both velocity profiles and skin friction. As far as the authors are aware turbulence measurements in the cylindrical wall jets are not yet available. To augment the existing experimental data on the cylindrical wall jets the present investigation was undertaken.

This paper describes the experimental set up and presents measurements of mean velocity profiles and streamwise development of the cylindrical wall jet. The Reynolds number $\left(\frac{U_m y_{m/2}}{\nu}\right)$ for the present investigation was 30,000.

2. THEORETICAL ANALYSIS

The following similarity analysis although it neglects the skin friction, predicts the gross features (downstream variations of a length scale and a velocity scale) of the cylindrical wall jet. Figure (2) defines the notation used in the analysis.

For carefully aligned cylinders the flow would be symmetrical and hence independent of θ.

The equation of motion in the x-direction with the boundary layer approximations and neglecting the term containing the difference of normal Reynolds stresses is:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y} + \frac{1}{\rho}\frac{\tau}{y} \qquad (1)$$

where $\tau = \mu\frac{\partial U}{\partial y} - \rho\overline{uv}$

The continuity equation is:

$$\frac{\partial U}{\partial x} + \frac{1}{y}\frac{\partial (yV)}{\partial y} = 0 \qquad (2)$$

The boundary conditions are at $y = y_m$, $V = 0$, (assumed), $U = U_m$, and $U = 0$ when y tends to infinity. Assuming self preserving flow with a length scale l_o and a velocity scale U_m which are both functions of x only, the mean velocity U and turbulent shear stress $-\rho\overline{uv}$ are given by

$$U = U_m f(\eta)$$

and $$\overline{uv} = U_m^2 g(\eta) \qquad (3)$$

where f and g are universal functions of

$$\eta = (y-y_m)/(y_{m/2} - y_m) = (y - y_m)/\ell_o$$

Substituting the similarity forms into equations (1) and (2) one obtains

$$\frac{\ell_o}{U_m}\frac{dU_m}{dx}\left[f^2 - \frac{1}{(\eta + \frac{y_m}{\ell_o})}\left\{f'\int_o^\eta f\eta d\eta + f'\int_o^\eta f d\eta\right\}\right]$$

$$+ \frac{d\ell_o}{dx}\left[\frac{2f'}{(\eta + \frac{y_m}{\ell_o})}\int_o^\eta f\eta d\eta + \frac{f'}{(\eta\frac{\ell_o}{y_m} + 1)}\int_o^\eta f d\eta\right]$$

$$+ \frac{dy_m}{dx}\left[\frac{f'}{(\eta\frac{\ell_o}{y_m} + 1)}\left\{\int_o^\eta f d\eta - f + 1\right\} - \frac{\eta f f'}{(\eta + \frac{y_m}{\ell_o})}\right]$$

$$= g' + \frac{1}{(y_m/\ell_o + \eta)} g \qquad (4)$$

where (') represents differentiation with respect to η.

Equation (4) can be made independent of downstream distance, x, by imposing following conditions provided (ℓ_o/d) is not a relevant parameter. A consequence of this restriction is that the rate of growth for a cylindrical wall jet would be the same as that in a plane wall jet. Experimental results to be presented later indicate that for the outer part of the cylindrical wall jet (ℓ_o/d) is unimportant and the rate of growth is nearly the same as that for a plane wall jet.

$$\frac{\ell_o}{U_m}\frac{dU_m}{dx}, \quad \frac{dy_m}{dx}, \quad \frac{d\ell_o}{dx}, \text{ and } \frac{y_m}{\ell_o} \text{ are}$$

independent of x.

Hence $y_m \propto (x + x_o)$

$\ell_o \propto (x + x_o)$

and $U_m \propto (x + x_o)^{-1}$

where x_o is a constant of integration and is identified as a hypothetical origin for the cylindrical wall jet.

From equations (5) it is concluded that the cylindrical wall jet grows linearly with the downstream distance and that the velocity scale, U_m, varies inversely with the downstream distance. The experimental results presented here sustantiate these conclusions.

3. EXPERIMENTAL ARRANGEMENT

The experimental apparatus used to investigate the cylindrical wall jet is shown in Figure (3). It consisted of 2.54 cms. diameter polished stainless steel tube fitted concentrically in a convergent nozzle having contraction ratio of about 5. The exit diameter of the nozzle was 3.81 cms. The annular gap through which air ejected was 6.35mm. The length of the stainless steel tube was approximately 1.8m.

To avoid duplication of the experimental arrangement of Starr and Sparrow[4] in the present investigation the cylinder was kept in the horizontal position. The mounting system for the cylinder was carefully designed to facilitate its alignment.

The air flow was provided by a centrifugal blower driven by a 1 H.P. variable speed electric motor. The blower exhausted in a plenum chamber containing three screens to remove non-uniformity in the flow. The nozzle assembly was attached to the plenum chamber as shown in Figure (3).

All measurements reported here were made with a 0.762mm outside diameter hypodermic stainless steel tubing with internally sharpened lips.

4. RESULTS AND DISCUSSION

It has been reported by Starr and Sparrow[4] that perfect symmetry of the flow field in a cylindrical wall jet was difficult to achieve. Also to avoid sagging of the circular rod they used the rod in the vertical position. In the present investigation a cylinder was placed in the norizontal position and very carefully aligned with the nozzle axis. In the preliminary tests it was observed that the cylinder in the horizontal position was vibrating at certain jet exit velocities. To overcome this difficulty a large weight was attached to the rear stand supporting the cylinder and the jet exit velocity was selected such that the cylinder did not exhibit any vibrations.

The symmetry of the flow field was checked by measuring velocities at two downstream stations, one close to the nozzle exit and another far away from the nozzle exit. Figures (4) shows the non-dimensional mean velocity profiles around the cylinder at $x/d = 36$. From this figure it can be seen that the flow field is roughly symmetrical. The deviations from the symmetry at this station appears to be of the same order as in published results on plane wall jets. The flow field was therefore assumed to be symmetrical and velocity measurements were made at fixed θ at various downstream stations.

The mean velocities measured at various downstream stations are presented in non-dimensional form as shown in Figure (5). For comparison measurements of Starr and Sparrow are also included in this figure. The present measurements at $x/d = 2$ indicate that the flow field at this station is not fully developed. Beyond $x/d = 7$ the non-dimensional mean velocity profiles in the outer part collaspe on a single curve thus providing an experimental support that the outer part of the cylindrical wall jet is self-preserving. The present data is in very good agreement with the mean velocity measurements of Starr and Sparrow. It is, therefore, concluded that the outer part (i.e. $y > y_m$) of the cylindrical wall jet is self-preserving. This then substantiates the assumption of self-preserving flow in the similarity analysis presented in section 2.

The streamwise development of the cylindrical wall jet is given by equation (5). The experimental results are presented in Figures (6) and (7). In Figure (6) both (y_m/d) and $(y_{m/2}/d)$ are plotted against (x/d). The results of Starr and Sparrow are also replotted in this figure. Their results at higher Reynolds number (i.e. $\frac{U_m Y_{m/2}}{\nu} \simeq$ 22450) are in good agreement with the present results. From the figure it can be seen that both y_m and $y_{m/2}$ grow linearly with the downstream distance as predicted by the approximate similarity analysis. The rate of growth for the cylindrical wall jet is found to be 0.070. As mentioned in section (2) note that (ℓ_o/d) is an unimportant parameter for the outer part of the cylindrical wall jet because the rate of growth is nearly the same as that for a plane wall jet and the results for various diameter of cylinder congregate on the straight line. For a plane wall jet the rate a growth varies between 0.0664 and 0.0737 (see collected values by Patel)[5].

Figure (7) shows the variation of the non-dimensional velocity (i.e. $\frac{U_m \text{ at } x/d = 2)}{U_m}$ with the downstream distance $(\frac{x}{d})$. In this figure the velocity scale, U_m, is made non-dimensional with the maximum velocity at $\frac{x}{d} = 2$. From the momentum consideration it can be shown that $U_m^2 l_o^2$ = constant. Since $l_o = (y_{m/2} - y_m)$ the variation of the maximum velocity can be given by U_m = constant/0.07 (x + 1.5) where $\frac{dy_{m/2}}{dx}$ = 0.07 and the hypothetical origin for the cylindrical wall jet was found to be at x/d = 1.5 upstream of the nozzle exit (see Figure (6)). The constant in this relation is evaluated by considering the station x/d = 7 where U_m = 28.45 m/s. Hence the non-dimensional variation of the maximum velocity is given by

$$\frac{(U_m)\ x/d = 2}{U_m} = \frac{(x/d + 1.5)}{5.61}$$

For comparison this relation is plotted in Figure (7). It can be seen that the measured values are in satisfactory agreement with the predicted variation of the velocity scale. It is therefore concluded that the maximum velocity varies inversely as the downstream distance. The results of Starr and Sparrow for higher Reynolds number are also re-plotted in this figure. Their results are in good agreement with the present measurements. It should be noted that in replotting the results of Starr and Sparrow (their Figure 3) in accordance with the present Figure (7) the value of U_{max} at x/d = 2 was obtained by extrapolation. The value was $(\frac{U_m}{U_j})$ x/d = 2 = 0.9. The small variations which are noticeable is, therefore, attributed to the inaccuracies in obtaining results from their Figure 3.

5. CONCLUSIONS

From the present investigation on the cylindrical wall jet in quiescent surroundings following conclusions are drawn:

The simple similarity analysis indicates that the outer part of the cylindrical wall jet grows linearly with the downstream distance and the maximum velocity varies inversely with the downstream distance. These predictions are substantiated very well by experimental results. The rate of growth for the cylindrical wall jets was found to be 0.07 which is nearly the same as that for a plane wall jet . This implies that for the outer part of the cylindrical wall jet (ℓ_o/d) is not a relevant parameter.

The present measurements are in good agreement with those of Starr and Sparrow[4]. In spite of the different experimental set up in the present investigation no apparent discrepancies were observed in the results presented here. It is noted that the gross features of the cylindrical wall jets were independent of the orientation, i.e. vertical or horizontal, of the cylinders as would be expected.

Also from the comparison of the present results with those of Starr and Sparrow it transpired that the outer part of the cylindrical wall jet was substantially independent of the radius of the cylinder.

ACKNOWLEDGEMENTS

The authors would like to thank Mr. G. O. Glasspell, Mr. E. Ndirangu and other members of the Mechanical Engineering Workshop who provided help in the construction of the experimental apparatus. The last two authros take this opportunity to record that the analysis presented here is due to Professor R. P. Patel.

REFERENCES

1. Bakke, P. An experimental investigation of a wall jet. Journal of Fluid Mechanics, Vol. 2, p.467, 1957.

2. Bradshaw, P. Love, E.M. The normal impingement of a circular air jet on a flat surface. Aero Res. Counc. Lond., R & M No. 3205, 1961.

3. Lawrence, R.L. Velocity profiles from compressible wall jets, AIAA Journal. Vol. 2, p.574, 1964.

4. Starr, J. B. Sparrow, E.M. Experiments on a turbulent cylindrical wall jet. Journal of Fluid Mechanics, Vol. 29, part 3, pp. 495-512, 1967.

5. Patel, R.P. A study of two-dimensional symmetric and asymmetric turbulent shear flows.

Ph.D. Thesis, Mechanical Engineering Department, McGill University, July 1970.

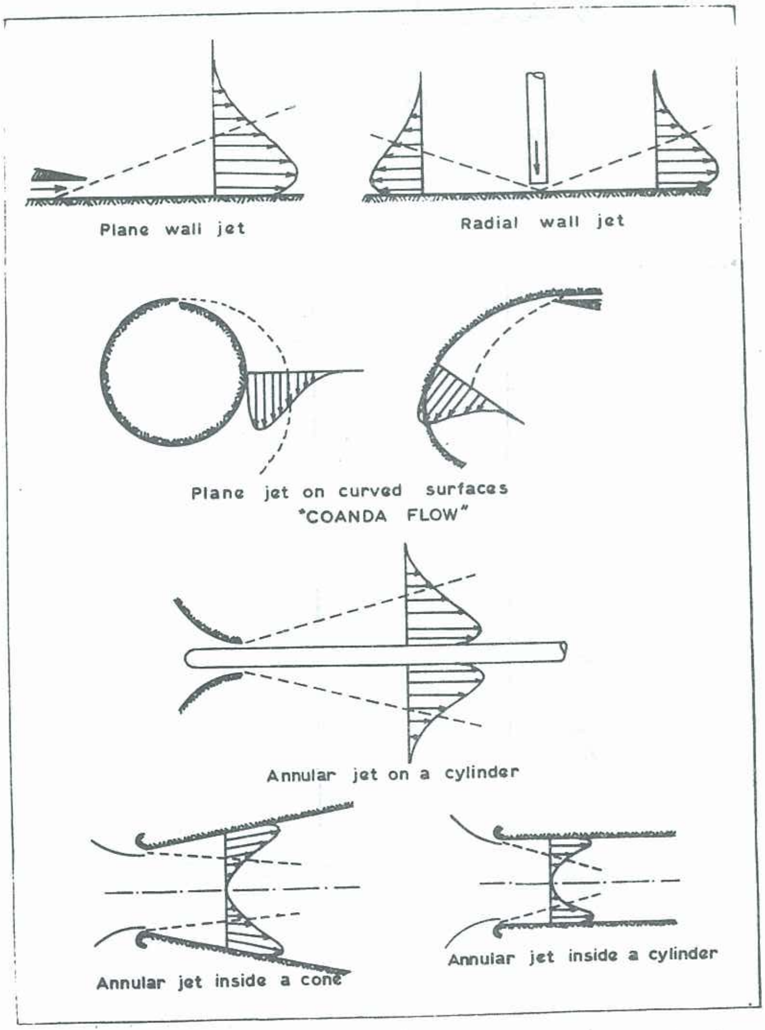

FIGURE 1: Various wall jet flow configurations.

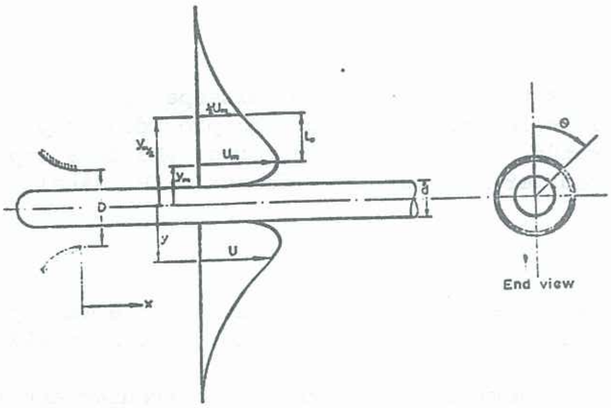

FIGURE 2: Definition sketch of a cylindrical wall jet.

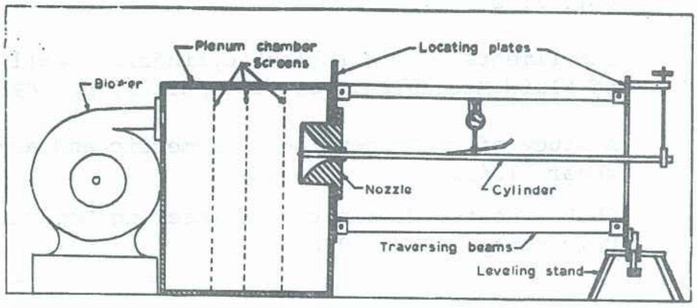

FIGURE 3: General layout of the experimental apparatus.

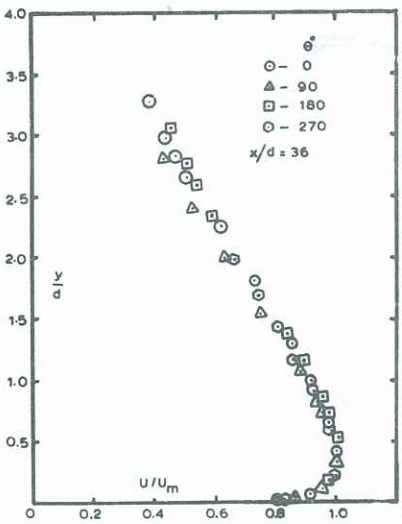

FIGURE 4: Symmetry check at x/d = 36

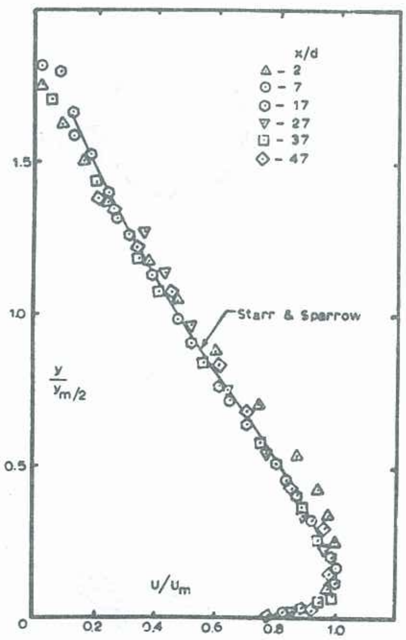

FIGURE 5: Non-dimensional velocity distributions at various downstream distances.

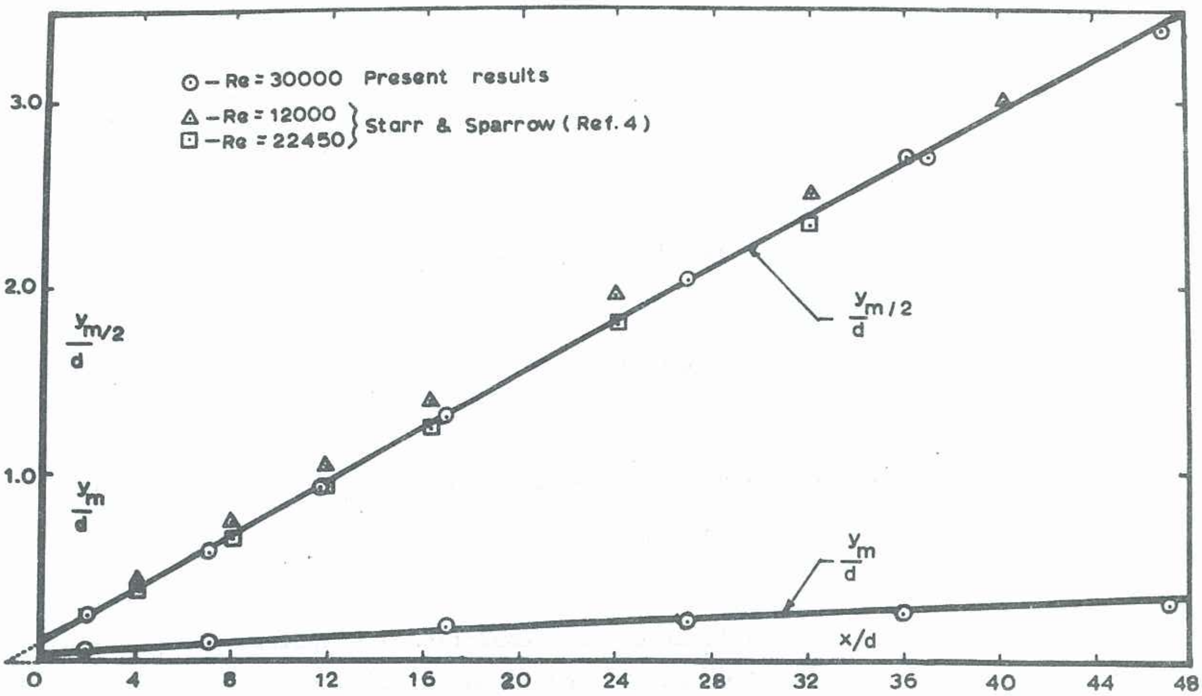

FIGURE 6: Variation of $(y_{m/2}/d)$ and (y_m/d) versus (x/d).

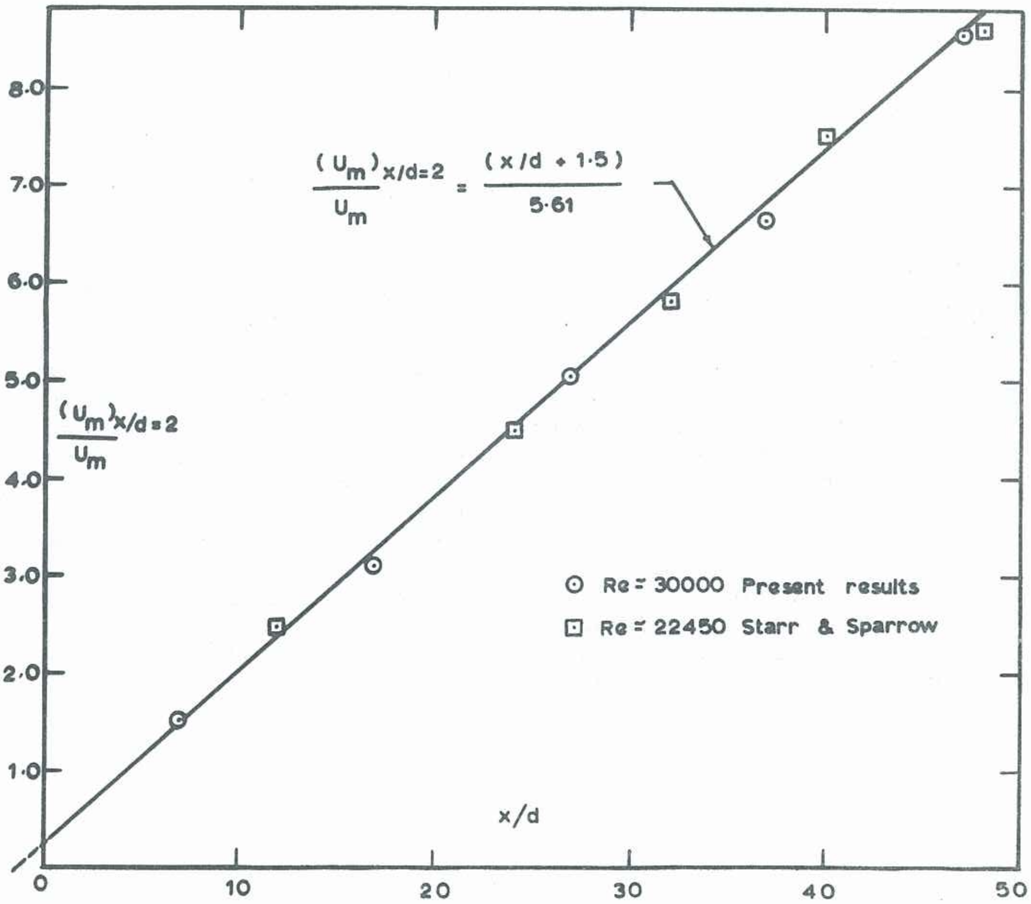

FIGURE 7: Variation of maximum velocity with downstream distance.